Project 7 - Solutions

Statistics 401: Fall 2006

1. (2 pts) Problem 10.4 on page 409:

   (a) To investigate the question "Do state laws that allow private citizens to carry concealed weapons result in a reduced crime rate?" the hypotheses to be tested are

   H_0 : concealed weapons laws do not reduce crime

   H_a: concealed weapons laws reduce crime

   This is the correct set of hypotheses of the two listed since the null hypothesis is always a statement of no difference or no effect.

   (b) The stated conclusion, "The strongest thing I could say is that I don't see any strong evidence that they are reducing crime," indicates that the null hypothesis was not rejected.

2. ((1 pt) Problem 10.10 on page 410) The hypotheses of interest to the circuit breaker manufacturer are

   H_0 : the mean amperage of the circuit breakers is equal to 40-amps

   H_a: the mean amperage of the circuit breakers is not equal to 40-amps

3. (2 pts) Problem 10.16 on page 415:

   (a) To make the conclusion less confusing, let's say that if a TV does not need service during the first three years of operation, then it is a "quality TV."

   A Type I error is finding that less than 90% of the TV sets are quality TV's when in fact this IS NOT true. Thus, the "consumer agency" would incorrectly accuse the television manufacturer of making false claims, causing an unnecessary court battle or a high dollar pay-off to the head of the consumer agency.

   On the other hand, a Type II error is failing to find that less than 90% of the TV sets are quality TV's when in fact this IS true. Thus, the "consumer agency" fails to find that the TV manufacturer is falsely advertising how good its TV's are, and consumers are being lied to.

   (b) Decreasing the significance level α from .1 to .01 will decrease the probability of a Type I Error from .1 to .01 But this will increase the probability of a Type II Error. So we need to decide which is more important to control against, Type I Error or Type II Error. Since making false accusations can be costly in prosecution costs and court battles, it can be argued that protecting against Type I Error is more important - we want to be very confident if we reject the null hypothesis. Using an $\alpha = .01$ protects against this Type I Error.

4. ((2 pts) Problem 10.18 on page 415) A Type I Error is finding that the plant is discharging water hotter than 150 degrees when in fact that is not true. In this case, a government agency may lean on the plant to decrease the temperature of the spent water when in fact the temperature of the water was not over the specified limit.

   A Type II Error is failing to find that the plant is discharging water hotter than 150 degrees when in fact it is. The consequences of this to the river ecosystem could be dire: loss of aquatic animals and plants.

   Due to the adverse effects on the ecosystem, it appears that a Type II Error is more serious, and so we ought to use a larger significance level so that the probability of a Type II Error is smaller.

5. (a) (4 pts) Problem 10.30 on page 427:
   i. The individuals in this study are car accidents.
   ii. The variable being measured is whether the automobile accident involved a teen.
   iii. The test of whether teenagers are in a disproportion number of automobile accidents is below. The R-code is in the Appendix.
      A. **Hypotheses:** $H_0: \pi = 0.07$ versus $H_a: \pi \neq 0.07$
      B. **Assumptions:** We do have a random sample of size $n = 500$, which is presumably less than 5% of the total population of teenagers who are driving. To check that the sample is large enough, observe that 500(0.07)=35 > 10 and 500(1-0.07)=465 > 10.
      C. **Test statistic:** $z = \frac{.14-.07}{\sqrt{\frac{.07(1-.07)}{500}}} = \frac{.07}{0.01141052} \approx 6.1347.$
      D. **p-value:** $2P(Z > |6.134688|) = 8.532649 \times 10^{-10}$
      E. **Decision:** Since the p-value is way less than any reasonable significance level, reject H_0 in favor of H_a.
      F. **Conclusion:** The evidence suggests that teenagers are involved a disproportionate incidences of driving accidents. That is, the true proportion of accidents involving teenage drivers differs from 7%, the proportion of teens in the driving population.

   (b) (4 pts) Problem 10.32 on page 427:
   i. The individuals are leukemia patients.
   ii. The variable being measured is whether the patient is in remission.
   iii. The sample must have been randomly assigned to the two treatments: arsenic and conventional treatments.
   iv. The steps to test whether arsenic is more effective than the conventional treatment are below. The R-code is in the Appendix.
      A. **Hypotheses:** $H_0: \pi = .15$ versus $H_a: \pi > .15$
      B. **Assumptions:** We probably do not have a random sample of size $n = 100$ (i.e. these are patients who have volunteered). This sample is presumably less than 5% of the total population of all leukemia patients.

To check that the sample is large enough, observe that 100(0.15)=15 > 10 and 100(1-0.15)=85 > 10.

C. **Test statistic:** $z = \frac{.42-.15}{\sqrt{\frac{.15(1-.15)}{100}}} \approx 7.5615.$

D. p**-value:** $P(Z > 7.56) \approx 2.02 \times 10^{-14}$

E. **Decision:** Since the p-value is way less than any reasonable significance level, reject H_0 in favor of H_a.

F. **Conclusion:** The evidence suggests that leukemia patients receiving arsenic were in remission more often than 15%, the remission rate for patients who receive the conventional leukemia treatment.

v. Since the sample was composed of volunteers, then we can not make conclusions to the population of all leukemia patients - the results only apply to the sample. If a CRD was performed, we can conclude that the arsenic treatment caused the leukemia patients in the sample to go into remission more often than patients receiving the conventional treatment.

6. (a) ($6\frac{1}{2}$ pts) Problem 10.48 on page 438:

i. The individuals in the study are blue collar workers with ischemic heart disease.

ii. The variable being measured is systolic blood pressure.

iii. The sample space for systolic blood pressure is $S = [0, \infty)$.

iv. Since the sample space is continuous, then the variable being measured is continuous.

v. Since there are no treatments, then this is an observational study.

vi. The test is:

A. **Hypotheses:** $H_0: \mu = 138$ versus $H_a: \mu > 138$

B. **Assumptions:** We have a random sample of size $n = 48$ This sample is presumably less than 5% of the total population of all IHD blue collar workers. The sample is large enough since $n = 48 > 30$.

C. **Test statistic:** $T = \frac{145.3-138}{\frac{17.6}{\sqrt{42}}} \approx 2.688.$

D. p**-value:** Since $T \sim t(41)$, p-value $= P(Z > 2.688) \approx .0037$

E. **Decision:** Since the p-value is way less than any reasonable significance level, reject H_0 in favor of H_a.

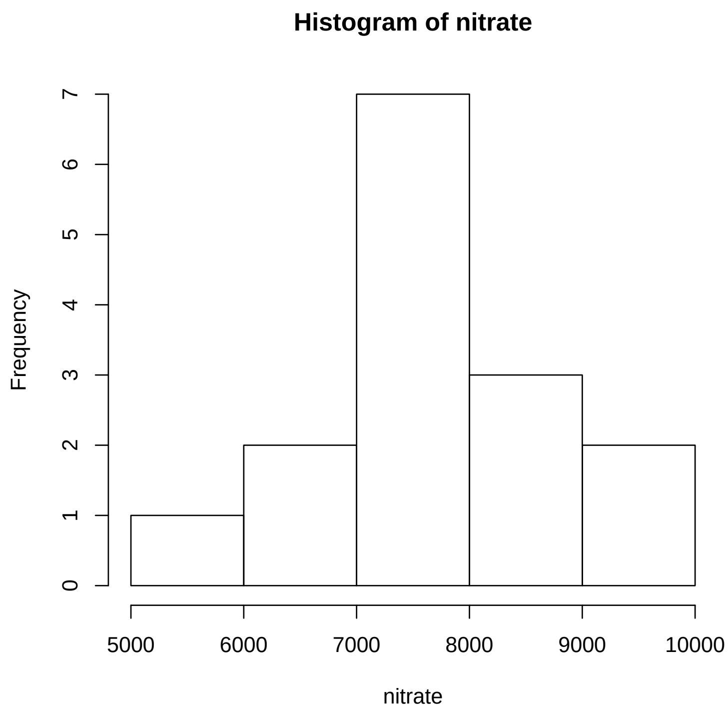
F. **Conclusion:** The evidence suggests that blue collar workers suffering from ischemic heart disease have a higher mean systolic blood pressure than 138, the mean systolic blood pressure for blue collar workers without the disease.

vii. Since we have a SRS, then the results apply to the whole population of IHD blue collar workers. Since this is an observational study, we can not make cause and effect conclusions.

(b) (4 pts) Problem 10.58 on page 440:

i. One can use t-tests for samples smaller than 30 if the population distribution is symmetric. The symmetric histogram in Figure 1 suggests that the data is from a symmetric distribution. Since the sample size is 15, a t-test is appropriate.

Figure 1: The data is from a symmetric distribution



ii. The test:

A. **Hypotheses:** $H_0: \mu = 8000$ versus $H_a: \mu < 8000$

B. **Assumptions:** We will assume that we have a random sample of size $n = 15$ This sample is less than 5% of the total population of all cultures, which is infinite. The sample is large enough since $n = 15$ and the data is assumed to be from a symmetric distribution.

C. **Test statistic:** $T = \frac{7788.8-8000}{\frac{1002.431}{\sqrt{15}}} \approx -0.816$.

D. p**-value:** Since $T \sim t(14)$, then p-value $= P(Z < -.816) \approx 0.2073$

E. **Decision:** Since the p-value $> .1$, fail to reject H_0.

F. **Conclusion:** The evidence fails to suggest that the mean uptake for cultures with nitrates is less than 8000, the mean uptake for cultures without nitrates.

iii. See the Appendix for the R-code.

7. ($2\frac{1}{2}$ pts) Problem 10.60acde on page 448:

(a) The use of a z test statistic is appropriate since σ is known and because the SRS is large enough, $n = 50 > 30$, the Central Limit Theorem assures that $\overline{X}$ is approximately normal.

(c) The desired significance level is $\alpha = P(z \geq 1.8) \approx 0.03593$.

(e) First, we need to find the mean $\bar{x}$ which corresponds to the z test statistic of $z = 1.8$. Since $z = 1.8 = \frac{\bar{x}-150}{\frac{10}{\sqrt{50}}}$, solving for $\bar{x}$ shows that $\bar{x} = 152.5456$. Now assuming that $\mu = 153$,

$$\beta = P(\overline{X} < 152.5456) = P\left(z < \frac{152.5456 - 153}{\frac{10}{\sqrt{50}}}\right) \approx P(z < -0.3213) \approx .374.$$

8. ($\frac{1}{2}$ pt) By definition, the power of the test in Problem 10.60acde when $\mu = 153$ is $1 - \beta = .626$.

9. ($2\frac{1}{2}$ pts) Problem 10.62 on page 449:

   (a) Using R to conduct the test of $H_0\colon \pi = .75$ versus $H_a\colon \pi > .75$, the p-value $\approx .04481$. Thus, reject the null hypothesis. So the evidence (barely) suggests that more than 75% of all apartments exclude children.

   (b) An $\alpha = .05$ test corresponds to rejecting the null hypothesis when $z > 1.645$. The corresponding sample proportion p is found by solving $1.645 = \frac{p-.75}{\sqrt{\frac{.75(1-.75)}{125}}}$, so $p \approx 0.8137$. Assuming that $\pi = .8$, the probability of a Type II Error is $\beta = P(p < .8137) = P\left(z < \frac{.8137-.8}{\sqrt{\frac{.8(1-.8)}{125}}} = .03578\right) \approx P(z < 0.3829) \approx .6491$. By definition, Power$= 1 - \beta = .3509$. The test will only be able to detect a difference between .75 and .8 35% of the time. This test is not very powerful.

Appendix

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> # PROBLEM 5a
> prop.test(.14*500,500,p=.07,conf.level=.95,alternative="two.sided",correct=F)

        1-sample proportions test without continuity correction

data:  0.14 * 500 out of 500, null probability 0.07
X-squared = 37.6344, df = 1, p-value = 8.533e-10
alternative hypothesis: true p is not equal to 0.07
95 percent confidence interval:
 0.1123227 0.1731669
sample estimates:
   p
0.14


> # PROLEM 5b
> prop.test(42,100,p=.15,conf.level=.95,alternative="greater",correct=F)

        1-sample proportions test without continuity correction

data:  42 out of 100, null probability 0.15
X-squared = 57.1765, df = 1, p-value = 1.992e-14
alternative hypothesis: true p is greater than 0.15
95 percent confidence interval:
 0.341973 1.000000
sample estimates:
   p
0.42



> #PROBLEM 6b
> nitrate=c(7251,6871,9632,6866,9094,5849,8957,7978,7064,7494,7883,8178,7523,87
> nitrate
 [1] 7251 6871 9632 6866 9094 5849 8957 7978 7064 7494 7883 8178 7523 8724 7468
> hist(nitrate)
>
> # Calculate the test stat and p-value by hand:
> mean(nitrate);sd(nitrate)
[1] 7788.8
[1] 1002.431
> (7788.8-8000)/(1002.431/sqrt(15))
[1] -0.8159904
> pnorm(-0.8159904)
[1] 0.2072528
```

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>
> # Perform the test using t.test:
> t.test(nitrate,mu=8000,conf.level=.9,alternative="less")

        One Sample t-test

data:  nitrate
t = -0.816, df = 14, p-value = 0.2141
alternative hypothesis: true mean is less than 8000
90 percent confidence interval:
    -Inf 8136.93
sample estimates:
mean of x
   7788.8

>
>
> # PROBLEM 9a
> prop.test(102,125,p=.75,conf.level=.95,alternative="greater",correct=F)

        1-sample proportions test without continuity correction

data:  102 out of 125, null probability 0.75
X-squared = 2.904, df = 1, p-value = 0.04418
alternative hypothesis: true p is greater than 0.75
95 percent confidence interval:
 0.7525097 1.0000000
sample estimates:
    p
0.816
```