# MATH 441 HW Assignment 7

**Due Tuesday, Dec. 7, 2004**

1. Let $F(x) = x^2 - 1$. Compute 2 iterations of Newton's method *by hand* to solve $F(x) = 0$ with initial guess $x_0 = 4$. Also, plot $F(x)$ together with the linear approximations

$$L_k(x) = F(x_k) + F'(x_k)(x - x_k), \quad k = 0, 1,$$

on the interval $0 \le x \le 4$. Clearly label the initial guess $x_0$ and the Newton approximations $x_1$, $x_2$ in your plot. You may use MATLAB to generate the plots if you'd like.

2. Rework problem 1 with the secant method in place of Newton's method. At the initial iteration you don't have two points with which to compute a secant approximation to the slope, so use the true slope. In other words, take $A_0 = F'(x_0)$.

3. You are to formulate and implement a hybrid bisection-Newton algorithm to solve one dimensional nonlinear equations $F(x) = 0$. Hand in

   a. A written description of the strategy that you use to combine bisection with Newton's method to find a solution to $F(x) = 0$ on a given interval $a \le x \le b$.

   b. Pseudo-code which corresponds to the description in part a.

   c. MATLAB code to implement the pseudo-code in part b. Include stopping criteria.

   d. A demonstration that your MATLAB codes works by finding the solution $x_*$ to $F(x) = \arctan(x) - .1 = 0$ on the interval $-2 \le x \le 5$ with initial guess $x_0 = 5$. Stop the iteration when $|F(x_k)| \le 10^{-9}$. Put together a table that contains the iteration counts $k$, the approximate solutions $x_k$ and function values $F(x_k)$, the solution errors $e_k = x_k - x_*$, and whether a Newton or bisection step was taken.

4. Modify the quasi-Newton convergence analysis in the handout of 11/30/04 to handle the case of nonlinear systems of equations $\mathbf{F}(\mathbf{x}) = \mathbf{0}$, where $\mathbf{F} : R^n \to R^n$. The properties of vector and matrix norms in Section 2.1 of the text may prove useful.

5. Apply one iteration of Newton's method with initial guess $\mathbf{x}_0 = (2, 1)$ to solve $\mathbf{F}(\mathbf{x}) = \mathbf{0}$, where $\mathbf{F} : R^2 \to R^2$ has components

$$\begin{aligned} f_1(x_1, x_2) &= x_1 + x_2 - 1 \\ f_2(x_1, x_2) &= x_1^2 + x_2^2 - 1 \end{aligned}$$

You are to do the work by hand, but you may use MATLAB to check your results. Clearly label $\mathbf{F}(\mathbf{x}_0)$, $\mathbf{F}'(\mathbf{x}_0)$, and $\mathbf{x}_1$. Also, clearly state what method you use to do the linear algebra, and provide details.