MATH 441 HW Assignment 3

Due Thursday, Oct. 7, 2004

1. To work Exercise 7 of HW Assignment #2, you should have provided pseudo-code for the solution of a tridiagonal system $A\mathbf{x} = \mathbf{b}$, given the lower bidiagonal matrix L and upper bidiagonal matrix U for which $A = LU$. You are to implement this pseudo-code in MATLAB. Write a MATLAB function named `trilu_solve.m` which has the following first line of code:

   ```
   function x = trilu_solve(Lsub,Udiag,Usuper,b)
   ```

   Here `Lsub` contains the lower subdiagonal of L, `Udiag` contains the main diagonal of U, and `Usuper` contains the upper superdiagonal of U. I have provided MATLAB code to test your code. Be sure to hand in (i) a listing of your matlab function `trilu_solve.m`, and (ii) output obtained by running your code on the test problem.

2. You are to compute the induced $1-$, $2-$, and $\infty-$norms and corresponding condition numbers of the following matrix A:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} \quad \text{Note: } A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & -4 & 3 \\ -2 & 11 & -6 \\ 3 & -6 & 3 \end{bmatrix}.$$

3. Find vectors for which each of the induced norms in Exercise 2 are attained, i.e., find a particular vector $\mathbf{x}$ for which $||\mathbf{x}||_1 = 1$ and $||A\mathbf{x}||_1 =$ the max absolute column sum of A, and similarly for the other 2 norms. You may use MATLAB to compute the eigenvalues and corresponding eigenvectors of $A^T A$. The proof on p. 119 and the results of the excercise at the top of p. 120 of the text may prove useful.

4. As in the notes of 9/30/04, consider the matrix

$$A = \begin{bmatrix} \alpha & 1 \\ 1 & 2 \end{bmatrix},$$

   where $0 < \alpha \ll$ machine epsilon, and both α and $1/\alpha$ are machine representable. Give the matrices $\hat{L}$, $\hat{U}$, and E for which $A + E = \hat{L}\hat{U}$ and $\hat{L}$, $\hat{U}$ are the computed lower and upper triangular factors obtained when LU-factorization with partial pivoting is applied to A. For this matrix and this algorithm, would you expect an accurate computed solution to a system $A\mathbf{x} = \mathbf{b}$, using your computed LU factorization with partial pivoting? Justify your answer in terms of the error bound (2.3.10) on the bottom of p. 135 of the text.