

**Final Exam** : Tuesday, December 12, 2006.

Problem	1	2	3	4	5	6	7	8	9	10	Total
Value	10	10	10	10	10	10	10	10	10	10	100
Points											

**Instructions** : All work must be shown to receive full credit.

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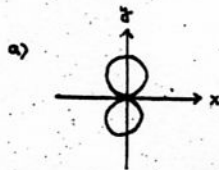
1. [10pts] Determine the first three terms of the Taylor Series of  $f(x) = \sqrt{3+x}$  about  $a = 1$ .

2. [10pts] Use integration by parts to evaluate the following integral:

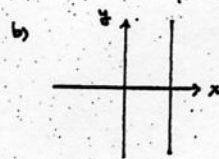
$$I = \int x e^{3x} dx$$

3. [10pts] In the left column below are equations of curves defined using polar coordinates  $(r, \theta)$ . In the right column are graphs of various curves. Match the letter of each graph to its appropriate equation in the left column.

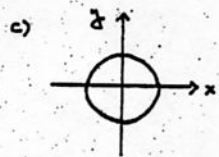
\_\_\_\_\_  $\theta = \frac{\pi}{4}$



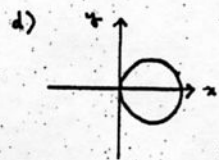
\_\_\_\_\_  $r = 2$



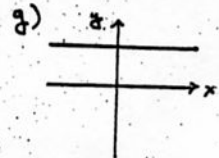
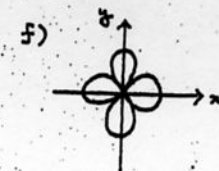
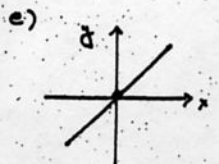
\_\_\_\_\_  $r = \frac{1}{\sin(\theta)}$



\_\_\_\_\_  $r = 2 \cos(\theta)$



\_\_\_\_\_  $r = \cos(2\theta)$



4. [10pts] Evaluate the following definite integral (Hint: first expand the integrand out)

$$I = \int_0^{2\pi} (1 + \cos(x))^2 dx$$

5. [10pts] Use the inverse substitution  $x = 3 \sec(\theta)$  to evaluate

$$I = \int \frac{dx}{x^2 \sqrt{x^2 - 9}}$$

6. [10pts] Use partial fraction expansions to evaluate

$$I = \int \frac{3}{(x+2)(x-1)} dx$$

7. [10pts] Determine the radius of convergence  $R$  of the following power series

$$S(x) = \sum_{n=1}^{\infty} \frac{x^n e^{-n}}{3n^2}$$

8. [10pts] Use the Limit Comparison Test to determine if the following series converges or diverges.

$$S = \sum_{n=0}^{\infty} \frac{5^n}{3^n + 4^n}$$

9. [10pts] Use any method to determine if the following series converges or diverges. State and verify all hypotheses of the theorem you used.

$$S = \sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{n \ln(n)}$$

10. [10pts] Use the Comparison Test to determine if the following series converges or diverges.

$$S = \sum_{n=3}^{\infty} \frac{\sin(n) + 2}{\ln(n)}$$

## Indefinite integrals and Trigonometric Identities

Some trigonometric identities which may or may not be needed include:

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\sin(x) \cos(y) = \frac{1}{2}(\sin(x+y) + \sin(x-y))$$

$$\cos(x) \cos(y) = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$

$$\sin(x) \sin(y) = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$

Some integrals which may or may not be needed include:

$$\int \sec(u) \, du = \ln |\sec(u) + \tan(u)| + c$$

$$\int \csc(u) \, du = \ln |\csc(u) - \cot(u)| + c$$

**3.** [10pts] In the left column below are equations of curves defined using polar coordinates  $(r, \theta)$ . In the right column are graphs of various curves. Match the letter of each graph to its appropriate equation in the left column.

\_\_\_\_\_  $\theta = \frac{\pi}{4}$

\_\_\_\_\_  $r = 2$

\_\_\_\_\_  $r = \frac{1}{\sin(\theta)}$

\_\_\_\_\_  $r = 2 \cos(\theta)$

\_\_\_\_\_  $r = \cos(2\theta)$