Math 450 (2009) – Homework 2

Due: October 2, 2009. NAME: _____

1. [6pts] Deep water oceanic waves of wavelength λ are observed to travel at a constant speed v. Because these waves are deep they are thought to be primarily a gravitational phenomena. Assume that λ, v and the gravitational constant g are related:

$$f(\lambda, v, g) = 0$$

a) Find all the dimensionless parameters π associated with the problem where

$$\pi = \lambda^{\alpha_1} v^{\alpha_2} g^{\alpha_3}$$

b) Use the π-theorem to derive a formula for v in terms of the other dimensional quantities. What power of λ is v proportional to?

2. [12pts] Bubbles rise in fluids. If you've ever seen videos of scuba divers you should have noticed large bubbles rise faster. Thus, the bubble speed v depends on the bubble volume V. The density ρ_0 of the gas in the bubble, the fluid density ρ and gravity g all affect the bubble velocity. Assume all the aforementioned dimensional quantities are related:

$$f(\rho, \rho_0, V, v, g) = 0$$

a) Find all the dimensionless parameters π associated with the problem where

$$\pi = \rho^{\alpha_1} \rho_0^{\alpha_2} V^{\alpha_3} v^{\alpha_4} g^{\alpha_5}$$

b) Use the π-theorem to derive a formula for v in terms of the other dimensional quantities.

c) If a bubble of volume V rises at $4cm/sec$ then how fast does a bubble of four times the volume rise?

3. [4pts] Let q_1, q_2 be two dimensional quantities and $[q_1] = L_1^a L_2^b$, $[q_2] = L_1^c L_2^d$ $(a, b, c, d > 0)$ in terms of some fundamental dimensions L_1, L_2. Show that there are nontrivial dimensionless $\pi = q_1^{\alpha_1} q_2^{\alpha_2}$ if and only if $a/b = c/d$ and that any resulting physical law can be written $q_1 = k\ q_2^{\mu}$ for some constants k and μ.

4. [8pts] Consider an ecosystem of herbivores (plant eaters) and plants. Let H and P be the total carbon biomass (kg) of the herbivores and plants, respectively. Plants produce carbon through photosynthesis at a rate ϕ. A dimensional model of the herbivore-plant carbon mass dynamic is

$$\frac{dP}{d\tau} = \phi - aP - bHP \tag{1}$$

$$\frac{dH}{d\tau} = \epsilon bHP - cH \tag{2}$$

where τ is dimensional time and a, b, c, ϵ, ϕ are all positive constants. The term ϵbHP represents the rate at which the herbivores gain carbon biomass by eating the plants. Likewise, the term bHP represents the loss of biomass in the the total plant population due to the herbivores eating them. The term cH represents the rate at which the herbivores secrete biomass. Plants also lose biomass due to death and other mechanisms as reflected in the term aP.

a) Determine the units of a, b, c, ϵ and ϕ in terms of the fundamental dimensions mass M and time T.

b) Nondimensionalize the model using the lowercase naming convention:

t = dimensionless time

p = dimensionless plant biomass

h = dimensionless herbivore biomass

Choose your scaling so the equation for $\frac{dp}{dt}$ has no parameters whatsoever.