Math 182 , Fall 2006 Section: ______ Name: ______________

Midterm 2 : Tuesday, October 24, 2006.

Problem	1	2	3	4	5	6	7	8	Total
Value	10	15	10	15	15	10	15	10	100
Points									

Instructions : All work must be shown to receive full credit.
In all questions simplify your answers.

1. [10pts] Use integration by parts to evaluate the following indefinite integral:

$$I = \int \underbrace{2x}_{u} \underbrace{\sin(3x)\,dx}_{dv}$$

$u = 2x \qquad du = 2\,dx$

$v = -\frac{1}{3}\cos(3x) \qquad dv = \sin(3x)\,dx$

$$I = -\frac{2}{3}x\cos(3x) + \frac{2}{3}\int \cos(3x)\,dx$$

$$I = -\frac{2}{3}x\cos(3x) + \frac{2}{9}\sin(3x) + C$$

2. [15pts] Use partial fraction expansions to evaluate the following integral:

$$\int \frac{8x^2+3}{x(x^2+1)}\,dx$$

$$\frac{8x^2+3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+x(Bx+C)}{x(x^2+1)} = \frac{(A+B)x^2+Cx+A}{x(x^2+1)}$$

Thus

$$\left.\begin{aligned} A+B &= 8 \\ C &= 0 \\ A &= 3 \end{aligned}\right\} \Rightarrow A = 3,\ B = 5,\ C = 0$$

$$I = \int \left(\frac{3}{x} + \frac{5x}{x^2+1}\right) dx$$

$$I = 3\ln|x| + \frac{5}{2}\ln|x^2+1| + C$$

3. [10pts] Evaluate the following definite integral. In your answer simplify any trigonometric functions evaluated at 0 and $\frac{\pi}{3}$.

$$I = \int_0^{\pi/3} \underbrace{\tan^2(x)}_{u^2} \overbrace{\sec^2(x)\, dx}^{du}$$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{3} \Rightarrow u = \tan(\frac{\pi}{3})$

$u = \sqrt{3}$

$$I = \int_0^{\sqrt{3}} u^2 du$$

$$I = \frac{1}{3} u^3 \Big|_0^{\sqrt{3}} = \frac{1}{3}(\sqrt{3})^3 = \sqrt{3}$$

4. [15pts] Use inverse trigonometric substitution to evaluate the following integral. Your answer must be expressed as a function of x.

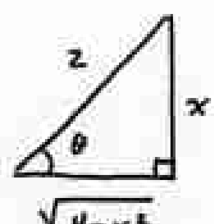
$$I = \int \sqrt{4 - x^2}\, dx$$

$x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

$$I = 2\int \sqrt{4(1-\sin^2\theta)} \cdot \cos\theta \, d\theta$$

$$I = 4\int \cos^2\theta \, d\theta$$



$$I = 2\int (1 + \cos 2\theta) d\theta$$

$$I = 2\theta + \sin 2\theta + C$$

$$I = 2\theta + 2\sin\theta\cos\theta + C$$

$\sin\theta = \frac{x}{2}$

$\cos\theta = \frac{1}{2}\sqrt{4-x^2}$

$$I = 2\arcsin\left(\frac{x}{2}\right) + \frac{1}{2} x\sqrt{4-x^2} + C$$

5. [15pts] (Improper Integrals) In the following problems full credit will not be given if there is an absence or misuse of limits or limit notation. For the Comparison Theorem problem, all details must be explained.

a) Determine if the following improper integral is convergent or divergent. If it is convergent, find its value.

$$I = \int_0^\infty xe^{-x^2}\,dx$$

Since $\int xe^{-x^2}dx = -\frac{1}{2}e^{-x^2} + c$ then

$$I = \lim_{t\to\infty}\int_0^t xe^{-x^2}dx = \lim_{t\to\infty}\left(-\frac{1}{2}e^{-x^2}\right)\Big|_0^t$$

$$= \lim_{t\to\infty}\left(\frac{1}{2} - \frac{1}{2}\cancelto{0}{e^{-t^2}}\right)$$

$$= \frac{1}{2}$$

Integral converges. Its value is $\frac{1}{2}$.

b) Use the Comparison Theorem to clearly explain why the following integral is convergent or divergent.

$$I = \int_1^\infty \frac{\sin^2(x)}{x^3-2}\,dx$$

Since $0 \le \frac{\sin^2 x}{x^3+2} \le \frac{1}{x^3+2} \le \frac{1}{x^3}$ for $x \ge 1$

and $\int_1^\infty \frac{dx}{x^3}$ converges, $\int_1^\infty \frac{\sin^2 x}{x^3+2}dx$ converges

by the Comparison Theorem.

6. [15pts] Find the surface area of the surface formed by revolving the curve

$$y = \underbrace{\sqrt{2x-1}}_{f(x)}, \quad 1 \le x \le 4$$

about the x-axis.

$$f'(x) = \frac{1}{\sqrt{2x-1}}$$

$$1 + f'(x)^2 = 1 + \frac{1}{2x-1}$$

$$1 + f'(x)^2 = \frac{2x}{2x-1}$$

$$S = \int_a^b 2\pi f(x)\sqrt{1 + f(x)^2}\,dx$$

$$S = \int_1^4 2\pi \cancel{\sqrt{2x-1}} \cdot \frac{\sqrt{2x}}{\cancel{\sqrt{2x-1}}}\,dx$$

$$S = 2^{3/2}\pi \int_1^4 x^{1/2}\,dx = 2^{3/2}\pi \cdot \frac{2}{3}x^{3/2}\Big|_1^4$$

$$= 2^{3/2}\pi \cdot \frac{2}{3}\,(\underbrace{4^{3/2}-1}_{7})$$

$$= \frac{28}{3}\sqrt{2}\,\pi$$

7. [10pts] Find the x coordinate $\bar{x}$ of the center of mass of the centroid of the region bounded by $y = \sqrt{1-x}$, $y = 0$, $x = 0$ and $x = 1$. (you need not compute the corresponding y component)

$$\bar{x} = \frac{1}{A}\int_0^1 x\,f(x)\,dx \qquad f(x) = \sqrt{1-x}$$

where

$$A = \int_0^1 (1-x)^{1/2}\,dx = \frac{2}{3}(1-x)^{3/2}\Big|_1^0 = \frac{2}{3}$$

and

$$M_y = \int_0^1 x\sqrt{1-x}\,dx \qquad u = 1-x \qquad x = 1-u$$

$$du = -dx$$

$$M_y = \int_0^1 (1-u)\,u^{1/2}\,du$$

$$M_y = \int_0^1 (u^{1/2} - u^{3/2})\,du = \left(\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right)\Big|_0^1 = \left(\frac{2}{3} - \frac{2}{5}\right) = \frac{4}{15}$$

Thus

$$\bar{x} = \frac{M_y}{A} = \frac{4}{15}\cdot\frac{3}{2} = \frac{2}{5}$$

8. (10pts) Use any methods to evaluate the following integral:

$$I = \int \frac{dx}{x(2+\sqrt{x})}$$

There are many ways. Here's one:

$$u = x^{1/2}, \quad x = u^2$$
$$2u\, du = \underline{dx}$$

$$I = \int \frac{2u\, du}{u^2(2+u)} = \int \frac{2}{u(2+u)}\, du$$

Then partial fractions

$$\frac{2}{u(2+u)} = \frac{A}{u} + \frac{B}{2+u} = \frac{\overbrace{(A+B)}^{=0} u + \overbrace{2A}^{=2}}{u(2+u)}$$

From which, $A+B=0$ and $2A=2$ so that $A=1$, $B=-1$

$$I = \int \left(\frac{1}{u} - \frac{1}{2+u}\right) du$$

$$I = \ln|u| - \ln|2+u| + C$$

$$I = \ln\sqrt{x} - \ln(2+\sqrt{x}) + C$$

Alternate methods

$u = 2+\sqrt{x}$ then partial fractions

$x = 4\tan^4\theta$ unusual but it works.