Name ________________________ Score __________________

Fall 2008

Math 560 Midterm

Due Tuesday, December 2, 2008

1. Let $A \in \mathbb{R}^{N\times N}$ be a symmetric positive definite matrix. Recall the quadratic form used in Section 1.3, $q(x) = \langle Ax, x\rangle$. Keener states that *if* A *is SPD, then the level surface* $q(x) = 1$ *is an ellipsoid.*

   (a) First show that this is true for the case when A is a diagonal matrix.

   (b) Then show that this is true for any SPD matrix A when the appropriate variable transformation is introduced.

   Note that we have had much discussion about this in class. And we weren't really sure that it was true. As it turns out, the statement is true, but we have to introduce the correct variable transformation to write $q(x)$ in a form that is algebraically recognizable as an ellipsoid. Note that the definition of ellipsoid that I am referring to is the set of points $(y_1, y_2, \ldots, y_N) \in \mathbb{R}^N$ which satisfy

   $$\frac{y_1^2}{\ell_1^2} + \frac{y_2^2}{\ell_2^2} + \cdots + \frac{y_N^2}{\ell_N^2} = 1,$$

   where $\ell_1, \ell_2, \ldots \ell_N \in \mathbb{R}$.

   For the case when A is diagonal, then the ellipsoid is recognizable as having its major/minor axes aligning with the Cartesian coordinate axes. For the case when A is a general SPD matrix, then we have to find the correct coordinate directions that describe our major/minor axes. These are closely related to eigenvectors of A.

   It may help for you to illustrate these concepts with the matrix

   $$A = \begin{bmatrix} 3.5 & -1.5 \\ -1.5 & 3.5 \end{bmatrix}$$

2. On page 53 of Keener textbook: Problem Section 1.4, Number 1.

3. On page 53 of Keener textbook: Problem Section 1.4, Number 3.

4. On page 53 of Keener textbook: Problem Section 1.4, Number 4.

5. On page 94 of Keener textbook: Problem Section 2.1, Number 10.