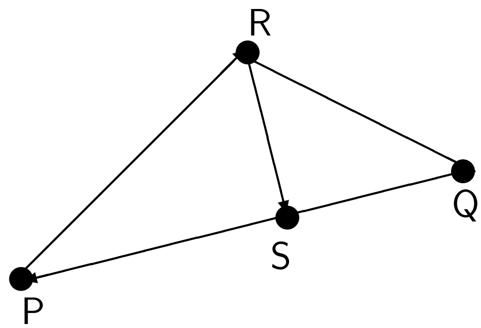
MATH 224-02 — Sample Test A
for Chapter 12

(1) A vector $\overrightarrow{u}$ of length 1 points northeast, and a vector $\overrightarrow{v}$ of length 2 points southeast. Suppose that the north direction points in the usual way — up! Sketch these vectors and the vector $\overrightarrow{v} - 2\overrightarrow{u}$.

(2) Suppose that $\overrightarrow{QP} = -3\,\overrightarrow{i} - \overrightarrow{j}$ and $\overrightarrow{RQ} = 2\,\overrightarrow{i} - \overrightarrow{j}$ and that the vector $\overrightarrow{RS}$ is perpendicular to $\overrightarrow{PQ}$. Find the vectors $\overrightarrow{PR}$ and $\overrightarrow{RS}$ indicated in the picture below.



(3) Find the symmetric equations of the line joining $(0, 1, 1)$ and $(-2, 2, 4)$.

(4) Find the equation of the plane P that contains the point $(0, 1, 0)$ and also contains the line with parametric equations

$$x = 2t - 1, \quad y = t + 3, \quad z = -t + 1.$$

(5) Find the volume of the box generated by the three vectors
$\overrightarrow{a} = 2\overrightarrow{\imath} + \overrightarrow{\jmath} - 2\overrightarrow{k}$, $\overrightarrow{b} = -2\overrightarrow{\jmath} + \overrightarrow{k}$, and $\overrightarrow{c} = \overrightarrow{\imath} + \overrightarrow{\jmath}$

SELECTED SOLUTIONS

(1) I will leave # 1 to you.

(2) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = -\overrightarrow{QP} - \overrightarrow{RQ} = 3\overrightarrow{\imath} + \overrightarrow{\jmath} - 2\overrightarrow{\imath} + \overrightarrow{\jmath} = \overrightarrow{\imath} + 2\overrightarrow{\jmath}$
$\overrightarrow{RS} = \overrightarrow{PS} - \overrightarrow{PR} = \overrightarrow{proj}_{\overrightarrow{PQ}}\overrightarrow{PR} - \overrightarrow{PR}$
$= \dfrac{(3\overrightarrow{\imath} + \overrightarrow{\jmath}) \cdot (\overrightarrow{\imath} + 2\overrightarrow{\jmath})}{10}(3\overrightarrow{\imath} + \overrightarrow{\jmath}) - (\overrightarrow{\imath} + 2\overrightarrow{\jmath}) = (1/2)\overrightarrow{\imath} - (3/2)\overrightarrow{\jmath}$
Check Work: This is perpendicular to $\overrightarrow{PQ}$.

(3) Line is parallel to $\langle -2, 1, 3\rangle$ (subtract the two points) and goes through $(0, 1, 1)$. So we get parametric equations, and solve for t to get symmetric equations:
$x = -2t + 0, \quad y = 1t + 1, \quad z = 3t + 1 \Longrightarrow$

$$t = \frac{x-0}{-2} = \frac{y-1}{1} = \frac{z-1}{3}$$

(4) The line is parallel to $\langle 2, 1, -1\rangle$ and contains the point $(-1, 3, 1)$ (for example, put $t = 0$). The plane also contains $(0, 1, 0)$. So the vector between those points $\langle 1, -2, -1\rangle$ is parallel to the plane. If we cross the line vector with $\langle 1, -2, -1\rangle$ we get a normal vector to the plane. That cross product is $\langle -3, 1, -5\rangle$ (you can check). Now we have both a point on the plane $(0, 1, 0)$ and a normal vector. The plane is $3x - (y - 1) + 5z = 0$.

(5) This is the absolute value of the triple scalar product of the three vectors. The volume equals 5.