

# Math 450 Assignment 1

Due Monday, Sept. 15, 2003

1. Compute the solution to the ODE IVP

$$\begin{aligned}\frac{d^2u}{dt^2} + 4u &= 0, \quad t > 0 \\ u(0) &= 2 \\ \frac{du}{dt}(0) &= 1.\end{aligned}$$

Then plot the solution for  $0 \leq t \leq 4\pi$ .

2. Compute the solution to the ODE IVP

$$\begin{aligned}\frac{d^2u}{dt^2} + 6\frac{du}{dt} + 25u &= 0, \quad t > 0 \\ u(0) &= 1 \\ \frac{du}{dt}(0) &= 0.\end{aligned}$$

Then plot the solution for  $0 \leq t \leq \pi$ .

3. Find the general solution to the ODE

$$\frac{d^4u}{dt^4} + 2\frac{d^2u}{dt^2} + u = 0.$$

Hint: In the characteristic equation, substitute  $\gamma = \lambda^2$ .

4. Let  $D = \frac{d}{dt}$  and let  $\lambda$  be a (possibly complex) constant. Verify that

$$(D - \lambda)^2(te^{\lambda t}) = 0.$$

First show that  $(D - \lambda)(te^{\lambda t}) = e^{\lambda t}$ .

5. Consider the ODE

$$m\frac{d^2u}{dt^2} + c\frac{du}{dt} + ku = 0,$$

where  $m, k > 0$  and  $c \geq 0$  are constants. Let  $\lambda_1, \lambda_2$  be solutions to the characteristic equation

$$m\lambda^2 + c\lambda + k = 0.$$

Give conditions on  $m, c, k$  for which

- a.  $\lambda_1, \lambda_2$  are complex conjugate pairs (in this case the mechanical system modeled by the ODE is called *underdamped*).
- b.  $\lambda_1, \lambda_2$  are real and distinct (the system is called *overdamped*).
- c.  $\lambda_1, \lambda_2$  are real and equal (the system is called *critically damped*).

Then in each of the 3 cases, give 2 linearly independent solutions to the ODE.

6. Use the eigenvalue approach to solve the following first order linear ODE IVP system:

$$\begin{aligned}\frac{du}{dt} &= -u, \quad t > 0 \\ \frac{dv}{dt} &= u - 3v, \\ u(0) &= 1, \\ v(0) &= 0.\end{aligned}$$

Then for  $0 \leq t \leq 4$ , plot both components  $u(t), v(t)$  of the solution.