M451 Homework Assignment 8

Due Friday, April 23, 2004

1. Use the methods of characteristics to solve

$$u_t - xu_x + 2u = 0, \quad -\infty < x < \infty, \ t > 0,$$
$$u(x,0) = f(x) = \begin{cases} 1, & -1 \le x \le 1, \\ 0, & |x| > 1 \end{cases}, \qquad -\infty < x < \infty.$$

   Be sure to (i) give analytic formulas for the characteristic curves $(x(\tau), t(\tau)),\ \tau \ge 0$, and plot them for the 4 initial values $x(0) = x_0 = 0,\ 1,\ -1$, and 2; and (ii) generate a mesh plot of the solution $u(x,t)$ for $-2 \le x \le 2,\ 0 \le t \le 2$.

2. Use the method of characteristics to solve the two-dimensional linear advection equation

$$u_t - yu_x + xu_y = 0, \quad -\infty < x, y < \infty, \ t > 0,$$
$$u(x,y,0) = f(x,y), \qquad -\infty < x, y < \infty.$$

   (a) For $f(x,y) = \exp[-(x-1)^2/4 - (y-1)^2/4]$, plot "snapshots" of your solution at times $t = 0, \pi/4, \pi/2$.

3. Work Exercise 1.9 on p. 323.

4. Work Exercise 1.10 on p. 323.

5. Let $\vec{V}(\mathbf{x}) = (R(\mathbf{x}), S(\mathbf{x}), T(\mathbf{x}))$, where $\mathbf{x} = (x_1, x_2, x_3)$. Confirm the vector identity

$$\text{curl (curl } \vec{V}) = \text{grad (div } \vec{V}) - \Delta \vec{V},$$

   where Δ denotes the Laplacian operator applied to each component of $\vec{V}$.

6. Consider Maxwell's equations for a nonconducting medium with no free charges. Show that with nonconstant permitivity $\epsilon = \epsilon(\mathbf{x})$, the electric field satisfies

$$\Delta \vec{E} - \epsilon(\mathbf{x})\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \text{grad } (\vec{E} \cdot \text{grad } \ln \epsilon(\mathbf{x})).$$