Math 182 , Fall 2006 Section: ________ Name: ______________________

Final Exam : Tuesday, December 12, 2006.

Problem	1	2	3	4	5	6	7	8	9	10	Total
Value	10	10	10	10	10	10	10	10	10	10	100
Points											

Instructions : All work must be shown to receive full credit.

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1. [10pts] Determine the first three terms of the Taylor Series of $f(x) = \sqrt{3+x}$ <u>about</u> $a = 1$.

$f(x) = (3+x)^{1/2}$ $\quad$ $f(1) = \sqrt{4} = 2$

$f'(x) = \frac{1}{2}(3+x)^{-1/2}$ $\quad$ $f'(1) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f''(x) = -\frac{1}{4}(3+x)^{-3/2}$ $\quad$ $f''(1) = -\frac{1}{4}\cdot\frac{1}{4^{3/2}} = -\frac{1}{32}$

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \cdots$$

$$f(x) = 2 + \frac{1}{4}(x-1) - \frac{1}{64}(x-1)^2 + \cdots$$

2. [10pts] Use integration by parts to evaluate the following integral:

$$I = \int x\, e^{3x}\, dx \qquad u = x \qquad v = \tfrac{1}{3}e^{3x}$$

$I = uv - \int v\,du$

$I = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}dx$

$I = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$

$I = \frac{1}{3}e^{3x}(x - \frac{1}{3}) + C$

3. [10pts] In the left column below are equations of curves defined using polar coordinates (r, θ). In the right column are graphs of various curves. Match the letter of each graph to its appropriate equation in the left column.

_e_ $\theta = \frac{\pi}{4}$

_c_ $r = 2$

_g_ $r = \frac{1}{\sin(\theta)}$

"$r \sin\theta = y = 1$"

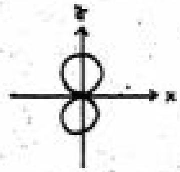
_d_ $r = 2\cos(\theta)$

$(x-1)^2 + y^2 = 1$

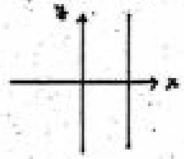
_f_ $r = \cos(2\theta)$

(4 leaves)

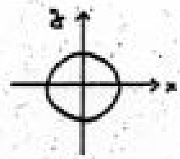
a)



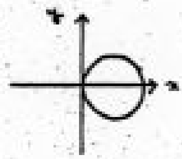
b)



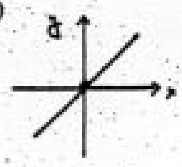
c)



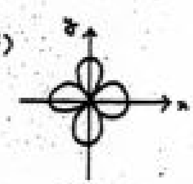
d)



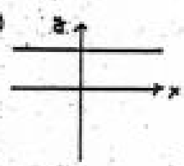
e)



f)



g)



4. [10pts] Evaluate the following definite integral (Hint: first expand the integrand out)

$$I = \int_0^{2\pi} (1+\cos(x))^2 \, dx$$

$$I = \int_0^{2\pi} (1 + 2\cos x + \cos^2 x)\,dx \qquad \cos^2 x = \tfrac{1}{2} + \tfrac{1}{2}\cos 2x$$

$$I = \int_0^{2\pi} 1 + 2\cos x + \tfrac{1}{2} + \tfrac{1}{2}\cos 2x \, dx$$

$$I = \int_0^{2\pi} \left(\tfrac{3}{2} + 2\cos x + \tfrac{1}{2}\cos 2x\right)dx$$

$$I = \tfrac{3}{2}x + 2\cancel{\sin x} + \tfrac{1}{4}\cancel{\sin 2x} \Big|_0^{2\pi}$$

$$I = 3\pi$$

5. [10pts] Use the inverse substitution $x = 3\sec(\theta)$ to evaluate

$$I = \int \frac{dx}{x^2\sqrt{x^2-9}}$$

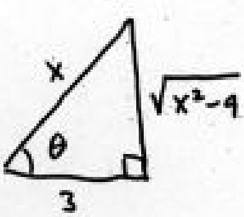
$$x = 3\sec\theta$$

$$dx = 3\sec\theta\tan\theta\,d\theta$$

$$\sqrt{x^2-9} = \sqrt{9\tan^2\theta} = 3\tan\theta$$

$$I = \int \frac{\cancel{3}\sec\theta\,\cancel{\tan\theta}\,d\theta}{9\sec^2\theta \cdot \cancel{3}\cancel{\tan\theta}}$$

$$I = \frac{1}{9}\int \frac{d\theta}{\sec\theta} = \frac{1}{9}\int \cos\theta\,d\theta = \frac{1}{9}\sin\theta + C$$



$$\sec\theta = \frac{x}{3}$$

$$\sin\theta = \frac{\sqrt{x^2-9}}{x}$$

$$I = \frac{\sqrt{x^2-9}}{9x} + C$$

6. [10pts] Use partial fraction expansions to evaluate

$$I = \int \frac{3}{(x+2)(x-1)}\,dx$$

Partial Fractions: $\frac{3}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$

yields $B = 1,\ A = -1$ thus

$$I = \int\left(\frac{1}{x-1} - \frac{1}{x+2}\right)dx$$

$$I = \ln|x-1| - \ln|x+2| + C$$

7. [10pts] Determine the radius of convergence R of the following power series

$$S(x) = \sum_{n=1}^{\infty} \frac{x^n e^{-n}}{3n^2}$$

$a_n = \frac{x^n e^{-n}}{3n^2}$ Use Ratio Test

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x|^{n+1} e^{-n-1}}{\cancel{3}(n+1)^2} \cdot \frac{\cancel{3}n^2}{\cancel{|x|^n} e^{-n}} = e^{-1}|x| \frac{n^2}{(n+1)^2}$$

Thus $S(x)$ converges absolutely if

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = e^{-1}|x| \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2 = \frac{|x|}{e} < 1$$

or $|x| < e$ so that $R = e$ is the radius of convergence.

8. [10pts] Use the Limit Comparison Test to determine if the following series converges or diverges.

$$S = \sum_{n=0}^{\infty} \frac{5^n}{3^n + 4^n} \Big\} a_n \qquad b_n = \left(\frac{5}{4}\right)^n$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5^n}{3^n + 4^n} \cdot \frac{4^n}{5^n} = \lim_{n\to\infty} \frac{1}{\left(\frac{3}{4}\right)^n + 1} = 1 \neq 0$$

Since $\sum b_n$ is a divergent geometric series and $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = 1 \neq 0$ then by LC-Test the series S diverges.

9. [10pts] Use any method to determine if the following series converges or diverges. State and verify all hypotheses of the theorem you used. Alternating Series

$$S = \sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{n \ln(n)} = \sum_{n=3}^{\infty} (-1)^{n+1}\, b_n$$

(1) For $n \geq 3$, $b_n > 0$.

(2) Clearly $\lim\limits_{n\to\infty} b_n = 0$

(3) Is $b_{n+1} \leq b_n$ for all $n \geq 3$?

(a) Both n and $\ln n$ are positive increasing fns (differentiable) then $\frac{1}{n \ln n}$ decreases.

(b) or, ...

$f(x) = x \ln x \qquad f'(x) = 1 + \ln x > 0$

$\Rightarrow f(x) \uparrow$ so $b_n \downarrow$.

Since S satisfies all hypotheses of AST then it converges.

10. [10pts] Use the Comparison Test to determine if the following series converges or diverges.

$$S = \sum_{n=3}^{\infty} \frac{\sin(n)+2}{\ln(n)}$$

Note for all $n \geq 3$

$$\ln n < n$$

$$\frac{1}{\ln n} > \frac{1}{n}$$

$$\underbrace{\frac{\sin(n)+2}{\ln n}}_{a_n} > \frac{\sin(n)+2}{n} \geq \underbrace{\frac{1}{n}}_{b_n}$$

Since $a_n \geq b_n \geq 0$ for all n and $\sum_{n=3}^{\infty} \frac{1}{n}$ is a divergent p-series ($p=1$) then $\sum_{n=3}^{\infty} a_n = S$ diverges by the Comparison Test.