Math 152, Spring 2007   Section: ________   Name: Solutions

Midterm 1 : Thursday February 8, 2007.

Problem	1	2	3	4	5	6	7	8	Total
Value	10	10	10	15	15	15	15	10	100
Points									

Instructions All work must be shown to receive full credit.
In all questions simplify your answers.

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1. [10pts] Use the method of substitution to evaluate the following indefinite integral:

$$\int \frac{(\ln x)^2}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \int u^2\,du$$

$$= \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}(\ln x)^3 + C$$

2. [10pts] Evaluate the following definite integral:

$$\int_0^1 9x^2\sqrt{1+3x^3}\,dx$$

$u = 1+3x^3$

$du = 9x^2\,dx$

$x = 0 \Rightarrow u = 1$

$x = 1 \Rightarrow u = 4$

$$= \int_1^4 u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2}\Big|_1^4 = \frac{2}{3}\left(4^{3/2} - 1^{3/2}\right) = \frac{14}{3}$$

3. [10pts] Find the average of the function $f(x) = \sin(x)\cos^4(x)$ over the interval $[0, \pi/2]$.

$$\bar{f} = \frac{1}{\left(\frac{\pi}{2}\right)}\int_0^{\pi/2} \sin x \cos^4 x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

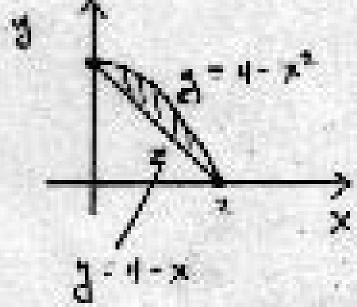
$$\bar{f} = \frac{2}{\pi}\left(-\frac{1}{5}\cos^5 x\right)\Big|_0^{\pi/2}$$

$$\bar{f} = \frac{2}{\pi}\cdot\frac{1}{5}\cos^5 x\Big|_{\pi/2}^{0}$$

$\cos 0 = 1$

$\cos\frac{\pi}{2} = 0$

$$\bar{f} = \frac{2}{5\pi}$$



$$A = \int_0^2 (4-x^2) - (4-2x)\,dx$$

$$A = \int_0^2 (2x - x^2)\,dx$$

$$A = \left(x^2 - \tfrac{1}{3}x^3\right)\Big|_0^2$$

$$A = 4 - \tfrac{8}{3}$$

$$A = \tfrac{4}{3}$$

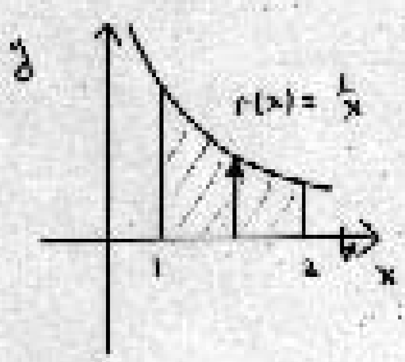
Intersection Pts:

$4 - 2x = 4 - x^2$

$2x = x^2$

$x = 0, 2$

5. [15pts] A solid is formed by revolving the region R bounded by $y = \frac{1}{x}$, $x = 1$, $x = 2$ and $y = 0$ about the x-axis. Sketch the region R and then compute the volume of the solid.



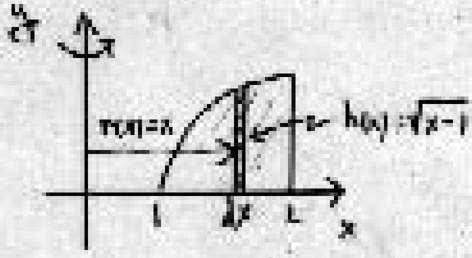
$$V = \int_1^2 \pi\, r(x)^2\,dx$$

$$V = \int_1^2 \frac{\pi}{x^2}\,dx$$

$$V = -\frac{\pi}{x}\Big|_1^2$$

$$V = \frac{\pi}{2}$$

... $x = 2$ about the y-axis. Use the method of cylindrical shells to compute the volume of the resulting solid.



$$V = \int_1^2 2\pi\, r(x)\, h(x)\, dx$$

$$V = \int_1^2 2\pi x \sqrt{x-1}\, dx \qquad \begin{array}{l} u = x-1 \\ du = dx \end{array}$$

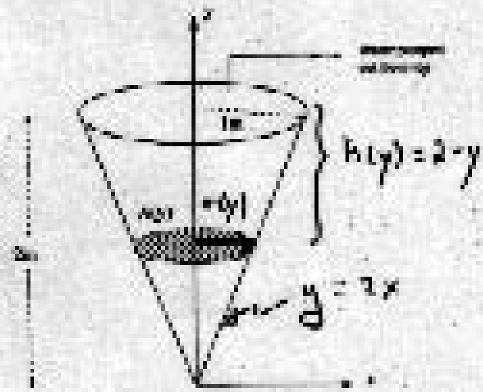
$$V = \int_0^1 2\pi (u+1) u^{1/2}\, du$$

$$V = \int_0^1 2\pi \left(u^{3/2} + u^{1/2}\right) du$$

$$V = 2\pi \left(\tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\right)\Big|_0^1$$

$$V = 2\pi\left(\tfrac{2}{5} + \tfrac{2}{3}\right) = \frac{32\pi}{15}$$

7. [10pts] The cone illustrated below is full of water (of density $\rho = 1000kg/m^3$). Compute the amount of work done in pumping the water out of the cone from its top as illustrated. You may leave your answer as a multiple of ρ and the gravitational constant $g = 9.8m/sec^2$



Since $r(y) = x = y/2$ by edge,

$$A(y) = \pi r(y)^2 = \frac{\pi}{4} y^2$$

Each section pumped a height of $h(y) = 2-y$:

$$W = \int_0^2 \rho g\, h(y) A(y)\, dy$$

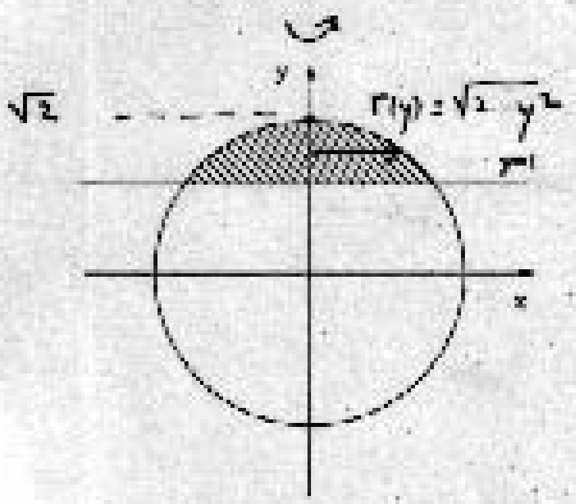
$$W = \int_0^2 \rho g \cdot (2-y) \frac{\pi}{4} y^2\, dy$$

$$W = \frac{\rho g \pi}{4} \int_0^2 (2y^2 - y^3)\, dy$$

$$W = \frac{\rho g \pi}{4} \left(\tfrac{2}{3} y^3 - \tfrac{1}{4} y^4\right)\Big|_0^2$$

$$W = \frac{\rho g \pi}{3} \text{ Joules}$$

with $y > 1$. Compute the volume of the solid obtained by revolving this region about the y-axis (the solid is the cap of a sphere).



Washer Method Easiest

Cross sections have area

$$A(y) = \pi\, r(y)^2$$

$$A(y) = \pi\,(2 - y^2)$$

Thus

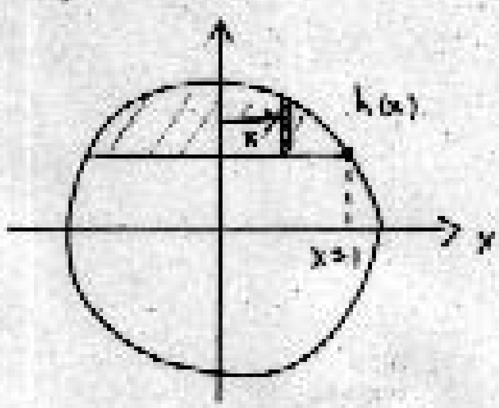
$$V(y) = \int_1^{\sqrt{2}} \pi\,(2 - y^2)\,dy$$

Some extra steps

$$V(y) = \pi\left(2y - \tfrac{1}{3}y^3\right)\Big|_1^{\sqrt{2}}$$

$$V(y) = \frac{\pi}{3}\left(4\sqrt{2} - 5\right)$$

By shells its hard



$$V = \int_0^1 2\pi x\, h(x)\, dx$$

$$V = \underbrace{\int_0^1 2\pi x\left(\sqrt{2 - x^2} - 1\right) dx}_{\text{hard to do!}}$$