1. Integrate.

(a) $\int (x+3)\sin 2x\,dx$

(b) $\int \sin x \cos^7 x\,dx$

(c) $\int \sqrt{9-x^2}\,dx$

(d) $\int \frac{7x+6}{(2x+1)(x+3)}\,dx$

(e) $\int x^2 e^{-4x}\,dx$

(f) $\int \sec^4 4\theta \tan^4 4\theta\,d\theta$

(g) $\int \frac{dt}{t^4+2t^2+1}$

(h) $\int \frac{4x^2+3x+2}{x^3+x^2}\,dx$

(i) $\int_1^e t\ln t\,dt$

(j) $\int \sin^2 \pi x \cos^2 \pi x\,dx$

(k) $\int \frac{\sqrt{4x^2-9}}{x}\,dx$

(l) $\int \frac{5x+6}{(x-2)(x^2+4)}\,dx$

(m) $\int (x+\sin x)^2\,dx$

(n) $\int \frac{x^2}{x^2+4x+13}\,dx$

(o) $\int \sin 2\theta e^{\sin\theta}\,d\theta$

(p) $\int \sec^2 x \sec^3(\tan x)\tan^3(\tan x)\,dx$

(q) $\int \frac{e^{2x}}{e^{2x}+5e^x+6}\,dx$

(r) $\int_0^{\pi/2} \cos x\sqrt{1+3\sin^2 x}\,dx$

2. Eliminate the parameter to express each of the parametric curves in the form $y = f(x)$.

(a) $x(t) = t+1,\ y(t) = t^3+1$

(b) $c(t) = \left(e^{2t}+1, e^{6t}+1\right)$

(c) $x(t) = \frac{2+t}{1+t},\ y(t) = \frac{1}{(1+t)^3}+1$

(d) $c(t) = (5\cos t, 5\sin t)$ for $t \in [\pi, 2\pi]$

3. Find the length of the following parametric curves over the given interval.

(a) $x(t) = 4t+1,\ y(t) = 3t-1,\ 0 \le t \le 2$

(b) $x(t) = 3t^2+3,\ y(t) = t^3-2,\ 0 \le t \le \sqrt{5}$

(c) $c(t) = \left(e^{2t}\cos t, e^{2t}\sin t\right),\ t \in [0,\pi]$

(d) $c(t) = (t-\sin t, 1-\cos t),\ t \in [0, 2\pi]$

4. Find the slope, $\frac{dy}{dx}$, and the speed, $\frac{ds}{dt}$, of a particle traveling along each curve.

(a) $x(t) = 2\sin t,\ y(t) = t,\quad t = \pi/6$

(b) $x(t) = \frac{t+2}{4},\ y(t) = \frac{t-3}{3},\quad t = \frac{-\pi^3}{e^{37}}$

(c) $c(t) = (2\cos 2t, \sin 2t),\quad t = 3\pi/8$

(d) $c(t) = \left(\frac{t}{t+1}, t^2\right),\quad t = 1$

5. Convert the following polar equations into rectangular coordinates.

(a) $r = 2$

(b) $r = \frac{3}{\cos\theta}$

(c) $r = 2\csc\theta$

(d) $r = \frac{6}{2\cos\theta - 3\sin\theta}$

(e) $r = -2\sin\theta$

(f) $\theta = \frac{\pi}{4}$

6. Sketch the following polar curves.

(a) $r = 3$

(b) $r = 2\cos\theta$

(c) $r = \sin 2\theta$

(d) $r = 1 - 2\sin\theta$

7. Find the length of the following polar curves.

(a) $r = 3$

(b) $r = \theta,\quad \theta \in [0,\pi]$

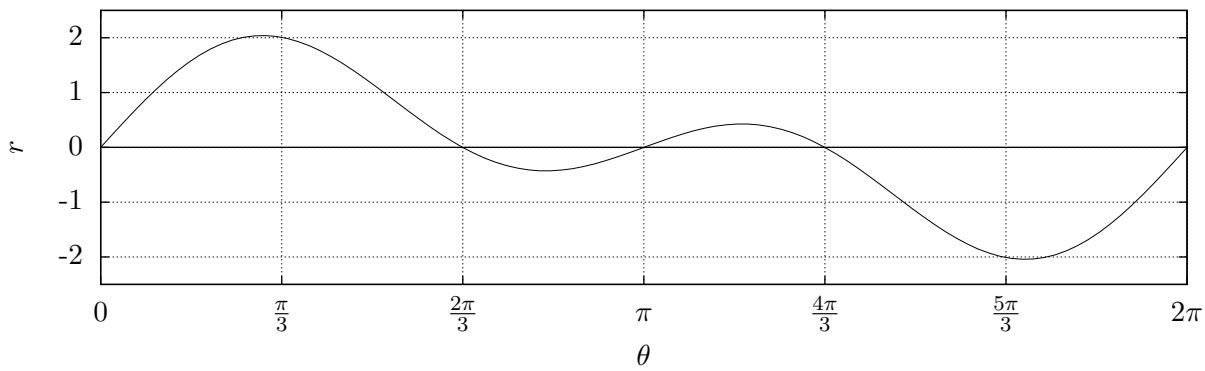
(c) $r = \theta^2,\quad \theta \in [0,\pi]$

(d) $r = \theta^2 - 1,\quad 0 \le \theta \le \pi$

(e) $r = e^{2\theta},\quad 0 \le \theta \le \pi$

(f) $r = \cos^2\theta$

8. The figure below shows a graph of r as a function of θ in Cartesian coordinates. Use it to sketch the corresponding polar curve.



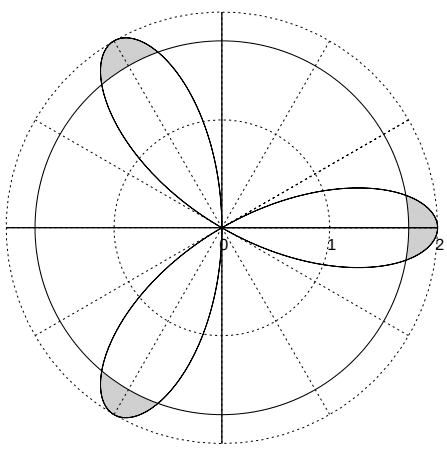
9. Sketch the polar region described and then find the area.

   (a) Inside $r = 2 + \cos\theta$.

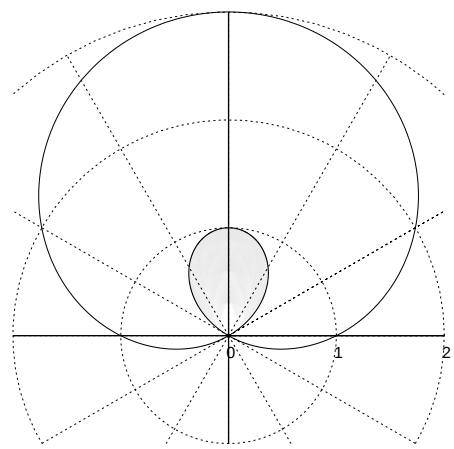
   (b) The line $r = 3\sec\theta$ cuts the area inside $r = 4\cos\theta$ into two pieces. Find the area of the smaller of the two.

10. Find the polar areas described below and shaded in the figures.

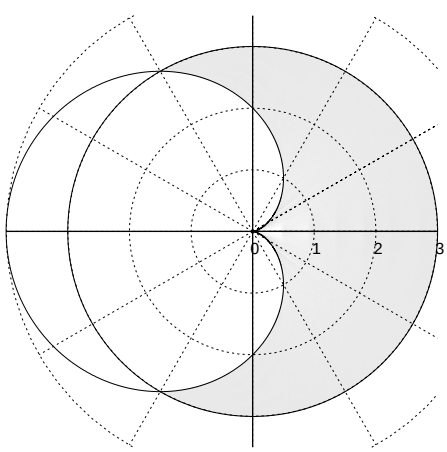
   (a) Inside $r = 2\cos 3\theta$ and outside $r = \sqrt{3}$.



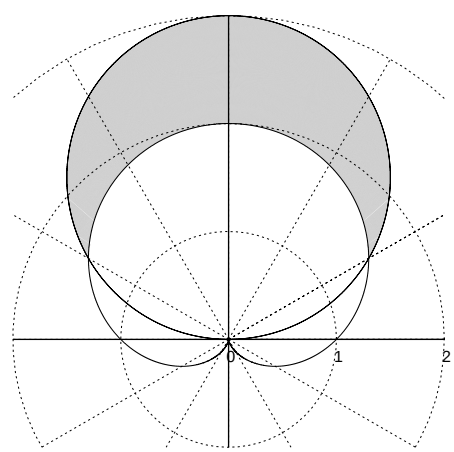
   (c) Inside the inner loop of $r = 1 + 2\sin\theta$.



   (b) Inside $r = 3$ and outside $r = 2 - 2\cos\theta$.



   (d) Inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$.



Practice Final A

1. Integrate.

   (a) $\displaystyle\int \sin^3 2x \cos^2 2x \, dx$

   (b) $\displaystyle\int_1^2 x^2 \ln 3x \, dx$

   (c) $\displaystyle\int \frac{4x}{x^2-1} dx$

   (d) $\displaystyle\int \frac{dt}{t^2\sqrt{4-t^2}}$

2. Express $\left(\frac{1}{\sqrt{t+1}}, \frac{t}{t+1}\right)$ in the form $y = f(x)$.

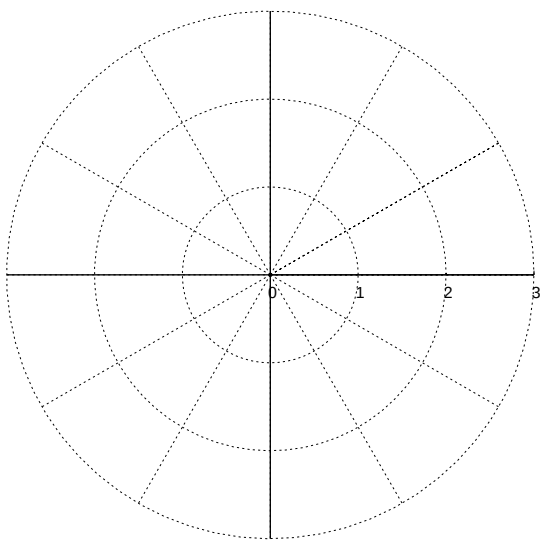
3. Find the length of the path $(3t^2, t^3+1)$ over $0 \le t \le \sqrt{5}$.

4. Find the speed of a particle traveling along the curve $(\tan t, 2t)$ at $t = \pi/4$.

5. Find the direction, $\frac{dy}{dx}$, of a particle traveling along the curve $(\tan t, 2t)$ at $t = \pi/4$.

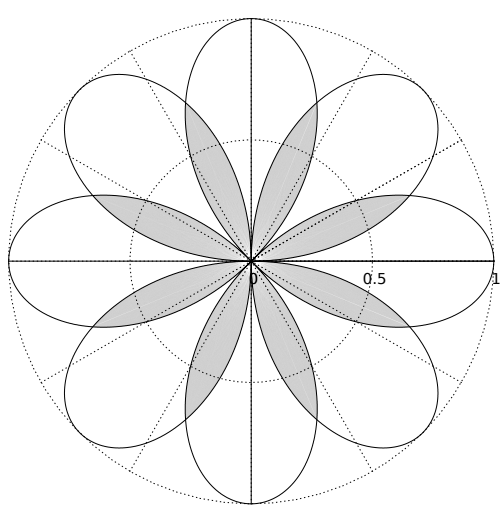
6. Convert to an equation in rectangular coordinates. $r = 2\sin\theta$

7. Carefully sketch the curve $r = 1 + 2\sin\theta$ on the provided polar grid.



8. Find the length of the path $r = \theta$ for $0 \le \theta \le \sqrt{3}$.

9. Find the area inside both $r = \sin 2\theta$ and $r = \cos 2\theta$. See the figure.



Practice Final A+

1. Integrate.

   (a) $\displaystyle\int x^2 e^{2x}\,dx$

   (b) $\displaystyle\int_0^1 \frac{dx}{(1+x^2)^2}$

   (c) $\displaystyle\int \tan^2\theta\sec^4\theta\,d\theta$

   (d) $\displaystyle\int \frac{2x}{(x+1)(x^2+1)}dx$

2. Express $\left(t+1, t^2-3\right)$ in the form $y = f(x)$.

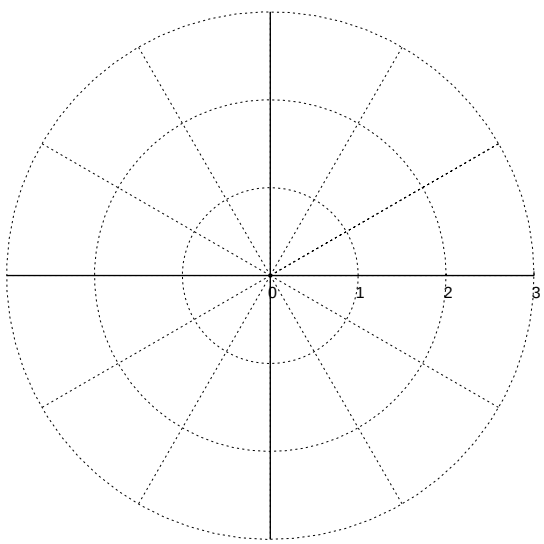
3. Find the length of the path $(2\arcsin t, \ln(1-t^2))$ over $0 \le t \le \frac{1}{2}$.

4. Find the speed of a particle traveling along the curve $(3t, e^{2t})$ at $t = \ln\sqrt{2}$.

5. Find the direction, $\frac{dy}{dx}$, of a particle traveling along the curve $(3t, e^{2t})$ at $t = \ln\sqrt{2}$.

6. Convert to an equation in rectangular coordinates. $r = 2\sec\theta$

7. Carefully sketch the curve $r = 3\sin 3\theta$ on the provided polar grid.



8. Find the length of the path $r = e^\theta$ for $-\infty < \theta \le 0$.

9. Find the area between the inner and outer loops of the limacon $r = 1 - 2\sin\theta$. See the figure.