Sample Test B for Chapter 13

(1) (15 pts) Match the vector function $\overrightarrow{r}(t)$ on the left with the correct description on the right. Some justification is required. A description (i)-(iii) might be used twice or not at all.

_____	(a)$\langle \cos t + \sin t, \cos t - \sin t, 5 \rangle$	$(-\infty, \infty)$	(i)A parabola
_____	(b)$\langle \sin t, -2\sin t, 0 \rangle$	$[0, \pi/2]$	(ii)intersection: cylinder and plane
_____	(c)$\langle t^3, t^6, 0 \rangle$	$(-\infty, \infty)$	(iii)A line segment

(2) (20 pts) Find parametric equations for the tangent line to the curve

$$\overrightarrow{r}(t) = \langle \cos t, te^t, \sin t \rangle, \quad , \ -\pi < t < \pi,$$

at the point $(1, 0, 0)$.

(3) (20pts) Given a helix $\langle 4\cos t, 4\sin t, 3t \rangle$ of length $L = 30\pi$, how many turns does it take around the z-axis?

(4) (30 pts) Given the position vector $\overrightarrow{r}(t) = \langle \cos 2t, \sin 2t, t^2 \rangle$, $-\infty < t < \infty$:

(4a) Find the unit tangent vector $\overrightarrow{T}(0)$.

(4b) Find the unit normal vector $\overrightarrow{N}(0)$.

(4c) Find the curvature $\kappa(0)$.

(5) (15 pts) A curve is given by $\overrightarrow{r}(t) = \langle 2e^t, -e^{-t}, 2e^t \rangle$

(5a) Find the velocity $\overrightarrow{v}$ at time t.

(5b) Find the acceleration $\overrightarrow{a}$ at time t.

(5c) Find $a_N = \dfrac{|\overrightarrow{v} \times \overrightarrow{a}|}{v}$.

SOLUTIONS

(1) Part (a) is matched by (ii). The surface $z = 5$ is a plane. We find that $x^2 + y^2 = 2(\cos^2 t + \sin^2 t) = 2$. So (x, y, z) forms a cylinder of radius $\sqrt{2}$. Part (b) is matched by (iii), since $y = -2x$, $z = 0$ is a straight line. It is a segment because x is bounded between 0 and 1. Part (c) is matched by (i) since $y = x^2$.

(2) To get a tangent line, we need the tangent vector $\overrightarrow{r'}(t)$ at $(1, 0, 0)$. If $(x, y, z) = (1, 0, 0)$, then $te^t = 0$ so $t = 0$ is the only solution. The derivative $\overrightarrow{r'}(0) = \langle 0, 1, 1\rangle$

(3) Line is parallel to $\langle -2, 1, 3\rangle$ (subtract the two points) and goes through $(0, 1, 1)$. So we get

$$x(t) = 1, \quad y(t) = t, \quad z(t) = t$$

for the parametric equations.

(4) If the helix starts at $t = 0$ and stops at $t = T$ then the length of the helix is

$$30\pi = \int_0^T \sqrt{x'^2 + y'^2 + z'^2}\, dt = \int_0^T \sqrt{4^2 + 3^2}\, dt = 5T$$

Frp, $5T = 30\pi$, we get $T = 6\pi$. Since one turn corresponds to a change in angle of 2π, this helix has exactly 3 turns.

(5) (a) The velocity is $\overrightarrow{v} = \overrightarrow{r'} = \langle 2e^t, e^{-t}, 2e^t\rangle$.

(b) The acceleration is $\overrightarrow{a} = \overrightarrow{v'} = \langle 2e^t, -e^{-t}, 2e^t\rangle$.

(c) $|\overrightarrow{a} \times \overrightarrow{v}| = |-4\overrightarrow{i} + 4\overrightarrow{k}| = 4\sqrt{2}$ and $v = \sqrt{8e^{2t} + e^{-2t}}$. So

$$a_N = \frac{4\sqrt{2}}{\sqrt{8e^{2t} + e^{-2t}}}$$