

## Practice Integration Problems

### MATH 182: Fall 2006

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The integrals practice problems on the following pages can all be evaluated using combinations of

- 1) The Method of Substitution
  - 2) Integration by Parts
  - 3) Trigonometric identities
  - 4) Inverse Trigonometric Substitutions
  - 5) Partial fraction expansions
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Some commonly used trigonometric identities are:

$$\begin{aligned}\sin^2(x) + \cos^2(x) &= 1 \\ \tan^2(x) + 1 &= \sec^2(x) \\ \cos^2(x) &= \frac{1}{2}(1 + \cos(2x)) \\ \sin^2(x) &= \frac{1}{2}(1 - \cos(2x)) \\ \sin(2x) &= 2 \sin(x) \cos(x) \\ \sin(x) \cos(y) &= \frac{1}{2}(\sin(x+y) + \sin(x-y)) \\ \cos(x) \cos(y) &= \frac{1}{2}(\cos(x+y) + \cos(x-y)) \\ \sin(x) \sin(y) &= \frac{1}{2}(\cos(x-y) - \cos(x+y))\end{aligned}$$

Some commonly integrals worth noting include:

$$\begin{aligned}\int \frac{1}{u^2 + 1} du &= \arctan(u) + c \\ \int \frac{1}{\sqrt{1 - u^2}} du &= \arcsin(u) + c \\ \int \tan(u) du &= \ln|\sec(u)| + c \\ \int \cot(u) du &= -\ln|\csc(u)| + c \\ \int \sec(u) du &= \ln|\sec(u) + \tan(u)| + c \\ \int \csc(u) du &= \ln|\csc(u) - \cot(u)| + c \\ \int \sec^3(u) du &= \frac{1}{2}(\sec(u) \tan(u) + \ln|\sec(u) + \tan(u)|) + c\end{aligned}$$

**(I) Quickies:**

$$a) \int 5e^{3x} dx = \frac{5}{3}e^{3x} + c$$

$$b) \int 2\cos(\pi x) dx = \frac{2}{\pi}\sin(\pi x) + c$$

$$c) \int \sec(2x) \tan(2x) dx = \frac{1}{2}\sec(2x) + c$$

$$d) \int 7\sec^2(5x) dx = \frac{7}{5}\tan(5x) + c$$

$$e) \int \frac{dx}{x^2+4} = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + c$$

$$f) \int \frac{x}{x^2+1} dx = \frac{1}{2}\ln|x^2+1| + c$$

$$g) \int \frac{2}{3x+1} dx = \frac{2}{3}\ln|3x+1| + c$$

**(II) Intermediate Difficulty Problems:**

$$1) \int_1^2 \frac{\ln(x)}{x} dx$$

$$2) \int_1^2 \frac{\ln(x)}{x^2} dx$$

$$3) \int \frac{2x+1}{x(1-x)} dx$$

$$4) \int xe^{x/2} dx$$

$$5) \int \frac{e^{\sqrt{z}}}{\sqrt{z}} dz$$

$$6) \int \tan^3(x) \sec^2(x) dx$$

$$7) \int \frac{dx}{\sqrt{9-x^2}}$$

$$8) \int \frac{dx}{\sqrt{x^2-9}}$$

$$9) \int \frac{3x+2}{x^2(x+2)} dx$$

$$10) \int \frac{x^3}{1+x^4} dx$$

$$11) \int \frac{3x^3+x^2+4}{3x+1} dx$$

$$12) \int_0^{\pi/2} \sin^2(x) dx$$

$$13) \int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx$$

$$14) \int (2x+1)\cos(x) dx$$

$$15) \int e^{3x} \cos(4x) dx$$

$$16) \int \frac{1}{x\sqrt{1+x^2}} dx$$

$$17) \int_0^1 \arcsin(x) dx$$

$$18) \int_0^{\pi/6} \frac{\cos(x)}{1+\sin(x)} dx$$

$$19) \int \sqrt{x^2-4} dx \text{ (need table)}$$

$$20) \int \sqrt{4-x^2} dx$$

21)  $\int \frac{4x+7}{(x+1)(2x+3)} dx$

22)  $\int \frac{x}{\sqrt{1+x^2}} dx$

23)  $\int \frac{\sin(\ln(x))}{x} dx$

24)  $\int x^2 e^x dx$

25)  $\int \sec^4(x) dx$

26)  $\int \frac{e^x}{e^{2x}+1} dx$

27)  $\int \cos^2(4x) dx$

28)  $\int \cos^2(x) \sin^3(x) dx$

29)  $\int_{\pi/4}^{\pi/3} \frac{\sec^2(x)}{\tan(x)} dx$

30)  $\int \arctan(2x) dx$

31)  $\int \frac{1}{x^2+4x+5} dx$

32)  $\int \sin(2x) \cos(4x) dx$

33)  $\int \frac{4x^2-2x}{(x-1)(x^2+1)} dx$

34)  $\int \frac{1}{(x^2+4)^{3/2}} dx$

35)  $\int x^2 \ln(x) dx$

36)  $\int x^2 e^{x^3} dx$

37)  $\int \tan(x) \sec^3(x) dx$

38)  $\int \frac{x}{\sqrt{1+x^2}} dx$

39)  $\int \frac{2x+1}{x^2-1} dx$

40)  $\int \sin(x) \cos^3(x) dx$

41)  $\int \frac{1}{\sqrt{x^2+2x+2}} dx$

42)  $\int \frac{3 \cos(x)}{\sqrt{1+3\sin(x)}} dx$

**Answers:**

1)  $u = \ln(x)$  ;  $\frac{1}{2}(\ln(2))^2$

3) *partial fraction* ;  $\ln|x| - 3\ln|x - 1| + c$

5)  $u = \sqrt{x}$  ;  $2e^{\sqrt{x}} + c$

7)  $u = x/3$ ;  $\arcsin(x/3) + c$

9) *partial fractions*;  $\ln|x| - \ln|x + 2| - x^{-1} + c$

11) *Long division*;  $1/3x^3 + 4/3 \ln|3x + 1| + c$

13)  $u = \sqrt{x} + 1$ ;  $2\ln(1 + \sqrt{x}) + c$

15) *IBParts twice*;  $3/25e^{3x}\cos(4x) + 4/25e^{3x}\sin(4x) + c$

17)  $u = \arcsin(x)$ ,  $v = x$ ;  $\pi/2 - 1$

19)  $x = 2\sec(\theta)$ ;  $\frac{1}{2} x\sqrt{x^2 - 4} - 2\ln|x + \sqrt{x^2 - 4}| + c$

21) *partial fraction*;  $3\ln|x + 1| - \ln|2x + 3| + c$

23)  $u = \ln(x)$ ;  $-\cos(\ln|x|) + c$

25) *trig.ident.* then  $u = \tan(x)$ ;  $\tan(x) + \frac{1}{3}\tan^3(x) + c$

27) *trig. ident.*;  $1/2x + 1/6 * \sin(8x) + c$

29)  $u = \tan(x)$ ;  $1/2\ln(3) = \ln(\sqrt{3})$

31)  $u = x + 2$ , *complete square*;  $\arctan(x + 2) + c$

33) *Partial. Frac.*;  $\ln|x - 1| + \arctan(x) + 3/2\ln(x^2 + 1) + c$

35) *IBP*  $u = \ln(x)$ ,  $v = 1/3x^3$ ;  $1/3x^3\ln|x| - 1/9x^3 + c$

37) *trig.*  $u = \sec(x)$ ;  $1/3 \sec^3(x) + c$

39) *partial frac.*;  $3/2\ln|x - 1| + 1/2\ln|x + 1| + c$

41)  $x + 1 = \tan(\theta)$ ;  $\ln|x + 1 + \sqrt{x^2 + 2x + 2}| + c$

2)  $u = \ln(x)$ ,  $v = -x^{-1}$  ;  $1/2(1 - \ln(2))$

4)  $u = x$ ,  $v = 2e^{x/2}$  ;  $2(x - 2)e^{x/2} + c$

6)  $u = \tan(x)$  ;  $1/4 \tan^4(x) + c$

8)  $x = 3 \sec(\theta)$ ;  $\ln|\frac{1}{3}x + \frac{1}{3}\sqrt{x^2 - 9}| + c$

10)  $u = 1 + x^4$ ;  $1/4 \ln(x^4 + 1) + c$

12) *trig ident.*;  $\pi/4$

14)  $u = 2x + 1$ ,  $v = \sin(x)$ ;  $2\cos(x) + (2x + 1) \sin(x) + c$

16)  $x = \tan(\theta)$ ;  $-\ln|(1 + \sqrt{1 + x^2})/x| + c$

18)  $u = 1 + \sin(x)$ ;  $\ln(3) - \ln(2)$

20)  $x = 2\sin(\theta)$ ;  $1/2x\sqrt{4 - x^2} + 2\arcsin(x/2) + c$

22)  $u = 1 + x^2$ ;  $\sqrt{1 + x^2}$

24) *IBP twice*,  $u = x^2$ ,  $v = e^x$ ;  $(2 - 2x + x^2)e^x$

26)  $u = e^x$ ;  $\arctan(e^x) + c$

28) *trig.ident.*  $u = \cos(x)$ ;  $1/5\cos^5(x) - 1/3\cos^3(x) + c$

30) *IBP*  $u = \arctan(2x)$ ,  $v = x$ ;  $x \arctan(2x) - 1/4\ln(1 + 4x^2) + c$

32) *trig. ident.*;  $1/4\cos(2x) - 1/12\cos(6x) + c$

34)  $x = 2\tan(\theta)$ ;  $x/(4\sqrt{x^2 + 4}) + c$

36)  $u = x^3$ ;  $1/3e^{x^3} + c$

38)  $u = 1 + x^2$ ;  $\sqrt{1 + x^2} + c$

40)  $u = \cos(x)$ ;  $-1/4\cos^4(x) + c$

42)  $u = 1 + 3\sin(x)$ ;  $2\sqrt{1 + 3\sin(x)} + c$