

## Exam 2

Statistics 401

November 3, 2006

Name \_\_\_\_\_

**TRUE/FALSE: (3 pts each) For each of the following, circle T or F. You need not justify your answer.**

1. **T / F** The following is a valid pair of statistical hypotheses:  $H_0: \pi > 0.6$  versus  $H_a: \pi = 0.7$ .
2. **T / F** The following is a valid pair of statistical hypotheses:  $H_0: \bar{X} = 4$  versus  $H_a: \bar{X} > 4$ .
3. **T / F** The larger the  $p$ -value, the stronger the evidence that  $H_0$  is true.
4. **T / F** Rejecting  $H_0$  implies  $H_a$  is true.
5. **T / F** A Type II Error is made if a false null hypothesis is rejected.
6. **T / F** The standard deviation of the population distribution decreases as the sample size  $n$  increases.
7. **T / F** The probability of a Type I Error decreases as the sample size increases.
8. **T / F**  $\bar{X}_1 - \bar{X}_2$  is an unbiased estimator of the population mean  $\mu_1 - \mu_2$ .
9. **T / F** A  $t$  distribution has skinnier tails than the standard normal distribution.
10. **T / F** The Power of a test increases as  $\alpha$  increases.

**MULTIPLE CHOICE: (3 pts each) Circle the single best answer.**

11. When concerned about normality of the sampling distribution of  $\bar{X}$ , one should consider transforming a sample if
  - A. The sample size is large and the data comes from an approximately normal distribution.
  - B. The sample size is large and the data does not come from a normal distribution.
  - C. The population variance is unknown and you must estimate it.
  - D. The sample size is small and the data comes from an approximately normal distribution.
  - E. The sample size is small and the data does not come from a normal distribution.
12. If a 95% confidence interval for  $\mu$  is (452, 470), then the probability is \_\_\_\_\_ that the true mean  $\mu$  falls in the interval (452, 470).
  - A. 0
  - B. 0.05
  - C. 0.95
  - D. 1
  - E. Either A. or D., but cannot determine which one is true.

13. Consider statements I, II, III, and IV. Which pair of statements is **TRUE**? Given the sample standard deviation  $s$  remains constant, the confidence interval for  $\mu$  will become
- |                 |   |
|-----------------|---|
| A. I. and III.  | I. Wider as the sample size increases.          |
| B. II. and IV.  | II. Narrower as the sample size increases.      |
| C. I. and IV.   | III. Wider as the confidence level increases.   |
| D. II. and III. | IV. Narrower as the confidence level increases. |
14. If the population distribution for some variable  $X$  is  $N(\mu, \sigma)$ , then the sample mean  $\bar{X}$  of from a small simple random sample is distributed as \_\_\_\_\_. Assume the population size is *small* relative to the sample size.
- A.  $N(\mu, \sigma)$   
 B.  $N(\mu, \frac{\sigma}{\sqrt{n}})$   
 C.  $N(0, 1)$   
 D.  $\mu_{\bar{x}} = \mu$  and  $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ , but the sampling distribution of  $\bar{X}$  is unknown.
15. For a random sample of 9 women, the average resting pulse rate is  $\bar{x} = 76$  beats per minute and the sample standard deviation is  $s = 5$  beats per minute. The *standard error* of the sample mean is
- A. 0.556  
 B. 0.745  
 C. 1.667  
 D. 3.333

**SHORT ANSWER AND COMPUTATION: Show all work to receive full credit! Write legibly.**

16. (4 pts) For a random sample of 9 women, the average resting pulse rate is  $\bar{x} = 76$  beats per minute and the sample standard deviation is  $s = 5$  beats per minute. A 95% confidence interval for  $\mu$  is (72.6, 79.4). Interpret this CI in **terms of the problem**.
17. (4 pts) State the Central Limit Theorem.
18. (4 pts) A real estate agent would like to estimate the proportion of offices in a large city that are currently occupied. How large of a sample is required to ensure that the estimate will be accurate to within 0.1 with 99% confidence?

19. (4 pts) A study released in September of 2006 found that 70% of the people who worked at ground zero in New York City after the terrorist attacks on September 11, 2001 suffer severe respiratory problems. Suppose that  $\pi = 0.7$ . If a random sample of  $n = 40$  workers at ground zero are selected, give the sampling distribution of  $p$ , the sample proportion out of these 40 who have severe respiratory problems. Be sure to give the mean and standard deviation of  $p$ .
20. Suppose that a population of  $N = 3$  female wolves have each had a litter of pups of size  $X = 2, 3$  and 6 pups respectively. An investigator plans on taking a random sample of  $n = 2$  wolves (without replacement) from these three and then computing the largest value in the sample. Denote the largest value of the sample by  $M$ . For example, if the sample of wolves has  $X = 2, 3$  then the largest value in this sample is  $M = 3$ .
- (a) (4 pts) Construct the sampling distribution of  $M$ , sampling without replacement. There are 3 different samples of size 2!
- (b) (3 pts) Calculate the mean of  $M$ ,  $\mu_M$ .
- (c) (3 pts) Is  $M$  unbiased for the largest value in the population? Justify your answer.

21. An industrial plant claims to discharge no more than 1000 gallons of waste water per hour, on the average, into a neighboring lake. The Environmental Protection Agency decides to monitor the plant, in case this limit is being exceeded. Based on a random sample of size  $n = 44$  they find  $\bar{x} = 1021$  gallons and  $s = 200$  gallons.

(a) (2 pts) What type of study is this? Circle one of the following:

Observational Study	Experiment
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(b) (4 pts) State the null and alternative hypotheses.

(c) (4pts) What assumptions do you need to check before conducting a hypothesis test? Are these assumptions satisfied?

(d) Conduct the hypothesis test.

i. (3 pts) Calculate the value of the test statistic.

ii. (2 pts) Give the distribution of the test statistic assuming that the null hypothesis is true.

iii. (2 pts) Give the  $p$ -value.

iv. (2 pts) Make a decision. Use a significance level of  $\alpha = .05$ .

v. (4 pts) Make a conclusion in terms of the problem.

(e) (4 pts) Explain what a Type I Error is in terms of this problem.

(f) (2 pts) For this problem, what is the probability that a Type I Error occurred?