MATH 441 HW Assignment 4

Due Tuesday, Oct. 19, 2004

1. Suppose you are given (t,y) data pairs (1,1), (2,2), (2,3), (3,5), (3,3), (4,7), (5,10). You are to fit a degree 2 (quadratic) polynomial $p(t) = c_1 + c_2 t + c_3 t^2$ to this data in a manner which minimizes the sum of squares of the residuals. You may wish to modify the MATLAB code in `Demo_ls.m` to accomplish this.

   a. Give the "design matrix" A and the data vector $\mathbf{y}$ for the least squares problem corresponding to this data fitting problem. Also, explain how the least squares solution vector $\mathbf{x}_{\mathrm{LS}}$ relates to the data fitting problem.

   b. Give the coefficient matrix and the right hand side vector whose solution gives the coefficients of the best-fit polynomial. In other words, compute the matrix and vector corresponding to the "normal equations".

   c. Use MATLAB's *chol* function, which gives the Choleski factorization, to solve the matrix equation in part b, giving $\mathbf{x}_{\mathrm{LS}}$.

   d. Plot the data and the best-fit polynomial.

   e. In addition to the material requested above, hand in a listing of MATLAB code used to solve this problem and generate plots.

2. You are to write a MATLAB function to compute the reflection matrix Q which, when applied to a vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, yields $Q\mathbf{x} = (r, 0, \ldots, 0)$. Your function should have the following first line of code:

   ```
   function [u,Q] = reflection_mat(x)
   ```

   The return argument `u` should contain the unit vector $\mathbf{u}$ for which $Q = I - 2\mathbf{u}\mathbf{u}^T$. You are to hand in (i) a listing of your MATLAB function `reflection_mat.m`, and (ii) MATLAB output that clearly demonstrates that your code works, by computing $Q\mathbf{x}$ for $\mathbf{x} = (1, 2, 3, 4)$.

3. You are to write a MATLAB function to solve linear least squares problems using reflection matrices. You are to then use this function to solve the least squares problem in Exercise 1. Your function should have the following first line of code:

   ```
   function xLS = ls_reflection(A,y)
   ```

   You are to hand in (i) a listing of your MATLAB function `ls_reflection.m`, and (ii) MATLAB output that clearly demonstrates that your code solves the least squares problem in Exercise 1.

4. Work Exercise 3.2.27 on p. 196 of the text.

5. Prove that the product of $n \times n$ orthogonal matrices is an orthogonal matrix.

6. *Extra Credit.* Use the orthogonality characterization

$$\langle \hat{\mathbf{y}} - \mathbf{y}, \mathbf{s} \rangle = 0 \quad \text{for all} \quad \mathbf{s} \in \text{Range}(A)$$

to derive the normal equations $A^T A\mathbf{x} = A^T\mathbf{y}$, which characterize least squares solutions,

$$\mathbf{x}_{\text{LX}} = \arg\min_{\mathbf{x}} ||A\mathbf{x} - \mathbf{y}||_2.$$