

- “Fixed” effects: a J level factor is coded as a baseline (in the intercept) and $J - 1$ indicators (adjustments to intercept)
- “Random” effects or Multilevel models: all coefficients have a common distribution (usually gaussian about a common mean, with common variance).

Note: With two levels of random effects, one group of coefficients will need to be centered at 0 so that we don't have two “intercept: columns in the data matrix.

Read footnote p 245 – five definitions of “random” effects.

When to use fixed vs random?

- Fixed if group-level coefficients are of interest, random if we care about the underlying population mean and variance of effects.
- Fixed if all groups are represented, random if we have a sample of groups.
- G&H advice: always use multilevel models (random effects) and separate out groups of coefficients for further modeling ($\alpha_1, \dots, \alpha_J$)

Classical Regression:

- Prediction for continuous or binomial (or Poisson) responses.
- Transformed response fixes nonconstant variance, nonlinear model.
- Categorical predictors as indicator variables.
- Interaction effects can be addressed.
- Causal inference for randomized (or ignorable) treatments.

Multilevel Models:

- Account for extra variation. City-child support example: classical regression could use city-level predictors, but city-to-city variation goes into overall error.
- Model distribution of individual-level coefficients.
- Blend estimates together as in radon in counties.

More levels \Rightarrow more complexity, but hey, that's the way life is.

Have to add more assumptions: random variables in each level must be independent of those in other levels. Equal spread, independence, proper model, normality.

Worth it?

Multilevel models fit more complex data in ways not suited to classical regression models. In limit ($\sigma^2 \rightarrow \infty$ or $\sigma_\alpha^2 \rightarrow 0$), multilevel model and classical often agree.