Math 450 Assignment 7

Due Friday, Nov. 21, 2003

1. (This is required for Math students who have had Advanced Calculus or Real Analysis. It is optional for everyone else.) Let J map a normed linear space V into $\mathbf{R}$. Assume that for each $v, h \in V$, $\tilde{J}(\epsilon) \equiv J(v+\epsilon h)$ is continuously differentiable. Prove that if $v^* = \arg\min_{v \in V} J(v)$ exists, then
$$\delta J(v^*, h) \equiv \frac{d}{d\epsilon} J(v+\epsilon h)|_{\epsilon=0} = 0 \quad \text{for all} \;\; h \in V.$$

2. Work Exercise 1.1 on p. 112 of Logan.

3. Let $J : C[0,\pi] \to \mathbf{R}$ be given by
$$J(f) = ||f - x||_2^2 \equiv \int_0^\pi [f(x) - x]^2 \, dx, \qquad f \in C[0,\pi].$$
Take the "admissible set" $\mathcal{A}$ to be the subspace of $C[0,\pi]$ spanned by (i.e., all linear conbinations of) $\sin x,\ \sin 2x,\ \sin 3x$. Find the best least squares approximation to x from $\mathcal{A}$, which is
$$f^* = \arg\min_{f \in \mathcal{A}} J(f)$$
Then plot $f^*(x)$ and x on the interval $[0,\pi]$.

4. Let A be an $m \times n$ matrix and let $\vec{y}$ be an $m \times 1$ vector. Let $J : \mathbf{R}^n \to \mathbf{R}^1$ be given by
$$J(\vec{x}) = ||A\vec{x} - \vec{y}||_2^2 = (A\vec{x} - \vec{y})^T (A\vec{x} - \vec{y}).$$
Derive a formula for $\vec{x}^* = \arg\min_{\vec{x} \in \mathbf{R}^n} J(\vec{x})$ in terms of A and $\vec{y}$. You may assume that $A^T A$ is nonsingular.