Math 450 (2009) – Homework 1

Due: September 18, 2009. NAME: _____

1. [20pts] Find the solution of the following initial value problems:

$$y' + tan(x)\ y = sec(x) \quad , \quad y(0) = 1 \tag{1}$$

$$xy' + y = xy^3 \quad , \quad y(1) = 1 \tag{2}$$

$$y'' + 2y' + y = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 1 \tag{3}$$

$$y'' - 4y' + 13y = 0 \quad , \quad y(0) = 0 \quad , \quad y'(0) = 1 \tag{4}$$

2. [5pts] Use the method of Variation of Parameters to find a particular solution of

$$\frac{d^2y}{dx^2} + y = tan(x)$$

3. [5pts] Level curves of a function $f(x, y)$ have tangents whose slope m is given by implicit differentiation:

$$f(x, y) = c \quad \Rightarrow \quad f_x + f_y \frac{dy}{dx} = 0 \quad \Rightarrow \quad m = \frac{dy}{dx} = -\frac{f_x(x, y)}{f_y(x, y)}$$

Thus, curves that are everywhere orthogonal to level curves have a slope that satisfies the differential equation

$$\frac{dy}{dx} = -\frac{1}{m} = \frac{f_y(x, y)}{f_x(x, y)}$$

Use this theory to find the equation of the curves orthogonal to the curves $x^4 + y^4 = c$. Sketch the level curves for a few values of $c > 0$ as well as a few orthogonal curves.

4. [5pts] For $a, b > 0$ the logistic equation

$$\frac{dP}{dt} = aP - bP^2 = birth\ rate - death\ rate$$

well models some organism populations $P(t)$ at time t. Find a formula for $P(t)$ given the initial population $P(0) = P_0$.