PROJECT 6

Math 441

Due: Thursday, October 30

All "by hand" calculations require that you show your work. All MATLAB code and output must be turned in.

1. Do exercise 2.1.16.
2. Do exercise 2.1.28.
3. Do exercise 2.2.10, only for the case $n = 10$, i.e. let **A=hilb(10)**.
   (a) Give the condition numbers $\kappa_1(A)$, $\kappa_2(A)$, $\kappa_\infty(A)$.
   (b) Is solving the problem $Ax = b$ (where b is non-zero) well conditioned or ill conditioned? Explain.
   (c) Let **b=randn(10,1)**, and use whatever method you'd like to give you an approximate solution $\hat{x} = x + \delta x$ to $Ax = b$.
   (d) Calculate the residual $\hat{r}$, which we have shown is equal to $-\delta b$. How big can the relative error $\frac{||\delta x||_2}{||x||_2}$ get?
   (e) Do you have confidence $\hat{x}$, your solution to $Ax = b$? Explain.
4. Do exercise 2.7.26.
   (a) Instead of "showing" using a mathematical proof, use MATLAB's **lu( )** function for several values of n to convince yourself that $||U||_\infty = 2^{n-1}$.
   (b) Explain why $||A_n||_\infty = n$.
   (c) Suppose that you are interested in solving $Ax = b$ where b is non-zero. Is Gauss Elimination (or LU decomposition) backward stable with respect to this problem? *Hint*: Theorem 2.7.14 (which we covered in class).
5. Do exercise 3.3.7, part (a) only. In addition to guessing the least squares solution (we talked about least squares solutions to problems of the form $y = a$ in class), actually compute the least squares solution, by hand, using the "classic method."
6. Do exercise 3.4.10. Give the explicit form of these vectors!
7. Do exercise 4.1.11.
8. Do exercise 4.4.16, parts (a)-(c) only.
   (a) Use MATLAB's **qr( )** function to find the QR factorization and then find the least squares solution using this factorization. Calculate the norm of the residual. Don't forget to get $\kappa_2(A)$.
   (b) Solve the normal equations using Cholesky's factorization. Calculate the norm of the residual. Give the condition number $\kappa_2(A^TA)$. How does it compare to $\kappa_2(A)$?
   (c) Do the plotting. Is one solution better than the other? Do your answers to (a) and (b) explain this?
   (d) Also, find the least squares solution using the singular value decomposition of A, and add this solution to the plot from (c). Is this solution better than either of the other two?