M451 Sample Exam Problems

Friday, March 5, 2004

1. Use Laplace transforms, combined with complex variables methods for inversion of the transform, to obtain an analytic expression for the solution to the following ODE IVP.

$$\frac{d^2u}{dt^2} + 4\frac{du}{dt} + 5u = 0, \quad t > 0,$$
$$u(0) = u_0,$$
$$\frac{du}{dt}(0) = v_0.$$

Express your solution in terms of real-valued quantities.

2. Find the eigenvectors and corresponding eigenvalues for the Sturm-Louisville operator

$$\mathcal{A}u = \frac{d^2u}{dx^2}, \; 0 < x < 1, \quad u(0) = \frac{du}{dx}(1) = 0.$$

Use these to obtain representations for the solutions to the following:

(i) $-\frac{d^2u}{dx^2} + 4u(x) = f(x)$, $0 < x < 1$, $\quad u(0) = \frac{du}{dx}(1) = 0$.

(ii) $u_t = \kappa u_{xx} + f(x,t)$, $0 < x < 1$, $t > 0$; $u(0,t) = u_x(1,t) = 0$, $t > 0$; $u(x,0) = u_0(x)$, $0 < x < 1$. κ is a positive constant.

3. Solve the initial-boundary value problem for the advection equation

$$u_t - u_x = 0, \quad -\infty < x < \infty, \; t > 0,$$
$$u(x,t) \to 0, \quad x \to \pm\infty, \; t > 0,$$
$$u(x,0) = f(x), \quad -\infty < x < \infty.$$

Provide a physical interpretation of your result.