# Math 450 (2011) – Homework 6

Due: December 7, 2011. NAME: _____

**1.** [8 pts] For the functionals $J(y)$ defined in a) and b)

i) Define the set of admissible variations $\mathcal{A}^*$ so that $y + h \in \mathcal{A}$ whenever $h \in \mathcal{A}^*$.

ii) Derive a formula for $\delta J(y, h)$ for general $y, h$.

iii) Use your result in ii) to compute $\delta J(\bar{y}, h)$ for the specific $\bar{y}, h$ listed.

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a) Let $J : \mathcal{A} \to \mathbb{R}$ where $\mathcal{A} = \{y \in C[0, \pi] : y'(0) = 1, y'(\pi) = -1\}$. For $\alpha \in [0, \pi]$ define

$$J(y) \equiv y(\alpha)y'(\alpha) \quad , \quad \bar{y}(x) = \sin x \quad , \quad h(x) = \cos x$$

Should get something that looks like the product rule for ii)

b) Let $J : \mathcal{A} \to \mathbb{R}$ where $\mathcal{A} = C^2[0, 1]$.

$$J(y) \equiv \int_0^1 \sqrt{x^2 + 2y(x)y'(x) + 1}\, dx \quad , \quad \bar{y}(x) = x \quad , \quad h(x) = \frac{1}{2}x + 2$$

**2.** [4 pts] Use the Euler Lagrange equations to find the extrema $\bar{y}(x)$ of

$$J(y) \equiv \int_0^1 y(x) + log(1 + y'(x))\, dx$$

over the admissible set

$$\mathcal{A} = \{y : \ y \in C^2[0, 1], \ y(0) = 0, \ y(1) = 0\}$$

**3.** [4 pts] Use the Euler Lagrange equations to find all possible extrema $\bar{y} \in \mathcal{A}$ of

$$\begin{aligned} J(y) &\equiv \int_0^\pi 4y'(x)^2 + 2y(x)y'(x) - y(x)^2\, dx \\ \mathcal{A} &= \{y : \ y \in C^2[0, \pi], \ y'(0) = A, \ y(\pi) = 0\} \end{aligned}$$

for $A = 1$ and $A = 0$. Ask yourself: are there extrema and how many?

**4.** [4 pts] Find the extrema of $J(y)$ defined by

$$\begin{aligned} J(y) &\equiv \int_0^{\pi/4} y'(x)^2\ cos^2 x\ dx \\ \mathcal{A} &= \{y : \ y \in C^1[0, \pi/4], \ y(0) = 1, \ y(\pi/4) = 2\ \} \end{aligned}$$