QUESTION THREE REMARKS

$$(1 - z)^{-2} = 1 + \tfrac{1}{2} z + \tfrac{3}{8} z^2 + O(z^3)$$

by the binomial theorem only if z is small!

If you wish to use that theorem write it as follows

$$\frac{1}{\sqrt{1 - (y_0 + \varepsilon y_1 + \cdots)^2}} = \cdots = \frac{1}{\sqrt{1 - y_0^2}} \sqrt{1 - \underbrace{\frac{2 y_0 y_1 \varepsilon + O(\varepsilon^2)}{1 - y_0^2}}_{z}}$$

Then z is small since $\varepsilon \ll 1$.

Clearly this is a messy way. Easier is:

$$F(\varepsilon) = \frac{1}{\sqrt{1 - (y_0 + \varepsilon y_1 + \cdots)^2}}$$

For fixed x the expansion of $F(\varepsilon)$ is

$$F(\varepsilon) = F(0) + F'(0)\,\varepsilon + O(\varepsilon^2)$$

$F(0)$ is easy! Ask yourself how many terms you really need to find the $O(\varepsilon)$ problem.

QUESTION FOUR (Guide)

The leading order problem is

(1) $\quad y_0' y_0'' = 18 \qquad y_0(0) = 1 \qquad y_0(1) = 5$

Show that the general soln is

(2) $\quad y_0(x) = 4(x + c_1)^{3/2} + c_2$

for arbitrary constants c_1, c_2.

* What I want you to do is verify the solution in (2) satisfies the boundary conditions if $c_1 = 0$, $c_2 = 1$.

That said, "deriving" what c_1 and c_2 must be is difficult. They satisfy

(3) $\quad 4 c_1^{3/2} + c_2 = 1$

(4) $\quad 4(1 + c_1)^{3/2} + c_2 = 5$

Subtract and rearrange yields

$$(1 + c_1)^{3/2} - c_1^{3/2} = 1$$

Squaring this one ultimately finds $c_1 = 0$.
Alternate is $c_1 = \tan^2 u \Rightarrow$

$$\sec^3 u - \tan^3 u = 1$$

can be solved using $\sin z = \frac{1}{2i}(e^{iz} - e^{-iz})$ etc.