MATH 224-02 Sample Test 2

Chapter 13

(1) (15 pts) Match the vector function $\vec{r}(t)$ on the left with the correct description on the right: No justification required.

______	$(\mathrm{a})\langle t^3, t^3, t^3\rangle$	$(-\infty, \infty)$	(i)A helix
______	$(\mathrm{b})\langle 2\sin t, 3t, 2\cos t\rangle$	$[0, 2\pi]$	(ii)intersection: cylinder and plane
______	$(\mathrm{c})\langle \cos t, \sin t, \cos t + \sin t\rangle$	$[0, 2\pi]$	(iii)A straight line

(2) (20 pts) Find parametric equations for the tangent line to the curve

$$\vec{r}(t) = \langle \sqrt{2-t}, e^{t-1}, \ln t\rangle, \quad 0 < t < 2,$$

at the point $(1, 1, 0)$

(3) Find the length of N turns of the helix $\vec{r}(t) = \langle R\sin at, R\cos at, bt\rangle$.Express answers in terms of a, b, N and R.

(4) (30 pts) Given $\vec{r}(t) = \langle \frac{t^2}{2}, t, t\rangle$, for all t:
  (a) Find the unit tangent vector $\vec{T}(0)$ (means $\vec{T}(t)$ at $t = 0$)
  (b) Find the unit normal vector $\vec{N}(0)$
  (c) Find the curvature $\kappa(0)$

(5) (15 pts) A curve is given by $\vec{r}(t) = \langle \cos t, \sin t, -\cos 2t\rangle$
  (a) Find the velocity at time t
  (b) Find the acceleration at time t
  (c) Find the tangential component of the acceleration at time t.

<u>*SOLUTIONS*</u>: Let me know if you find errors!

(1) (iii)↔(a) and (ii)↔(c) and (i)↔(b)

(2) The curve passes through $(1, 1, 0)$ when $t = 1$. $\vec{r}'(t) = \langle -(1/2)(2-t)^{-1/2}, e^{t-1}, 1/t\rangle$ so $\vec{r}'(1) = \langle -1/2, 1, 1\rangle$ is the tangent vector. The parametric equations are $x = 1 - t/2$, $y = 1 + t$, $z = t$.

(3) The helix has N turns when the angle is $at = 2\pi N$ so the maximal t-value is $t = 2\pi N/a$. The length is

$$L = \int_0^{2\pi N/a} |\vec{r}'(t)|\, dt = \int_0^{2\pi N/a} \sqrt{R^2a^2 + b^2}\, dt = \sqrt{R^2a^2 + b^2}\,(2\pi N)/a$$

(4) For this problem
  (a) $\vec{T} = \langle t, 1, 1\rangle/(\sqrt{t^2+2}$
  (b) First we need the direction

$$\vec{T}'(t) = \langle t, 1, 1\rangle(t^2+2)^{-3/2}(-1/2)(2t) + \langle 1, 0, 0\rangle(t^2+2)^{-1/2}$$

  At $t = 0$ we get $(\langle 0, 1, 1\rangle)(0) + \langle 1/\sqrt{2}, 0, 0\rangle$ So the unit normal is $\vec{N}(0) = \vec{i}$.
  (c) for curvature, use

$$\frac{|\vec{v} \times \vec{a}|}{v^3} = \left|\frac{1}{(t^2+2)^{3/2}}\langle t, 1, 1\rangle \times \langle 1, 0, 0\rangle\right|_{t=0} = \left|\frac{1}{2^{3/2}}(\vec{j} - \vec{k})\right| = 1/2\ .$$

(5) For this curve:
  (a) $\vec{v} = \langle -\sin t, \cos t, 2\sin 2t\rangle$
  (b) $\vec{a} = \langle -\cos t, -\sin t, 4\cos 2t\rangle$
  (c) $a_T = d^2s/dt^2 = (\sqrt{1 + 4(\sin 2t)^2})' = 8(\sin 2t)(\cos 2t)(1 + 4(\sin 2t)^2)^{-1/2}$