Math 449:
Assigned Homework 4
Due: February 27, 2008.

1. [5] Let $\mathbb{R}e(f(z)) = \mathbb{I}m(z)$ where $f : \mathbb{C} \to \mathbb{C}$. Is $f(z)$ analytic and if so what is $f(z)$, (i.e. a formula in z).
2. [5] The function $f : \mathbb{C} \to \mathbb{C}$, $f(z) = u(x, y) + iv(x, y)$ is entire and $v(x, y) = 3x^2y - y^3 - 1$. Compute $f(i)$.
3. [25] In the following, $f : \mathbb{C} \to \mathbb{C}$ and $f(z) = u(x, y) + iv(x, y)$. In each case you are given either u or v. Decide which functions $f(z)$ are analytic. When it is analytic, find a formula for $f(z)$ involving only z, i.e. not just x and y's.

a) $u(x, y) = x + y + x^2 + y^2$

b) $v(x, y) = 3y - zx^2 + 2y^2$

c) $u(x, y) = e^{2x}\ cos(2y)$

d) $u(x, y) = \frac{1}{x^2+y^2}$

e) $u(x, y) = 3Arg(z)$

4. [10] The function $f : \mathbb{C} \to \mathbb{C}$, $f(z) = u(x, y) + iv(x, y)$ has and $u(x, y) = \frac{x}{x^2+y^2}$.

a) Find a formula for $f(z)$.

b) Is $f(z)$ entire?

c) Compute $f''(i)$

5. [15] For each of the following, compute $f(i)$

a) $f(z) = sin(z/4)$

b) $f(z) = Log\left(\frac{1+z}{1-z}\right)$

c) $f(z) = tanh(z/6)$