1. Compute the following in exact form.
   (a) $\cos^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{3}$
   (b) $\cos^{-1} \left( \frac{\sqrt{3}}{2} \right) = \frac{\pi}{6}$
   (c) $\sin^{-1} \left( -\frac{\sqrt{3}}{2} \right) = -\frac{\pi}{3}$
   (d) $\sin^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{6}$
   (e) $(\tan \frac{\pi}{3})^{-1} = \frac{1}{\sqrt{3}}$

2. Find the exact values of the following expressions. Do not use a calculator.
   Hint: Think about the domains.
   (a) $\tan^{-1}(1) = \frac{\pi}{4}$
   (b) $\tan(\tan^{-1}(10)) = 10$
   (c) $\sin^{-1}(\sin(\frac{7\pi}{3})) = \frac{\pi}{3}$ [Note: $\frac{7\pi}{3}$ is not the answer]
   (d) $\tan(\sin^{-1}(0.8)) = \frac{4}{3}$

3. Give a simple expression for $\sin(\cos^{-1}(x))$ which has no trigonometric functions.
   Hint: Draw a right triangle.

\[
\sin(\cos^{-1}(x)) = \sqrt{1-x^2}
\]
4. When batted, a baseball leaves the bat at an angle of $\theta$ with the horizontal and an initial velocity of $v_0 = 100$ feet per second. An outfielder catches the ball 300 feet from home plate. Find $\theta$, given that the range, $R$, of the projected baseball is modeled by the equation

$$R = \frac{(v_0)^2 \sin(2\theta)}{32}$$

$$300 = \frac{100^2 \sin(2\theta)}{32}$$

$$\frac{32 \cdot 3}{100} = \sin(2\theta)$$

$$2\theta = \sin^{-1}(0.96)$$

$$\theta = \frac{1}{2} \sin^{-1}(0.96) \approx 36.9^\circ$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

5. A triangular walking course has 2 sides of 230 feet and 360 feet, and the angle between these sides measures $38^\circ$. Find the length of the third side of the course.

$$x^2 = 230^2 + 360^2 - 2(230)(360) \cos 38^\circ$$

$$x = \sqrt{230^2 + 360^2 - 2(230)(360) \cos 38^\circ} \text{ ft}$$

$$\approx 228 \text{ ft}$$

6. You are standing looking at a large painting on the wall. The bottom of the painting is 1 foot above your eye level. The painting is 10 feet tall. Assume you are standing $x$ feet from the painting and that angle $\theta$ is formed by the lines of vision to the bottom and to the top of the painting.

(a) Solve for $\theta$ in terms of $x$.

$$\theta = -\tan^{-1}\left(\frac{11}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$$

(b) If you are standing 10 feet from the painting, what is $\theta$?

$$\theta = -\tan^{-1}(1.1) - \tan^{-1}(1) \approx 42^\circ \text{ or } 0.733 \text{ rad.}$$