M551 Homework

Due Monday, March 31, 2003

1. You are to supply a direct proof of Theorems V.1.1 and V.1.2 on p. 156-157. With C_R as in the book,

   a. Using the fact that $|\int g| \leq \int |g|$, show that

   $$|f_n(z) - \frac{1}{2\pi i} \int_{C_R} \frac{f(w)}{w-z}\, dw| \to 0, \quad |z - z_0| \leq R/2,$$

   and explain why the integral expression above must be $f(z)$.

   b. Show (repeat of previous homework, but include for completeness) that as $h \to 0$,

   $$\frac{f(z+h) - f(z)}{h} \to \frac{1}{2\pi i} \int_{C_R} \frac{f(w)}{(w-z)^2}\, dw = f'(z), \quad |z - z_0| \leq R/2.$$

   c. Explain why $f_n'(z)$ has a (local) Cauchy-like integral representation and use it to verify that $f_n'(z) \to f'(z)$ uniformly for $|z - z_0| \leq R/2$.

2. Work Exercise V.1.1, p. 158. *Hint:* Expand $f(z)$ as a power series about z_0 and use the approach in the notes of 1/31/03 to construct a power series for the reciprocal of $f(z) - f(z_0)$. Explain why this power series converges uniformly for some (small) ball about z_0. Then explain why you can interchange $\sum$ and $\int$ to compute the integral.

3. Work Exercise V.1.2, p. 158. *Hint:* Use Theorem V.1.1 to obtain a contradiction.

4. Work Exercises V.1.3ac and 7 on p. 158-159.

Recommended exercises on p. 158-160: 4, 5, 6, 9, 10, 11.