STAT / M 421 **Homework #9** **Due: Monday, November 9, 2009**

You must justify your answers to receive credit. **Total: 18 points**.

1. (2pt) Assume that the time (in weeks) until failure of an electrical component is a random variable $X \sim \text{PAR}(100, 2)$.
   (a) Find the probability that $X > 20$ weeks.
   (b) Find the probability that $X > 100$ weeks.
   (c) If is observed that the component is still working at 80 weeks, find the conditional probability that $X > 100$ given that $X > 80$.
2. (2pt) Suppose $X \sim \text{PAR}(\theta, \kappa)$.
   (a) Derive $E(X)$ (assuming $\kappa > 1$).
   (b) Derive $E(X^2)$ (assuming $\kappa > 2$).
3. (2.5pt) The strength of a spot weld (in pounds) is a Weibull random variable $X \sim \text{WEI}(400, 2/3)$.
   (a) Find $P(X > 420)$.
   (b) Find the conditional probability $P(X > 420|X > 380)$.
   (c) Find $E(X)$ and $\text{Var}(X)$.
4. (1pt) Darrel likes to play darts. The distance (in centimeters) of Darrel's darts are from the center of the dartboard is a random variable $D \sim \text{WEI}(12, 2)$.
   (a) Find the probability that a dart thrown by Darrel is at least 24 cm from the target center.
   (b) Find the median distance of Darrel's darts from the center of the target.
5. (3pt) Suppose that $X \sim N(4, .25)$
   (a) Find $P(X > 4)$.
   (b) Find $P(X < 4.2)$.
   (c) Find $P(X < 3.6)$.
   (d) Find $P(4.1 < X \leq 4.3)$.
   (e) Find a such that $P(X > a) = .9370$.
   (f) Find b such that $P(4 - b < X < 4 + b) = .90$.
6. (1.5pt) The Rockwell hardness of a metal specimen is determined by impressing the surface of the specimen with a hardened point and then measuring the depth of the penetration. The hardness of a certain metal alloy is normally distributed with a mean of 70 Rockwell units and a standard deviation of 3 units.
   (a) Suppose a metal alloy specimen is acceptable if its hardness is between 64 and 76 units. What is the probability that a randomly chosen specimen is acceptable?
   (b) Suppose a metal alloy specimen is acceptable if its hardness is between $70 \pm c$ units. What is the value of c if 95% of all specimens are acceptable?
7. (2pt) Let $f(x) = \dfrac{1}{\pi} \dfrac{1}{1 + x^2}$ for $-\infty < x < \infty$.
   (a) Verify that $f(x)$ is a pdf.
   (b) Suppose random variable X has this pdf. Does $E(X)$ exist? If yes, then what is the value? If not, then justify why not.
8. (2.5pt) Exercise #52, page 133.
9. (1.5pt) Exercise #58 (b), page 134. Hint: you need to use the results from part (a). You do not have to prove (a).