I. Decide if the following are convergent or divergent integrals. Evaluate them if they are convergent.

$$\int_2^5 \frac{dx}{\sqrt{3x-6}} = 6 \ (convergent) \tag{1}$$

$$\int_0^\infty \frac{dx}{x^2+4} = \frac{\pi}{4} \ (convergent) \tag{2}$$

$$\int_0^1 ln(x)\ dx = -1 \ (convergent) \tag{3}$$

$$\int_0^\infty \frac{e^x}{e^{2x}+1}\ dx = \frac{\pi}{4} \ (convergent, u = e^x) \tag{4}$$

$$\int_{-\infty}^\infty \frac{x}{x^4+1}\ dx = 0 \ (convergent) \tag{5}$$

$$\int_0^\infty xe^x\ dx \qquad (divergent) \tag{6}$$

II. Use the Comparsion Test to decide if the following converge or diverge:

$$\int_1^\infty \frac{e^x}{\sqrt{x}}\ dx \quad diverges \quad \frac{e^x}{\sqrt{x}} \geq \frac{1}{\sqrt{x}}\ for x > 1 \tag{7}$$

$$\int_2^\infty \frac{\sqrt{x}}{(x^2+1)}\ dx \quad converges \quad 0 \leq \frac{\sqrt{x}}{(x^2+1)} \leq \frac{\sqrt{x}}{x^2} = \frac{1}{x^{3/2}}\ for x > 2 \tag{8}$$

$$\int_0^\infty \frac{\cos^2(x)}{x^2+2}\ dx \quad converges \quad 0 \leq \frac{\cos^2(x)}{x^2+2} \leq \frac{1}{x^2+2} \leq \frac{1}{x^2}\ for x > 0 \tag{9}$$

$$\int_0^\infty \frac{arctan(x)}{x^3+2}\ dx \quad converges \quad 0 \leq \frac{arctan(x)}{x^3+2} \leq \frac{\pi}{2}\frac{1}{x^3+2} \leq \frac{\pi}{2}\frac{1}{x^3}\ for\ x > 0 \tag{10}$$

$$\int_1^\infty \frac{\sin^2(x)+1}{\sqrt{x}+1}\ dx \quad diverges \quad \frac{\sin^2(x)+1}{\sqrt{x}+1} \geq \frac{1}{\sqrt{x}+1} \geq \frac{1}{2\sqrt{x}}\ for\ x \geq 1 \tag{11}$$

III. Find the arclength of the following curves:

$$y = \frac{1}{3}(1+2x)^{(3/2)} \quad 0 < x < 1 \quad ANS: \ L = \int_0^1 \sqrt{2+2x}\ dx = \frac{8}{3} - \frac{2\sqrt{2}}{3} \tag{12}$$

$$y = \frac{1}{3}(x^2+2)^{(3/2)} \quad 0 < x < 1 \quad ANS: \ L = \int_0^1 (1+x^2)\ dx = \frac{4}{3} \tag{13}$$

$$y = \frac{3}{\sqrt{2}}x^{(2/3)} \quad 1 < x < 8 \quad ANS: \ L = \int_1^8 \frac{\sqrt{x^{2/3}+2}}{x^{1/3}}\ dx = 3^{3/2}(2^{3/2}-1) \qquad u = x^{2/3} \tag{14}$$

$$y = \frac{x^6+8}{16x^2} \quad 1 < x < 2 \quad ANS: \ L = \int_1^2 \frac{4+x^6}{4x^3}\ dx = \frac{21}{16} \tag{15}$$

$$y = log(sec(x)) \quad 0 < x < \pi/6 \quad ANS: \ L = \int_0^{\pi/6} sec(x)\ dx = \frac{1}{2}ln(3) \tag{16}$$

$$y = f(x) = \int_0^x \sqrt{e^t-1}\ dt \quad 0 < x < ln(4) \quad ANS: \ L = \int_0^{ln(4)} 2e^{x/2}\ dx = 2 \tag{17}$$

x-axis:

$$y = \sqrt{3}x \quad 1 < x < 3 \quad ANS: \quad S = \int_1^3 4\pi\sqrt{3}\ x\ dx = 16\pi\sqrt{3} \tag{18}$$

$$y = \sqrt{3x+1} \quad -\frac{1}{3} < x < 1 \quad ANS: \quad S = \int_{-1/3}^1 \pi\sqrt{12x+13}\ dx = \frac{49}{9}\pi \tag{19}$$

$$y = \frac{2}{3}x^3 \quad 0 < x < 1 \quad ANS: \quad S = \int_0^1 \frac{4}{3}\pi x^3\sqrt{1+4x^2}\ dx = \frac{1}{18}\left(5^{3/2}-1\right)\pi \tag{20}$$

$$y = \sqrt{25-x^2} \quad 2 < x < 3 \quad ANS: \quad S = \int_2^3 10\pi\ dx = 10\pi \tag{21}$$

$$y = cos(x) \quad 0 < x < \frac{\pi}{4} \quad ANS: \quad S = \int_0^{\pi/4} 2\pi cos(x)\sqrt{1+sin^2(x)}\ dx \tag{22}$$

$$y = \frac{3}{2}x^{2/3} \quad 0 < x < 1 \quad ANS: \quad S = \int_0^1 3\pi x^{1/3}\ \sqrt{1+x^{2/3}}\ dx = \frac{6}{5}\left(\sqrt{2}+1)\right)\pi \tag{23}$$

$$use \quad u = x^{1/3} \tag{24}$$

$$y = \frac{1}{2}x^2 \quad 0 < x < 1 \quad ANS: \quad S = \int_0^1 \pi x^2\sqrt{1+x^2}\ dx \quad = \frac{\pi}{8}\left(ln(\sqrt{2}-1)+3\sqrt{2}\right) \tag{25}$$

8. [10pts] Find the x and y coordinates of the center of mass of the centroid (region) bounded by the curves:

$$y = \frac{1}{2}x^2, y = 0, x = 0, x = 2, \quad \bar{x} = \frac{3}{2}, \bar{y} = \frac{3}{5}$$

$$y = e^x, y = 0, x = 0, x = ln(2) \qquad \bar{x} = 2\ ln(2) - 1, \bar{y} = \frac{3}{4}$$

$$y = sin(x), y = 0, x = 0, x = \frac{\pi}{2} \quad 0 < x < \quad \bar{x} = 1\ , \bar{y} = \frac{\pi}{8}$$

Some trigonometric identities which may or may not be needed include:

$$\begin{aligned}
cos^2(x) &= \frac{1}{2}(1 + cos(2x)) \\
sin^2(x) &= \frac{1}{2}(1 - cos(2x)) \\
sin(2x) &= 2\ sin(x)\ cos(x) \\
sin(x)\ cos(y) &= \frac{1}{2}(sin(x+y) + sin(x-y)) \\
cos(x)\ cos(y) &= \frac{1}{2}(cos(x+y) + cos(x-y)) \\
sin(x)\ sin(y) &= \frac{1}{2}(cos(x-y) - cos(x+y))
\end{aligned}$$

Some integrals which may or may not be needed include:

$$\begin{aligned}
\int sec(u)\ du &= ln\,|sec(u) + tan(u)| + c \\
\int csc(u)\ du &= ln\,|csc(u) - cot(u)| + c
\end{aligned}$$