Project 8 Solutions

Statistics 401: Fall 2016, 30 pts

1. (6 pts) Regarding the numbers of elk and deer harvested in 2015 and 2016:

   (a) Since the measurements can be paired for each hunting check station, define the variable d =(count for 2015) - (count for 2016) as in Table.

   Table 1: Numbers of elk and deer counted at 6 hunting check stations in 2015 and 2016.

Station	counts in 2016	counts in 2015	difference (d)
1	54	68	14
2	75	100	25
3	53	71	18
4	56	74	18
5	74	100	26
6	71	83	12

   Thus, $\mu_d = \mu_{2015} - \mu_{2016}$. A one sample t-test on these differences is a matched pairs t-test:

   i. **Hypotheses:**
   $H_0 : \mu_{2015} = \mu_{2016}$
   $H_a : \mu_{2015} > \mu_{2016}$

   which is equivalent to testing
   $H_0 : \mu_d = 0$
   $H_a : \mu_d > 0.$

   ii. **Assumptions:** Since we have paired samples of size $n = 6$, a **very small sample size**, then we must assume that the differences d between the counts in 2015 and the counts in 2016 are normal. To generalize conclusions to all stations in Montana, we need to assume that the 6 stations are a **SRS** of all stations, which could occur if FWP listened to statistician when choosing these check stations. Because we have data from 6 check stations, the total number of check stations in Montana (i.e., the **population**) must be larger than $20 \times n = 120$ in order for the following statistical analyses to be valid. It is highly unlikely that there are this many check stations in our state (I looked for this on-line to no avail). So this assumption appears to be violated.

   iii. **Test statistic:** The mean of the differences is $\bar{d} = 18.8\bar{3}$ and $s_d = 5.6716$. Thus, the test statistic is $t = \frac{18.8-0}{\frac{5.6716}{\sqrt{6}}} \approx 8.1339$.

   iv. **p-value:** p-value $= P(Z > 8.1339) \approx 0$.

   v. **Decision:** Since the p-value is tiny, we reject H_0 in favor of H_a.

   vi. **Conclusion:** The preponderance of evidence suggests that the mean count of elk and deer was larger in 2015 than the mean counts in 2016.

   (b) See the Appendix for R-code and R-output.

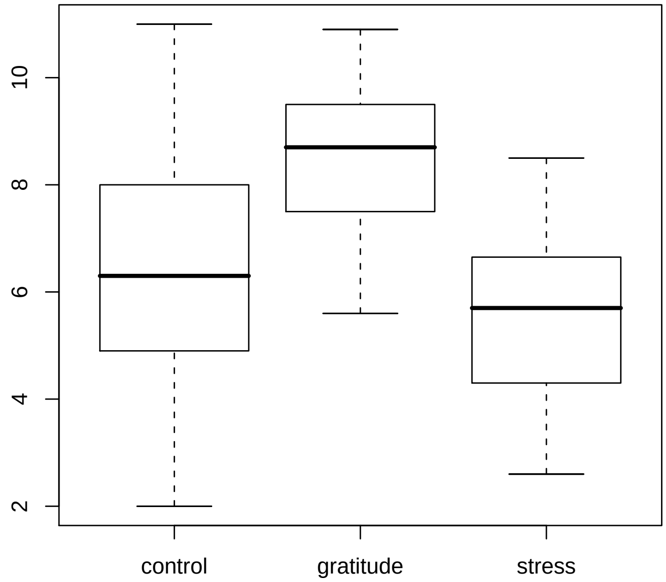
2. (19 pts) Regarding *In Praise of Gratitude* reported by Harvard Medical School in November of 2011:

   (a) To have been a CRD, the individuals must have been randomly assigned to the treatment groups.

   (b) The variable $x_{2,13} = 6.4$ hours, which means that the thirteenth subject in the group which kept the "stress journals" slept an average of 6.4 hours each night.

   (c) Side-by-side box-plots of the amount of sleep for each treatment group are given in Figure 1.

Figure 1: Mean amount of sleep (in hours) per treatment group



(d) <u>Hypotheses:</u>
Let μ_g be the mean amount of sleep for subjects who keep a “gratitude journal;” μ_s be the mean amount of sleep for subjects who keep a “stress journal;” μ_c be the mean amount of sleep for subjects in the control group who keep a “regular journal;”

$H_0: \mu_g = \mu_s = \mu_c$

$H_a: \mu_i \neq \mu_j$ for some i and j

(e) Fitting the ANOVA model yields the one-way ANOVA table in Table 2. The relevant R code and output are in the Appendix.

Table 2: One-way ANOVA Table for Sales by Design

Source	DF	Sum of Squares (SS)	Mean Squares (MS)	F	p-value
Treatments	2	96.756	48.378	13.265	1.401×10^{-5}
Error	67	244.347	3.647		
Total	69	341.103			

(f) The Chapter 5.5 notes make clear how to calculate DFG and DFE by hand:

- The degrees of freedom for the groups is $DFG = k - 1 = 3 - 1 = 2$ (k is the number of groups in this study).
- The degrees of freedom for error is $DFE = \sum n_i - k = (25 + 21 + 24) - 3 = 70 - 3 = 67$.

(g) The Chapter 5.5 notes also make clear how to calculate MSG and MSE by hand:

- The estimate for the variability among the group means is

$$MSG = \frac{\sum n_i(\bar{x}_i - \bar{\bar{x}})^2}{k-1} = \frac{SSG}{DFG} = \frac{96.756}{2} = 48.378.$$

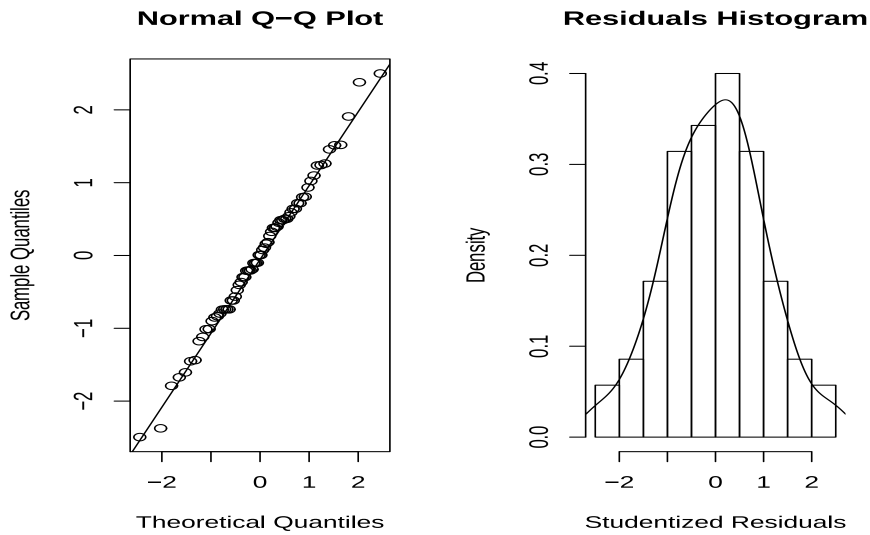
- The estimate for the variability within the groups is

$$MSE = \frac{\sum(n_i - 1)s_i^2}{\sum(n_i - 1)} = \frac{SSE}{DFE} = \frac{244.347}{67} = 3.647.$$

(h) Check the assumptions.

i. **Sample size and normality:** Because we have less than 30 data in each treatment group, then we will check for normality of the data. The evidence fails to suggest that the data for each group are not normal. The normal points in the normal probability plot have a linear pattern, and the smoothed histogram of the studentized residuals appears normal (see Figure 2). The correlation of the studentized residuals is .997, which is much larger than the critical correlation value of $r_{\text{critical}} = .976$.

Figure 2: Checking the normality assumption



ii. We know that the students who participated in this study were not a **SRS**, but rather a convenience sample of students in the researchers classes. However, the 3 treatment **groups are independent** of each other because the students were assigned to groups via a CRD. And lastly, regarding a large enough **population size**, we will assume that $0.05N > n$ because there are lots and lots of humans.

iii. Since $\frac{\text{largest s}}{\text{smallest s}} \approx \frac{2.5}{1.46} \approx 1.71 < 2$, the **constant variance** assumption appears to hold.

(i) The distribution of the test statistic assuming that H_0 is true is $F \sim F(2, 67)$.

(j) Since the p-value=1.4×10^{-5} is tiny and less than $\alpha = 0.05$, then we REJECT H_0.

(k) If this was a completely randomized experiment (CRD), then we can make cause-and-effect conclusions. And if the individuals were not from a SRS, then we can not make inferences to the entire human population. Thus, the evidence suggests that, among the subjects in the study, keeping a certain type of journal caused some of the subjects to have a longer night's sleep on average than some of the others.

3. (5 pts) Tukey's follow-up tests:

(a) It is appropriate to conduct a follow-up test since the ANOVA null hypothesis was rejected.

(b) Table 3 displays the 95% Tukey confidence intervals for the pairwise differences between means.

Table 3: 95% Tukey CI's

Comparison	Estimate	Lower	Upper
$\mu_g - \mu_c$	2.0611	0.7062	3.4161
$\mu_s - \mu_c$	-0.8085	-2.1166	0.4996
$\mu_s - \mu_g$	-2.8696	-4.2374	-1.5019

(c) From the Tukey's CI's, keeping a daily gratitude journal (and so focusing on being grateful) caused these subjects to have a longer night's sleep on average than either the subjects focusing on stress, or the subjects focusing on nothing in particular. There is no significant difference in mean amount of sleep between those focusing on stress and those focusing on nothing in particular.

(d) Table 4 gives the table of parameters and estimates:

Table 4: Estimate of some ANOVA Parameters

parameter	estimate	Explanation in English words of the parameter
Estimate for μ_3	6.496	The mean amount of sleep of people who do not focus on either gratitude or stress
Estimate of σ	$\sqrt{(3.647)} \approx 1.91$	The constant standard deviation for each group.

Appendix

```
> # PROBLEM 1 - elk and deer
> counts_2015=c(68,100,71,74,100,83)
> counts_2016=c(54,75,53,56,74,71)
> diffYears=counts_2015-counts_2016
> rbind(counts_2015,counts_2016,diffYears)
            [,1] [,2] [,3] [,4] [,5] [,6]
counts_2015   68  100   71   74  100   83
counts_2016   54   75   53   56   74   71
diffYears     14   25   18   18   26   12

> # Test statistic
> mean(diffYears)
[1] 18.83333
> sd(diffYears)
[1] 5.671567
> 18.83333/(5.671567/sqrt(6))
[1] 8.133916

> # p-value
> 1-pt(8.133916,5)
[1] 0.0002279798

> t.test(counts_2015,counts_2016,conf.level=0.95,alternative="greater",paired=T)

        Paired t-test

data:  counts_2015 and counts_2016
t = 8.1339, df = 5, p-value = 0.000228
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 14.16768      Inf
sample estimates:
mean of the differences
               18.83333
```

```
> # PROBLEM 2 - gratitude
> grat=read.table("http://www.math.montana.edu/parker/courses/STAT401/data/gratitude.txt",
      header=TRUE)
> attach(grat)

# Figure 1
> boxplot(sleep ~ group)


# Calculate the ANOVA table by hand
# Check means
> meani=tapply(sleep,group,mean)
> meani
  control gratitude    stress
 6.496000  8.557143  5.687500

# Check sample sizes
> ni = tapply(grat$sleep,grat$group,length)
> ni
  control gratitude    stress
       25        21        24

# Total number of students in this study
> sum(ni)
> ni
[1] 70

# Grand mean
> gmean=mean(grat$sleep)
> gmean
[1] 6.837143

> sum(ni*(mi-gmean)^2)    # SSG
[1] 96.75615

# Get the variance in each group
> vari = tapply(grat$sleep,grat$group,var)
> vari
  control gratitude    stress
 6.254567  2.141571  2.235054

> sum((ni-1)*vari)        # SSE
[1] 244.3473

> sum(ni*(mi-gmean)^2)/2     # MSG
[1] 48.37807
```

```
> sum((ni-1)*vari)/67        # MSE
[1] 3.646974


# Have R generate the ANOVA table:
> m=aov(sleep ~ group)
> summary(m)
            Df  Sum Sq Mean Sq F value    Pr(>F)
group        2  96.756  48.378  13.265 1.401e-05 ***
Residuals   67 244.347   3.647
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


# Check normaility of the 3 groups (i.e., of the residuals)
> par(mfrow=c(1,2))
> library(MASS)
> qqnorm(studres(m))
> qqline(studres(m))
> hist(studres(m),freq=FALSE,main="Residuals Histogram",xlab="Studentized Residuals")
> lines(density(studres(m)))
> xy=qqnorm(studres(m))
> cor(xy$x,xy$y)
[1] 0.9969766


# Check constant variance
> tapply(sleep,group,sd)
  control gratitude    stress
 2.500913  1.463411  1.495010



> # PROBLEM 3
> TukeyHSD(m)
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = sleep ~ group)

$group
                       diff        lwr        upr     p adj
gratitude-control  2.061143  0.7062274  3.4160583 0.0014940
stress-control    -0.808500 -2.1165831  0.4995831 0.3062210
stress-gratitude  -2.869643 -4.2373840 -1.5019017 0.0000116
```