Math 449:
Assigned Homework 3
Due: February 15, 2008.

1. [20] (Real and Imaginary parts of functions of a complex variable) For each of the following functions $f(z) = u(x, y) + iv(x, y)$ with $f : \mathbb{C} \to \mathbb{C}$ determine $u(x, y)$ and $v(x, y)$

a) $f(z) = z^3$

b) $f(z) = \frac{z}{\bar{z}+z}$

c) $f(z) = e^{2z}$

d) $g(z) = \mathrm{I\!Re}(z)i + \mathrm{I\!Im}(z)$

2. [20] (Images of Sets/Mappings) For each of the complex map $f(z)$ and set S defined below, sketch the set S in the z-plane and then the image of S under f in the $w = f(z)$ plane. For each, label at least three key points P, Q, R and their images $P', Q,' R'$.

a) $f(z) = 3z + i$, $S = \{z : y = x, z = x + iy\}$

b) $f(z) = z^6$, $S = \{z : |z| \geq 0, 0 \leq Arg(z) \leq \frac{\pi}{6}\}$

c) $f(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$ $S = \{z : |z| \geq 1, \mathrm{I\!Im}(z) \geq 0\}$

d) $f(z) = |z^2 - a^2|$, $S = \{z : z = \sqrt{2}\, a + iy\ ,\ y \in \mathrm{I\!R}\ \}$ $\quad$ (a *fixed real constant*)

3. [10] (Formal Proof of Continuity) For each function below prove it is continuous at the indicated point z_0. Make sure every detail is covered and as much as possible use the logic and set theory symbols: $\forall, \ni, \in, \exists, \Rightarrow$ .....

a) $f(z) = 4z + 3 + i, \quad z_0 = 1 + i$

b) $f(z) = z^2, \quad z_0 = 1$

4. [5] (Stereographic Projection) Find the coordinate P of the point on the Riemann (Unit) Sphere that corresponds to $z = r\ e^{i\ \alpha}$. Express P in spherical coordinates (θ, ϕ) where $\phi \in [0, \pi], \theta \in [0, 2\pi)$.