EXAM 1 REVIEW

Math 441

1. Effects of finite precision on calculations: machine epsilon, underflow, overflow
2. Orthogonal and orthonormal vectors and matrices
3. Understand the relationships between the following:
   (a) Linear dependence/independence of vectors (for example columns/rows of a matrix)
   (b) invertible/non-singular and singular matrices
   (c) null space and range space of a matrix
   (d) determinant of a matrix
   (e) solutions of linear systems: 0, 1, or ∞
4. eigenvalues and eigenvectors
5. singular values and singular vectors
6. When will Gauss Elimination work without requiring row permutations?
7. Perform Gauss Elimination to solve a linear system. Know when it is preferable to keep the LU decomposition (or LUP decomposition) to solve a linear system.
8. When will Cholesky factorization work? Know when to use Cholesky instead of Gauss Elimination.
9. Perform Cholesky factorization to solve a linear system.
10. A positive definite matrix A can be defined in any one of the following ways: A has only positive eigenvalues; $x^T Ax > 0$ for any x; or $A = R^T R$ for some matrix R.
11. Be familiar with the proofs we have done in class re: positive definite matrices. There will be a single proof on the exam, and it will be similar to the ones you have done already.
12. Principle Components analysis: Given data (of many variables for many individuals), the covariance matrix for the data, and the eigenvectors and eigenvalues, what is the direction of most variability in the data? What is the direction of least variability? How do you construct a new variable for each individual using an eigenvector?
13. Vandermonde matrices and polynomial interpolation.
14. How are sparse matrices stored in computer memory? Why is this important?
15. Banded matrices and envelopes, and why are they important?
16. Compare the computational cost of vector dot products, matrix-vector multiplies, forward/backward substitution, matrix-matrix multiplies, matrix inversion, Gauss Elimination/LU decomposition, Cholesky factorization.
17. Well conditioned versus ill conditioned problems.
18. condition number of a matrix
19. vector norms: $||x||_p$
20. matrix norms: spectral $||A||_2$; column sum $||A||_1$; row sum $||A||_\infty$; Frobenius $||A||_F$.
21. absolute error and relative error