Example of Effects of Floating Point Error on Numerical Computations

Here is MATLAB code for computing a divided difference approximation to the derivative of e^x at $x_0 = 1$,

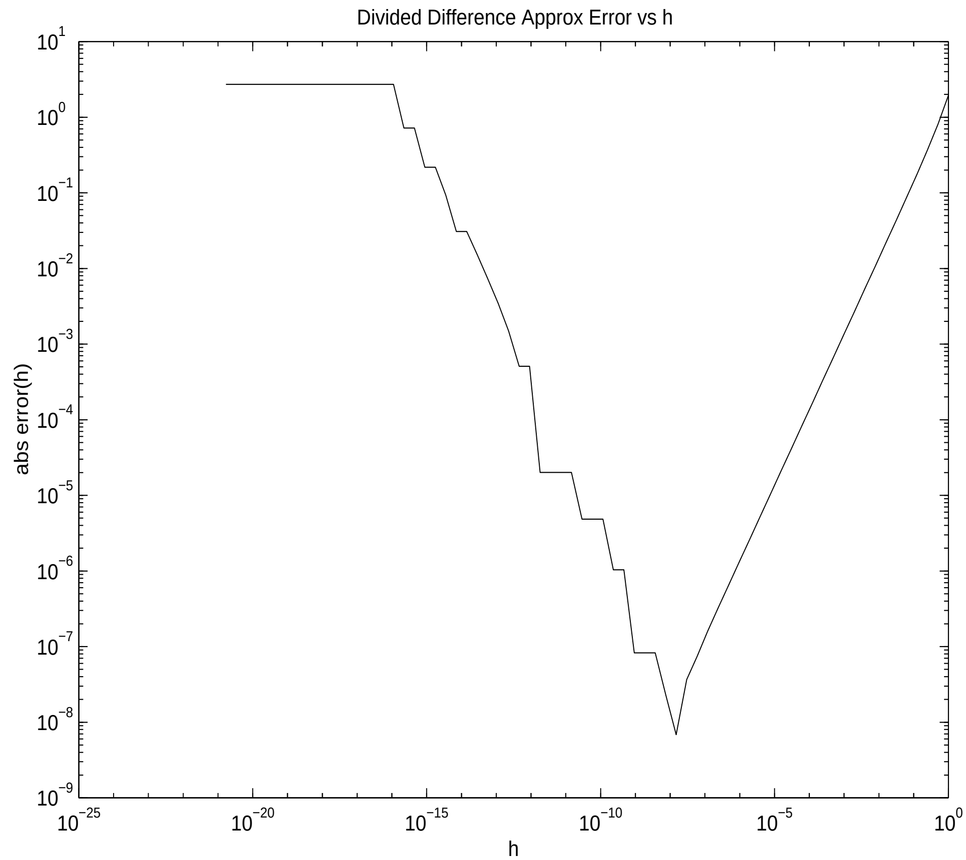
$$\frac{d}{dx} e^x|_{x=1} \approx \frac{e^{1+h} - e^1}{h}.$$

```
h = 1;  n = 70;
x0 = 1;
expx0 = exp(x0);
hvec = zeros(n,1);
evec = zeros(n,1);
true_deriv = expx0;
for i = 1:n
  ddif = (exp(x0+h) - expx0)/h;       %  Compute divided difference approx.
  abs_error = abs(ddif - true_deriv); %  Compute abs. value of approx. error.
  hvec(i) = h;
  evec(i) = abs_error;
  h = h/2;                            %  Decrease step size h.
end

loglog(hvec,evec)
xlabel('h'), ylabel('abs error(h)')
title('Divided Difference Approx Error vs h')
```

Here is a loglog plot of the resulting absolute value of the error of approximation,

$$\text{abs error}(h) = \left| \frac{e^{1+h} - e^1}{h} - e^1 \right|.$$



Note that when h is not "too small", the error of the divided difference approximation decreases linearly as h decreases. This is consistent with the results of an analysis based on Exercise 6 of HW#1. However, as h continues to decrease, the error begins to *increase*, and then it stagnates, i.e., it remains constant all h below some threshold.