```
> restart:
>
```

As derived in the handout on averaging, the system

y'' + y = ε f(y,y',t)

can be transformed to standard form

$$a' = -\,\varepsilon \sin(\psi)\, f(a\cos(\psi), -a\sin(\psi))$$
$$\phi' = -\,\frac{\varepsilon \cos(\psi)\, f(a\cos(\psi), -a\sin(\psi))}{a}$$

where $\psi = t + \phi(t)$ using

$y = a\cos(\psi)$
$y' = -a\sin(\psi)$

We use the equations for a, ϕ above to solve the averaged system in this worksheet.
The worksheet is taylored only for initial conditions

y(0) = a0   y'(0) = 0

```
> f:=(y,dy,tau)->y*dy*sin(tau)+1/2*(y^2-1)*cos(tau);
>
```

$$f := (y, dy, \tau) \rightarrow y\, dy \sin(\tau) + \frac{1}{2}(y^2 - 1)\cos(\tau)$$

The standard form equations are, for $\tau = \varepsilon\, t$

```
> SF1:=subs(psi=tau+phi,-sin(psi)*f(a*cos(psi),-a*sin(psi),tau)):
> SF2:=subs(psi=tau+phi,-1/a*cos(psi)*f(a*cos(psi),-a*sin(psi),tau))
  :
> DSF1:=diff(a(tau),tau)=SF1;DSF2:=diff(phi(tau),tau)=SF2;
```

$$DSF1 := \frac{d}{d\tau}\,\mathrm{a}(\tau) = -\sin(\tau+\phi)\left(-a^2\cos(\tau+\phi)\sin(\tau+\phi)\sin(\tau) + \frac{1}{2}(a^2\cos(\tau+\phi)^2 - 1)\cos(\tau)\right)$$

$$DSF2 := \frac{d}{d\tau}\,\phi(\tau) = -\frac{\cos(\tau+\phi)\left(-a^2\cos(\tau+\phi)\sin(\tau+\phi)\sin(\tau) + \frac{1}{2}(a^2\cos(\tau+\phi)^2 - 1)\cos(\tau)\right)}{a}$$

The averaged equations for A, Φ are

```
> SUB:={a=A,phi=Phi}:
> sf1:=subs(SUB,SF1):
> sf2:=subs(SUB,SF2):
> AV1:=factor(int(sf1,tau=0..2*Pi)/(2*Pi)):
> AV2:=factor(int(sf2,tau=0..2*Pi)/(2*Pi)):
> DAV1:=diff(A(tau),tau)=AV1;DAV2:=diff(Phi(tau),tau)=AV2;
```

$$DAV1 := \frac{d}{d\tau}\,\mathrm{A}(\tau) = -\frac{1}{128}\,\frac{A^2 \cos(3\,\Phi) + 3\cos(\Phi)\,A^2 - 4\,A^2\cos(\Phi)^3 - 32\sin(\Phi)\,\pi + 24\sin(\Phi)\,A^2\,\pi}{\pi}$$

$DAV2 :=$

$$\frac{d}{d\tau}\,\Phi(\tau) = \frac{1}{128}\,\frac{A^2\sin(3\,\Phi) + A^2\sin(\Phi) - 4\,A^2\sin(\Phi)\cos(\Phi)^2 + 32\cos(\Phi)\,\pi - 8\cos(\Phi)\,A^2\,\pi}{A\,\pi}$$

whose solution for $\mathrm{A}(0) = a_0$ and $\Phi(0) = \phi_0$

IF the averaged equations are UNCOUPLED use:

```
> SUB1:={A=A(tau),Phi=Phi(tau)}:
> dsolve({subs(SUB1,DAV2),Phi(0)=phi0},Phi(tau)):
Warning, it is required that the numerator of the given ODE depends on the highest
derivative. Returning NULL.
```

```
> dsolve({subs(SUB1,DAV1),A(0)=a0},A(tau)):
Warning, it is required that the numerator of the given ODE depends on the highest
derivative. Returning NULL.
```

```
>
```