

Math 450 (2009) – Homework 5

Due: November 20, 2009.

NAME: _____

1. [12 pts] Let $y(x, \epsilon)$ be the solution of the nonlinear boundary value problem:

$$\begin{aligned}\epsilon y'' + x(y')^2 - y &= 0 \quad , \quad 0 < x < 1 \\ y(0) &= A \quad , \quad y(1) = B\end{aligned}$$

where $()'$ denotes differentiation with respect to x and $0 <<< 1$.

- a) Let $A = -1$ and $B = 1$. Find all uniformly valid approximations $y_u(x, \epsilon)$ that have a boundary layer at $x = 0$. There are several (two or more) such solutions since the outer problem has several solutions, i.e., the solution is not unique.
- b) Sketch your solutions.

This problem is meant to illustrate that nonlinear problems may have non-unique solutions.

2. [8 pts] Find a uniformly valid approximation to the (singular) initial value problem:

$$\begin{aligned}\epsilon y'' + (t+1)^2 y' &= 1 \quad , \quad t > 0 \\ y(0) &= 1 \quad , \quad \epsilon y'(0) = 1\end{aligned}$$

where $()'$ denotes differentiation with respect to t and $0 < \epsilon << 1$.