Math 182 , Spring 2007 **Section:** ______ **Name:** SOLUTIONS

Midterm 2 : Thursday, March 22, 2006.

Problem	1	2	3	4	5	6	7	8	Total
Value	10	15	10	15	20	10	10	10	100
Points									

Instructions : All work must be shown to receive full credit.
In all questions simplify your answers.

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1. [10pts] Use integration by parts to evaluate the following indefinite integral:

$$I = \int 3x^2 \, ln(x) \, dx$$

$u = \ln x \qquad dv = 3x^2 dx$

$du = \frac{1}{x} dx \qquad v = x^3$

$$I = x^3 \ln x - \int \frac{1}{x} \cdot x^3 dx$$

$$I = x^3 \ln x - \frac{1}{3} x^3 + C$$

2. [15pts] Use partial fraction expansions to evaluate the following integral:

$$I = \int \frac{x^2+x+4}{x(x^2+4)} dx$$

$$\frac{x^2+x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} = \frac{\overset{=1}{(A+B)}x^2 + \overset{=1}{C}x + \overset{=4}{4A}}{x(x^2+4)}$$

$4A = 4$
$C = 1$
$(A+B) = 1$

$\Rightarrow \quad A = 1, \quad B = 0, \quad C = 1$

Thus

$$I = \int \frac{1}{x} dx + \int \frac{dx}{x^2+4} = \int \frac{1}{x} dx + \frac{1}{4} \int \frac{dx}{(\frac{x}{2})^2 + 1}$$

Letting $u = x/2$ in last integral:

$$I = \ln x + \frac{1}{2} \arctan(x/2) + C$$

3. [10pts] Evaluate the following definite integral. In your answer simplify any trigonometric functions evaluated at 0 and $\frac{\pi}{2}$.

$$I = \int_0^{\pi/2} 5\cos^2(x)\sin^3(x)\,dx$$

$$I = \int_0^{\pi/2} 5\cos^2 x\,(1-\cos^2 x)\sin x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$x = 0 \Rightarrow u = 1$

$x = \pi/2 \Rightarrow u = 0$

$$I = -\int_1^0 5u^2(1-u^2)\,du$$

$$I = \int_0^1 5u^2 - 5u^4\,du = \frac{5}{3}u^3\Big|_0^1 - u^5\Big|_0^1$$

$$I = \frac{5}{3} - 1 = \frac{2}{3}$$

4. [15pts] Use inverse trigonometric substitution to evaluate the following integral. Use an appropriate triangle to express your answer in terms of x.

$$I = \int \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$x = \sin\theta$

$dx = \cos\theta\,d\theta$

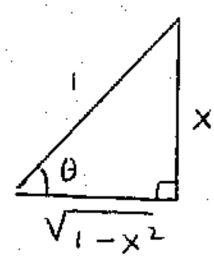
$$I = \int \frac{\sin^2\theta \cdot \cancel{\cos\theta}}{\cancel{\sqrt{1-\sin^2\theta}}}\,d\theta$$

$$I = \int \sin^2\theta\,d\theta$$

$$I = \frac{1}{2}\int (1-\cos 2\theta)\,d\theta$$

$$I = \frac{1}{2}\theta - \frac{1}{4}\underbrace{\sin 2\theta}_{2\sin\theta\cos\theta} + C$$

$$I = \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta + C$$



$\cos\theta = \sqrt{1-x^2}$

$\sin\theta = x$

$$I = \frac{1}{2}\arcsin\theta - \frac{1}{2}x\sqrt{1-x^2} + C$$

5. [20pts] (Improper Integrals) In the following problems full credit will not be given if there is an absence or misuse of limits or limit notation. For the Comparsion Theorem problem, all details must be explained.

a) Determine if the following improper integral is convergent or divergent. If it is convergent, find its value.

$$I = \int_0^1 \frac{(ln(x))^2}{x}\, dx \qquad u = \ln x$$

$$I = \lim_{t \to 0^+} \int_t^1 \frac{(\ln x)^2}{x} dx$$

$$I = \lim_{t \to 0^+} \tfrac{1}{3}(\ln x)^3 \Big|_t^1 \qquad (\ln 1 = 0)$$

$$I = \lim_{t \to 0^+} -\tfrac{1}{3}(\ln t)^3 \qquad (\text{and } \ln t \to -\infty \text{ as } t \to 0^+$$

$$I = +\infty$$ Thus integral is divergent.

b) Use the Comparison Theorem to clearly explain why the following integral is convergent or divergent.

$$I = \int_1^\infty \frac{e^{-x}}{x^3+1}\, dx$$

For $x > 1$ we have (since $e^{-x} \leq 1$)

$$(1) \qquad 0 \leq \frac{e^{-x}}{x^3+1} \leq \frac{1}{x^3+1} \leq \frac{1}{x^3}$$

Since $\int_1^\infty \frac{dx}{x^3} = \frac{1}{2}$ is a convergent integral then the comparison test for integrals and inequality in (1) imply

$$0 \leq \int_1^\infty \frac{e^{-x}}{x^3+1} dx \leq \frac{1}{2}$$

is a convergent improper integral.

6. [10pts] Find the arclength of the curve

$$y = f(x) = \frac{1}{3}(2x-1)^{3/2}, \quad 1 < x < 4.$$

$$f'(x) = \frac{1}{2}(2x-1)^{1/2} \cdot 2 = (2x-1)^{1/2}$$

$$1 + f'(x)^2 = 1 + 2x - 1 = 2x$$

Thus the arclength L is

$$L = \int_1^4 \sqrt{1+f'(x)^2}\,dx = \int_1^4 \sqrt{2x}\,dx = \sqrt{2}\int_1^4 x^{1/2}\,dx$$

$$L = \sqrt{2}\cdot\frac{2}{3}x^{3/2}\Big|_1^4 = \frac{2\sqrt{2}}{3}\big(\underbrace{4^{3/2} - 1^{3/2}}_{7}\big) = \frac{14\sqrt{2}}{3}$$

7. [10pts] Find the surface area of the surface formed by revolving the curve

$$y = f(x) = \sqrt{25 - x^2}, \quad 2 < x < 3$$

about the x-axis.

$$f'(x) = -x(25-x^2)^{-1/2} = \frac{-x}{\sqrt{25-x^2}}$$

$$1 + f'(x)^2 = 1 + \frac{x^2}{25-x^2} = \frac{25 - x^2 + x^2}{25-x^2} = \frac{25}{25-x^2}$$

$$\sqrt{1+f'(x)^2} = \frac{5}{\sqrt{25-x^2}}$$

Thus, the surface area of revolution S is:

$$S = \int_2^3 2\pi f(x)\sqrt{1+f'(x)^2}\,dx = \int_2^3 2\pi \cancel{\sqrt{25-x^2}}\,\frac{5}{\cancel{\sqrt{25-x^2}}}\,dx$$

$$S = \int_2^3 10\pi\,dx$$

$$S = 10\pi$$

8. [10pts] Use any methods to evaluate the following integral:

$$I = \int e^x \cos(2x)\, dx$$

$$I = \frac{1}{2} e^x \sin(2x) + \frac{1}{2}\int e^x(-\sin 2x)dx \qquad \begin{cases} u = e^x \\ u' = e^x \end{cases} \quad \begin{matrix} v = \frac{1}{2}\sin 2x \\ v' = \cos 2x \end{matrix}$$

$$I = \frac{1}{2} e^x \sin(2x) + \frac{1}{2}\left\{ \frac{1}{2}e^x \cos 2x - \frac{1}{2}\int e^x \cos 2x dx \right\} \quad \begin{cases} u = e^x \\ u' = e^x \end{cases} \quad \begin{matrix} v = \frac{1}{2}\cos 2x \\ v' = -\sin(2x) \end{matrix}$$

$$I = \frac{1}{2} e^x \sin(2x) + \frac{1}{4}e^x \cos 2x - \frac{1}{4}\underbrace{\int e^x \cos 2x dx}_{I} + C$$

$$I = \frac{1}{2} e^x \sin(2x) + \frac{1}{4}e^x \cos 2x - \frac{1}{4} I + C$$

$$\frac{5}{4} I = \frac{1}{2} e^x \sin(2x) + \frac{1}{4}e^x \cos 2x + C$$

$$I = \frac{2}{5} e^x \sin(2x) + \frac{1}{5}e^x \cos 2x + C$$