Weekly Write-Up on Sections 3.3-3.6

Problem 1. From the suggested homework:

[3.4 #34] Suppose \( \theta(t) \) measures the angle between a clock’s minute and hour hands where \( \theta \) is in radians and \( t \) in in hours. What is \( \theta'(t) \) at 3 o’clock?

\[
\frac{d\theta}{dt} = \left( \frac{\text{angular speed of hour hand}}{\text{angular speed of minute hand}} \right)
\]

\[
\frac{d\theta}{dt} = \frac{2\pi/12 \, \text{radians}}{1 \, \text{hr}} - \frac{2\pi \, \text{radians}}{1 \, \text{hr}}
\]

\[
\frac{d\theta}{dt} = -\frac{11\pi}{6} \, \text{radians/hr}
\]

Problem 2. From an old exam:

A peg on the end of a rotating crank slides freely in the vertical guide shown in the diagram. The guide is rigidly connected to a piston which moves horizontally. The crank rotates at a constant angular speed of 1 Rad/s in a circle of radius \( r \).

The position of the piston is given by

\( P(t) = L + r \cos(t) \).

a. Find the velocity, \( \frac{d}{dt} P(t) \).

\[
P'(t) = 0 + r (-\sin t)
\]

\[
P'(t) = -r \sin t
\]

b. Find the acceleration of the piston, \( \frac{d^2}{dt^2} P(t) \).

\[
P''(t) = -r \cos t
\]
Problem 3. For in-class discussion:
The growth rate of bacteria in bacterial cultures is often modeled by the Monod (1949) equation:

\[ G(S) = \mu \frac{S}{K + S} = \frac{\mu S}{K + S} \]

where \( G \) (1/seconds) is the specific growth rate of the microorganisms, \( \mu \) (1/seconds) is the maximum specific growth rate of the microorganisms, \( S \) (mol/L) is the concentration of the limiting substrate (food) for growth and \( K \) (mol/L) is the half-saturation constant, as above this is the value of \( S \) where \( G = \frac{1}{2} \mu \).

a. Find \( \frac{dG}{dS} = \frac{(K+S)\mu - \mu S}{(K+S)^2} = \frac{K\mu}{(K+S)^2} \)

b. What are the units of \( \frac{dG}{dS} \):

\[ \frac{1}{\text{sec}} \quad \frac{\text{mol}}{\text{L}} \]

c. For which value of \( S \) (\( \geq 0 \)) is \( \frac{dG}{dS} \) the largest. This is the value of the substrate at which the growth increases the most for a unit increase in the substrate.

\[ \frac{dG}{dS} = \frac{K\mu}{(K+S)^2} \]

For large \( S \), \( \frac{dG}{dS} \) is small

For small \( S \), \( \frac{dG}{dS} \) is large

\( \frac{dG}{dS} \) is largest at \( S = 0 \)

d. Compute \( \lim_{S \to \infty} \frac{dG}{dS}(S) = \lim_{S \to \infty} \frac{K\mu}{(K+S)^2} = \square \)

e. Sketch \( G(S) \). Be sure to include relevant information.