Understanding Winter Road Conditions in Yellowstone National Park Using Cumulative Sum Control Charts

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Contents

1 Introduction

During the winter, about 200 miles of snow-roads are maintained in Yellowstone National Park to support the administrative and commercial travel throughout the park. The task of understanding what factors may have an effect on the road conditions is important to the safety of both the staff and visitors of Yellowstone throughout the winter. Several types of vehicles must use these winter roads to transport visitors to the main attractions of Yellowstone. It is of interest to assess how the winter road conditions change both throughout the day and throughout the winter season. It is also of interest to assess evidence for certain vehicle types contributing more than others to unfavorable road conditions. In order to investigate, several exploratory plots will be considered. Additionally, quality control cumulative sum charts will be used to identify days for which the road conditions were exceptionally severe.

2 Data Collection and Background

In order to begin understanding the winter road conditions in Yellowstone, one location was chosen within each of three areas (Madison, Firehole, and Gibbon) for data collection during the winter of 2012/2013. Figure 1 below shows the snow-roads in Yellowstone and the approximate locations of where data were collected.

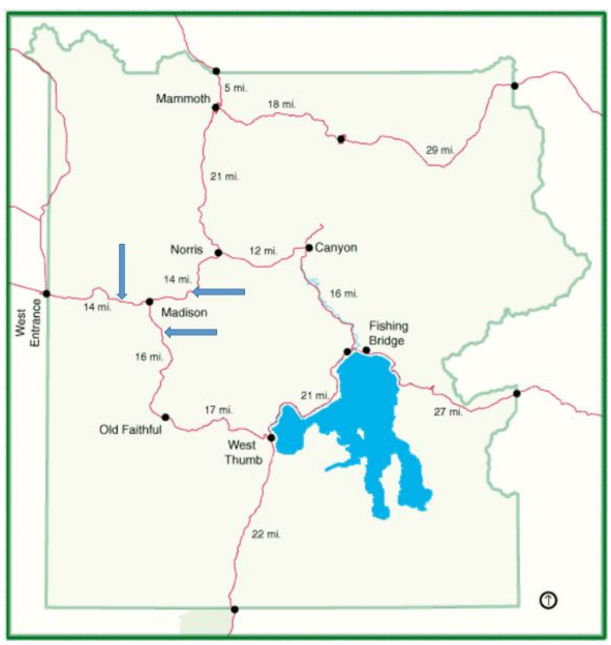

Figure 1: *The rough locations used for data collection in Yellowstone National Park.*

At each location a rope was attached to a post and was strung across the road several feet above the ground. Each location was chosen because it had both a lot of daily traffic and a post already located on the side of the road. Between one and four times per day, typically around 8:00 a.m., noon, and 4:00 p.m., a measurement of the distance between the rope and the top of the snow in centimeters was taken at approximately 60 centimeter intervals across the road for each location. Snow hardness measurements were also taken at each interval. The temperature at the location was also recorded at each time of measurement. Snow density was recorded twice per day at spots on the road nearby each location. Other variables such as snow-water-equivalent and year-to-date precipitation were recorded once per day (one measurement per day, but not for each location). It should also be noted that a groomer drove by each site almost every night and smoothed the surface of the snow-road.

Throughout the day a remote camera also captured the type of vehicle and time of day for each vehicle that drove by each location, including the groomer. The main types of vehicles that use these snow-roads throughout the day during the winter are: snowmobiles, purpose-built Bombardier snow-coaches, and vehicles converted from street use ranging from SUVs and vans to large touring buses. Figure 2 below shows a couple examples of the types of snow-coaches used on the snow-roads in Yellowstone.

Figure 2: *Examples of snow-coaches that were converted from street use for use on snow-roads.*

3 Exploratory Data Analysis

First, it is important to plot the raw data (measurements from the rope to the snow level) for each day for each location to see how the road conditions change throughout the day and throughout the season. The "depth" measurements could be multiplied by -1 so that the plots show more of a profile of the snow-road surface for each day with the more negative measurements corresponding to deeper "ruts" in the road. Figures 3, 4 and 5 show the snow-road profiles by day and location.

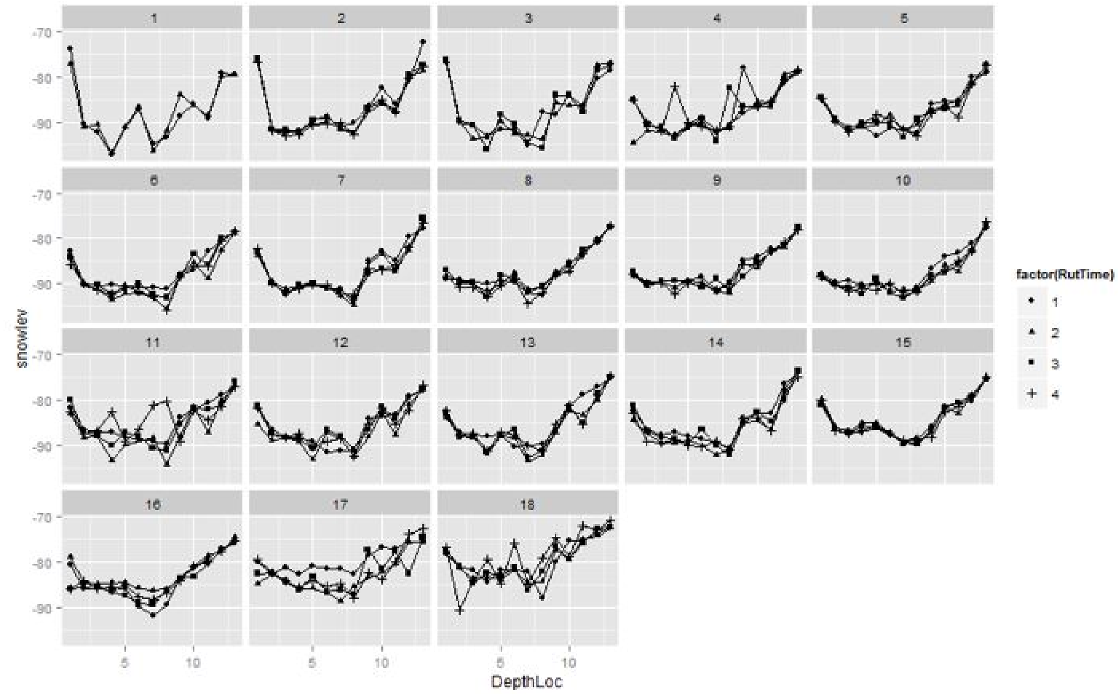

Figure 3: *Depth measurements from the Madison location by day.*

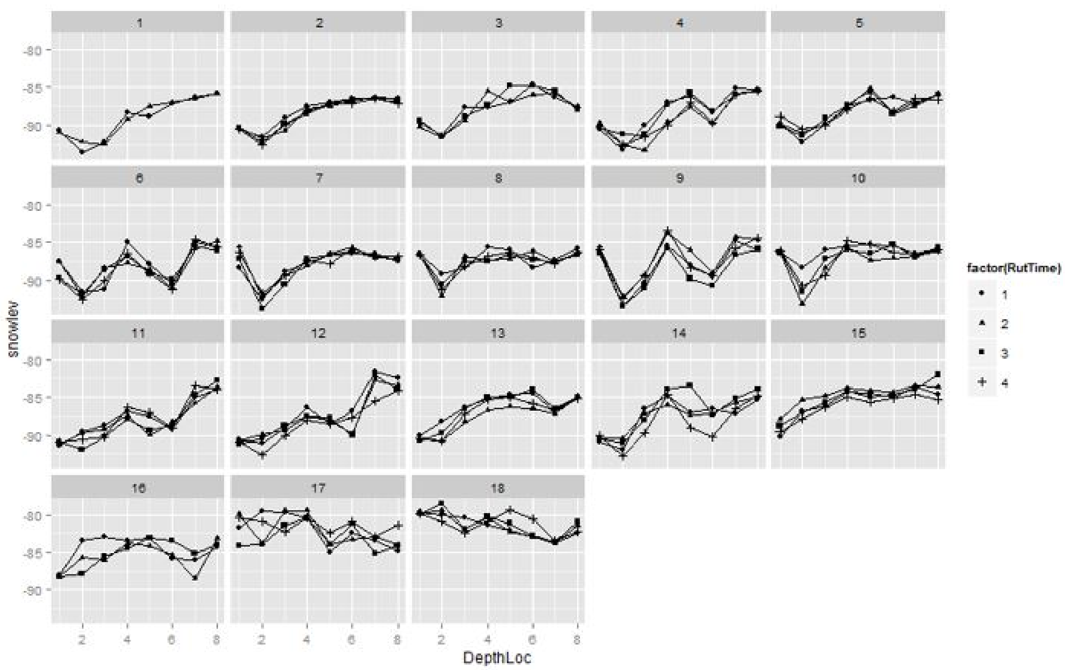

Figure 4: *Depth measurements from the Firehole location by day.*

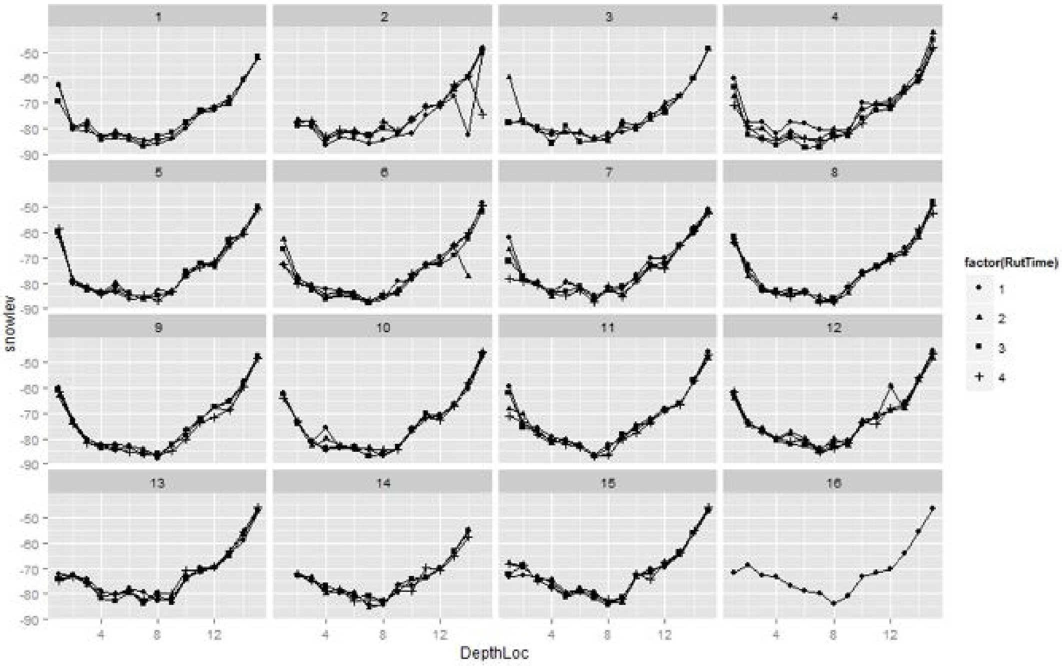

Figure 5: *Depth measurements from the Gibbon location by day.*

These plots can help us assess how the road is changing both throughout a given day and throughout the season. The lines on each plot show a general profile of the snow level across the road. We can then see days that show what looks like "ruts" in the snow-road. For example, on day 9 at Firehole, we see two distinct spots where the snow level is lower. We can also see that, for some days, the road condition can become more severe as the day progresses, For example, on day 18 at Madison, we see that the measurements taken at the first three times of measurement show a relatively more smooth road surface than the measurements taken the fourth time that day.

4 Defining Severity of the Road Conditions

In raw form, the data include all the measurements across the road for each time for each location. To simplify statistical analysis and inference it may be helpful to simplify the data by obtaining a summary statistic for each "road profile" (each time for each location). As the overall safety of the road may be of interest, it might be useful to somehow quantify the severity of the road conditions for each time measurements were taken.

One possibility is using the sum of the absolute values of the differences between consecutive measurements of a road profile as one way of quantifying the severity of the road condition at each time of measurement. That is, for each road profile (each time measurements were taken across the road), calculate a severity index as in Equation 1:

$$severity = \sum_{i=1}^{n_{location}-1} |depth_{i+1} - depth_i| \tag{1}$$

Using the values calculated from Equation 1 might be a good way to assess "severity" for the road condition because one would expect that, if consecutive depth measurements are very different, the road condition is more severe (i.e. less smooth). Using this severity index may also give us an idea of how the road profile is changing. Figures 6, 7 and 8 show the severity measurements for each time by day and location. Each point represents a sum of the absolute values of the differences between consecutive depth measurements taken across the road at one time. The lines connect the severity measurements from the same day.

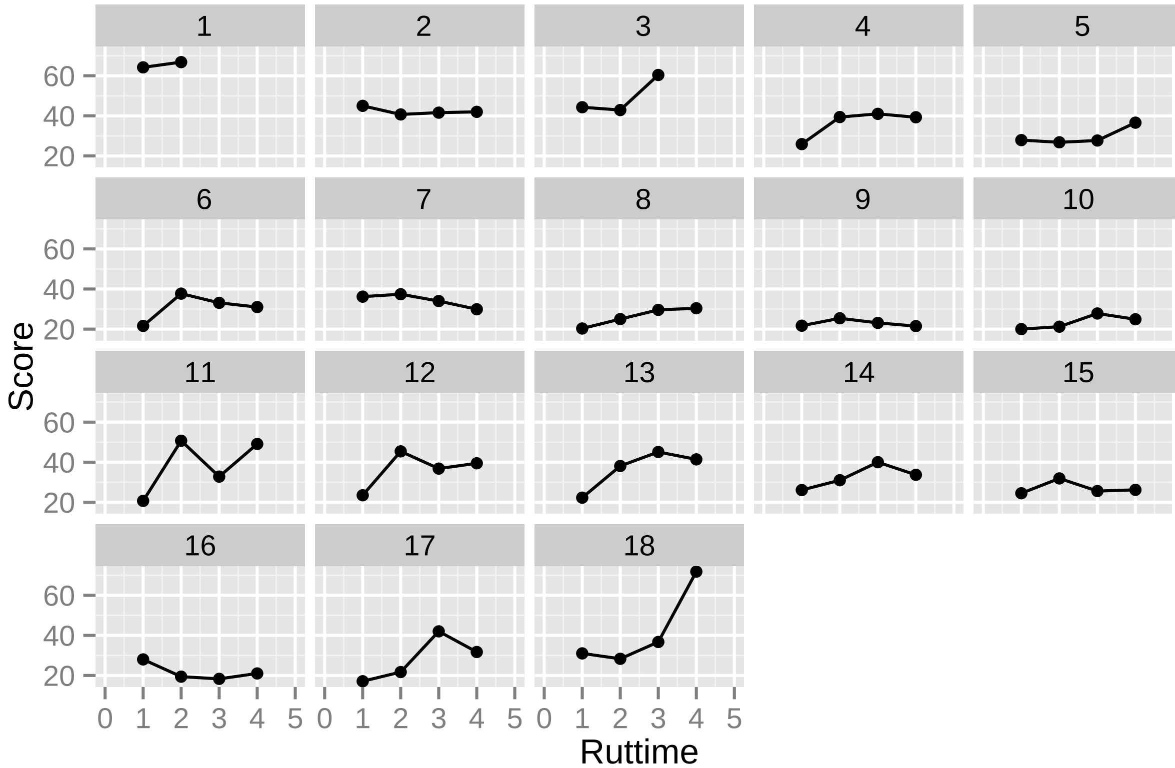

Figure 6: *Severity measurements by day for the Madison location.*

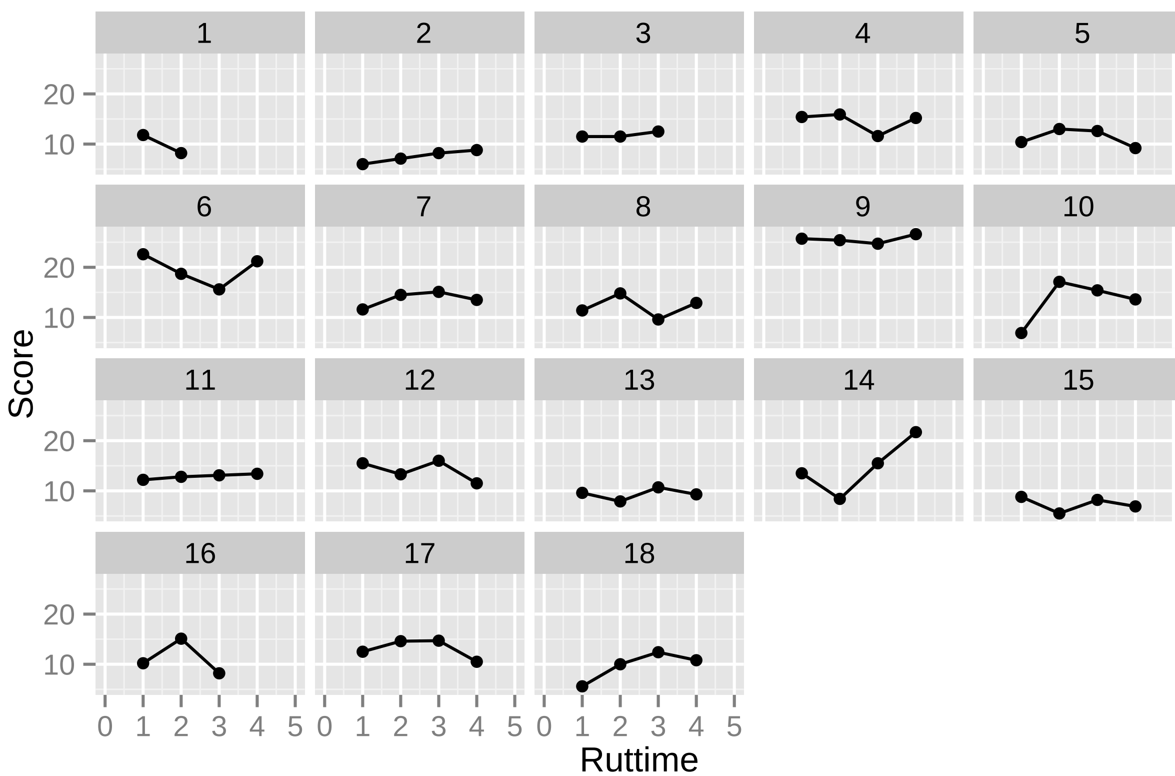

Figure 7: *Severity measurements by day for the Firehole location.*

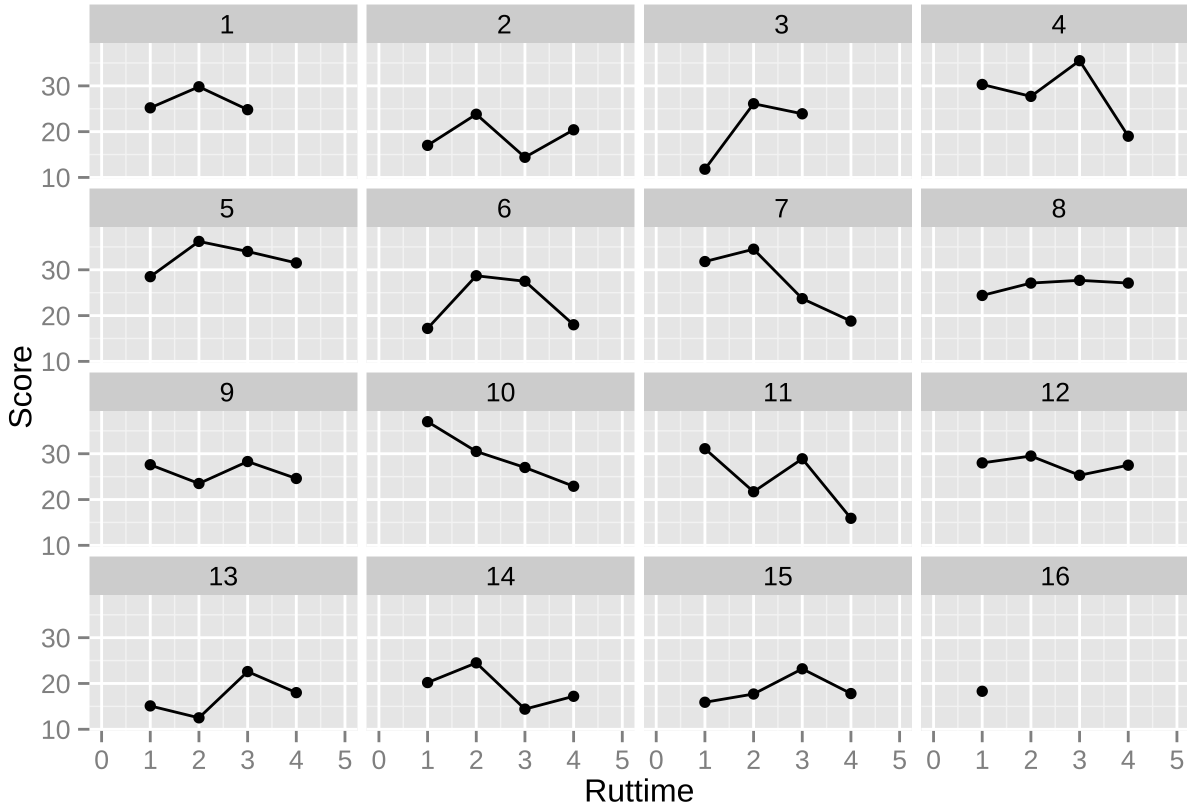

Figure 8: *Severity measurements by day for the Gibbon location.*

The height of the points for one day can be compared to the height of the points for a different day for the same location to compare the severity of the road conditions from different days. For example, the severities from Day 9 at Firehole are much higher than the severities from Day 2, indicating that the road conditions were worse on Day 9 at Firehole than on Day 2 at Firehole.

We can examine how the severities change throughout one day to see if there are drastic changes in the road condition. We see that the severity of the road conditions does not change that much throughout the day for some of the days. For example, consider Day 8 from the Gibbon location. The severity measurements are relatively consistent throughout the day. However, there are days for which the severity of the road conditions changes drastically. For example, consider Day 18 from the Madison location. The first three severity measurements stay about the same, but for the measurements taken across the road at the fourth time, the severity increases drastically. Note that the first point is not always the lowest, as would be expected if the groomer comes by each location every night and levels the snow-road. However, there were vehicles that passed by each location after the groomer, but before the first time of measurement, making it possible that the first severity is not the lowest. It is also possible that the road conditions improve throughout the day, like on Day 10 at Gibbon, potentially due to changing weather and snow conditions.

Using the severity index from Equation 1 may be a reasonable way to quantify severity. However, when using this method, it is not appropriate to compare severities from *different* locations. For example, it does not seem reasonable to compare the severities from Day 9 from Firehole to the severities from Day 9 from Gibbon. The road profiles from Day 9 at Firehole in Figure 4 show considerably worse road conditions than the road profiles from Day 9 from Gibbon in Figure 5. The locations might not be comparable for several reasons. The height of the rope above the pavement might have been different for each location. The level of the snow from the same day could also be quite different for the three locations. Each location also varies in the amount of traffic passing by each day. There are most likely many other reasons why it may be inappropriate to compare measurements from different sites. This may not be a big issue, however, since we are generally not interested in the differences between the locations. Thus, using these severity measurements may be a simple and useful way to compare the road conditions both within one day and across days for each location.

5 Application of Quality Control Charts

5.1 Cumulative Sum Charts

Using the severity measurements for each day and for each location, we might now consider using common quality control techniques to further investigate the Yellowstone winter road conditions. One option is using a Cumulative Sum Control Chart.

5.1.1 Introduction to Cumulative Sum Charts

In general, a Cumulative Sum Chart (CUSUM) can be used to monitor any quality characteristic. In the Yellowstone winter roads case, the quality characteristic of interest is the average severity of the road condition for each day. Cumulative Sum Charts track the cumulative deviations from a mean or target value, μ_0. We will create three CUSUM Charts, one for each location (Madison, Firehole, and Gibbon). In this case, we will use the sample mean of the severity measurements from each location as the target (μ_0) for that location. The purpose of using a Cumulative Sum chart is to quickly detect relatively small shifts in a process mean from the target value, while being sure to avoid falsely detecting a shift. Here we want to detect significant shifts in severity from the target severity for each location and note the days on which we observe a shift. Then we can attempt to find a potential reason why the shift occured on that day, which we call an "assignable cause."

Since it is desirable to have low road condition severity, we will want to focus on identifying severities that are too *large*. In order to detect shifts from a target value in one direction, we can use a *upper one-sided* CUSUM chart. The upper one-sided CUSUM chart will help determine the days for which the mean severity has deviated too far *above* the target value. For each day (day i), a cusum calculation is made using Equation 2:

$$C_i^+ = max[0, \overline{x}_j - (\mu_0 + K) + C_{i-1}^+] \qquad (2)$$

where $K = k\sigma$ (for a specified value of k) helps define the minimum shift in the mean severity that we would like to detect. The $\overline{x}_j$'s are the sample average severities for the days from the location of interest. Again, μ_0 is the "target" severity, for which we will use the overall sample mean severity for each location. If the average severity has not deviated too far from the target value, the values of the C_i^+'s will remain close to zero and the CUSUM chart will be relatively flat. If the average severity has deviated too far above the target value, the values of the C_i^+'s will increase and the CUSUM chart will show an increasing trend. Finally, if the average severity has deviated above the target value, but then is beginning to come back down to the target value, the values of the C_i^+'s will decrease and the CUSUM chart will show a decreasing trend. Thus, we will be interested in identifying the days on each CUSUM chart for which we observe an increasing trend that goes beyond some threshold, H.

5.1.2 Designing a Cumulative Sum Chart

An important consideration in the design of a CUSUM chart is the choice of the parameters H, K, and δ. $\Delta = \delta\sigma$ specifies the magnitude of the smallest shift in the average severity that we would like to detect. K specifies how far away from the target value an average severity would need to be in order to be considered "unacceptable," thereby increasing the cumulative sum, C_i^+. The other parameter, $H = h\sigma$ defines a bound for C_i^+ for signalling an "out-of-control" signal. If the value of C_i^+ exceeds H, then an out-of-control signal occurs. In this case, an out-of-control signal represents a day for which the average road condition severity is considered unacceptable.

Typically, the choice of H and K is important for obtaining reasonable values for the in-control average run length, ARL_0, and for the out-of-control average run length, ARL_δ. In this case, the in-control average run length, ARL_0 is the average number of days for an in-control process before a *false* out-of-control signal is detected. That is, it is the average number of days before we decide a road condition is too severe, when, in reality, it is not too severe. The out-of-control average run length, ARL_δ, is the average number of days before a shift in the mean severity exceeding a value of $\Delta = \delta\sigma$ away from the target value is detected. Thus, we desire the ARL_0 to be large, so as not to detect false signals too often, and the ARL_δ to be small, so that we can quickly detect average severities that are too large.

It is possible to find reasonable values of H and K by considering what values of the ARL_0 and the ARL_δ are desirable. For Yellowstone National Park, visitors can use the winter roads around between December 15th and March 15th, roughly 90 days. Thus, we might consider a desirable value of the ARL_0 to be around 90. In regards to the out-of-control ARL, it is probable that we would want to detect small shifts in the mean as quickly as possible. We might want to focus on detecting an average severity that is only half a standard deviation away from the target value as quickly as possible, rather than one or two standard deviations away. That is, we want to use $\delta = 0.5$. We want the value of the ARL_δ to be small, and we might consider several values of H and K that might produce low out-of-control average run lengths for a half of a standard deviaton shift. SAS can be used to generate reasonable values, as in Table 1. Table 1 shows us the in-control average run lengths (arl0) for different values of H and K, as well as the out-of-control average run lengths for a half standard deviation shift from the target value (arl_pt5), for a one standard deviation shift (arl1), and for a two standard deviation shift (arl2).

Table 1. Generated values of H and K.

Obs	k	h	arl0	arl_pt5	arl1	arl2
1	0.20	4.8	93.482	14.5085	6.74116	3.30206
2	0.20	4.9	98.525	14.8328	6.86617	3.35773
3	0.20	5.0	103.794	15.1576	6.99117	3.41346
4	0.25	4.3	92.994	14.4023	6.46001	3.09922
5	0.25	4.4	98.894	14.7769	6.59332	3.15601
6	0.25	4.5	105.117	15.1527	6.72663	3.21293
7	0.30	3.9	93.400	14.4914	6.27418	2.94457
8	0.30	4.0	100.231	14.9267	6.41688	3.00267
9	0.30	4.1	107.505	15.3646	6.55962	3.06086
10	0.35	3.6	96.687	14.8797	6.20399	2.84058
11	0.35	3.7	104.695	15.3890	6.35743	2.90043
12	0.40	3.3	96.532	15.1326	6.10992	2.72908
13	0.45	3.0	92.591	15.1779	5.98366	2.60828

We desire the ARL_0 to be above 90 days, while keeping the out-of-control average run lengths as small as possible. From this table, we might choose $H = 4.3$ and $K = 0.25$ since, for these values, the in-control average run length is more than 90 days and the out-of-control average run lengths are generally smaller.

One last consideration is whether or not to re-set the CUSUM after an out-of-control signal is detected. One might argue that we should re-set after an out-of-control signal is detected, since the groomer came by each location every night and smoothed the road. The groomer coming by every night might also provide support for assuming that the average severities are independent from each other. For investigating each location, we shall use an upper one-sided CUSUM with re-sets after each out-of-control signal.

5.1.3 Cumulative Sum Chart for the Madison Location

We can use SAS to produce an upper one-sided CUSUM chart for the average severities from the Madison location, with $H = 4.3$ and $K = 0.25$ to detect a half of a standard deviation shift ($\delta = 0.5$) from the target severity value. The target value we will use is the sample average severity from the Madison location, $\mu_0 = 33.56029$. Since we do not have any previous information on what a reasonable estimate of the Madison location process standard deviation, σ, we will let SAS estimate it using mean square successive difference (MSSD) given by Equation 3.

$$\hat{\sigma} = \sqrt{MSSD} = \sqrt{\frac{\sum\limits_{j=1}^{i-1} (\overline{x}_{j+1} - \overline{x}_j)^2}{2(i-1)}} \tag{3}$$

Figure 9 shows the desired CUSUM chart and Table 2 shows the tabular form of the CUSUM for Madison with re-sets after an out-of-control signal is detected.

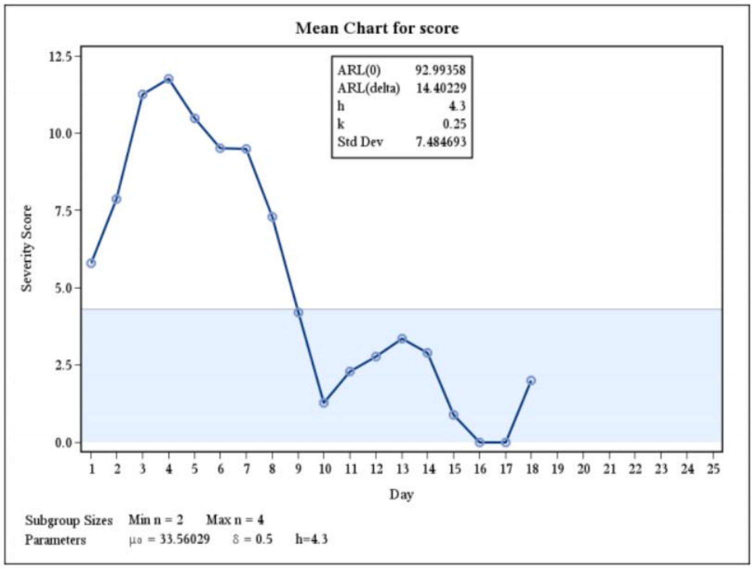

Figure 9: *Upper one-sided CUSUM chart for the Madison location.*

Table 2. Tabular CUSUM for Madison with re-sets.

day	xbar	n	cusum_l	hsigma	cusum_h	flag
1	65.500	2	0.0000	22.7586	30.3519	upper
2	42.325	4	0.0000	16.0928	15.4710	
3	49.200	3	0.0000	18.5823	21.9852	upper
4	36.400	4	0.0000	16.0928	3.6210	
5	29.750	4	5.5622	16.0928	0.0000	
6	30.850	4	6.0497	16.0928	0.0000	
7	34.375	4	0.5873	16.0928	0.0000	
8	26.325	4	14.7498	16.0928	0.0000	
9	22.925	4	17.9627	16.0928	0.0000	lower
10	23.475	4	18.1122	16.0928	0.0000	lower
11	38.325	4	0.0000	16.0928	7.4710	
12	36.290	4	0.0000	16.0928	7.0430	
13	36.725	4	0.0000	16.0928	9.5200	
14	32.700	4	0.0000	16.0928	3.5119	
15	27.050	4	10.9622	16.0928	0.0000	
16	21.675	4	16.1501	16.0928	0.0000	lower
17	28.125	4	8.8122	16.0928	0.0000	
18	41.950	4	0.0000	16.0928	14.7210	

Note that the CUSUM chart in Figure 9 does not reflect the signals we would see if we re-set the cusum after each out-of-control signal. We can use the tabular form in Table 2 to identify out-of-control signals using a cusum that has been re-set after a signal. We are only concerned with upper signals under the "flag" column, since we are only worried about average road condition severities that are too high.

The out-of-control signals for the Madison location occur on days 1 and 3. The next step is to attempt to find an assignable cause for these out-of-control signals. That is, we might try to figure out what made the average severity for these days so high.

5.1.4 Assignable Causes for High Severities at the Madison Location

The CUSUM chart revealed average severities that are too high on day 1 (February 5th) and on day 3, (February 7th) at the Madison location. We must now think about what factors may have contributed to these high average severities. For example, perhaps on these days there were many of a certain type of vehicle that drove by the Madison location. Or perhaps the average temperature was really high for these two days. We can investigate these types of questions by comparing the data observed on the high severity days to plots or charts of the values of different variables from the Madison location. Below are bar charts showing the number of each type of vehicle that went through the Madison location during Yellowstone's winter operating hours (9 a.m. to 4 p.m.) on day 1 and day 3. Also below are histograms showing the distribution of each of the climate variables that were recorded, with the values from day 1 and day 3 shown with vertical lines.

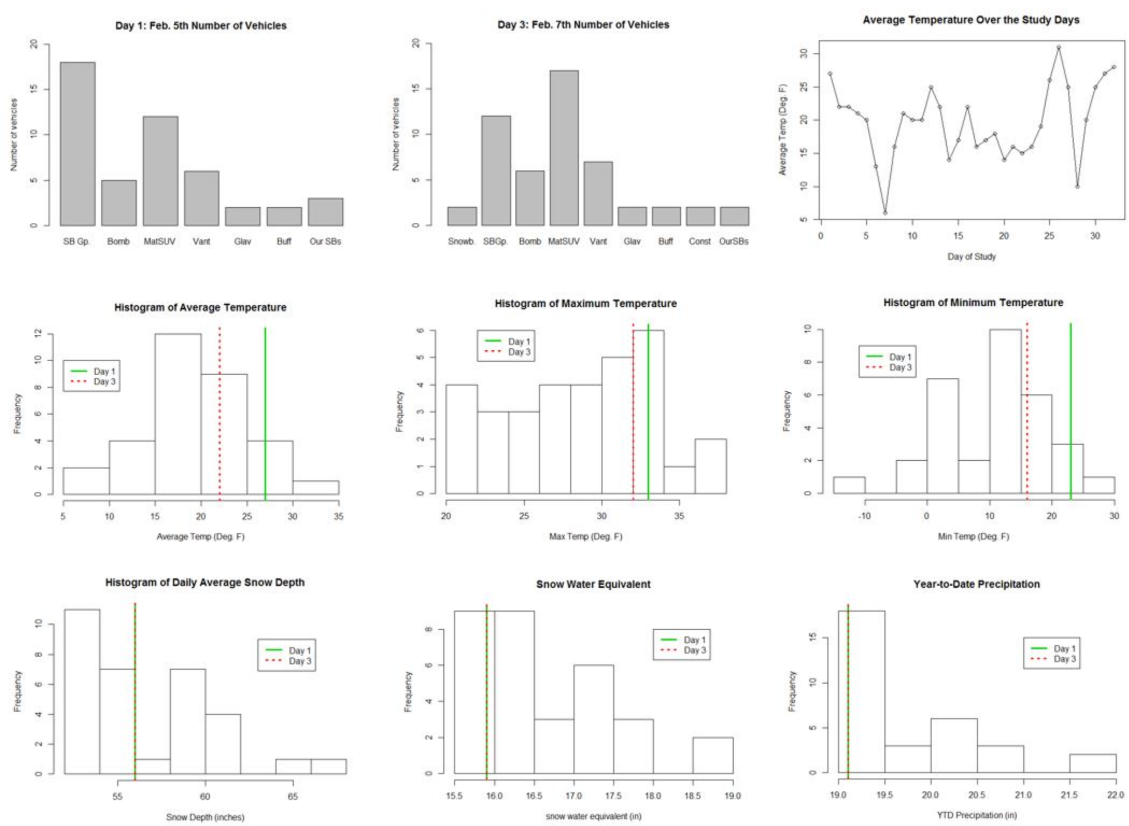

Figure 10: *Comparing Values from Days 1 and 3 at Madison to other observed values.*

From these plots we might hypothesize that the high average road condition severities from days 1 and 3 are associated with higher temperatures and/or many groups of snow-mobiles and Mattrack SUVs on the snow-roads. However, we must be careful *not* to say that high severities are *caused* by any one condition or combination of conditions, such as temperature or type of vehicle, since we did not randomly assign these conditions. That is, the statistics are derived from an observational study. It is not possible to randomly assign a weather condition and the vehicle conditions were not experimentally controlled; we simply observed the weather conditions and the number of vehicles that passed by each location for each day.

5.1.5 Cumulative Sum Chart for the Firehole Location

Next, we consider an upper one-sided CUSUM chart with the same H, K, and δ parameters for the Firehole location. The target severity we will use for this location is the overall average severity from the Firehole location, $\mu_0 = 13.00294$. Again, we will let SAS estimate the Firehole process standard deviation, σ. Figure 10 and Table 11 show the one-sided CUSUM chart and the tabular form of the CUSUM with re-sets after an out-of-control signal.

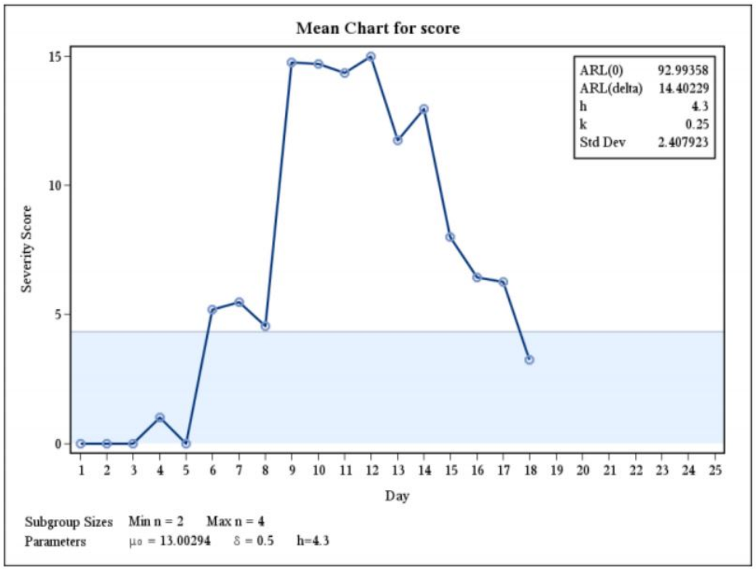

Figure 11: *Upper one-sided CUSUM chart for the Firehole location.*

Table 3. Tabular CUSUM for Firehole with re-sets.

day	xbar	n	cusum_l	hsigma	cusum_h	flag
1	10.0000	2	2.57728	7.32143	0.0000	
2	7.5250	4	7.75422	5.17703	0.0000	lower
3	11.8333	3	0.82205	5.97792	0.0000	
4	14.5250	4	0.00000	5.17703	1.2211	
5	11.3000	4	1.40195	5.17703	0.0000	
6	19.5250	4	0.00000	5.17703	6.2211	upper
7	13.6750	4	0.00000	5.17703	0.3711	
8	12.1750	4	0.52695	5.17703	0.0000	
9	25.6000	4	0.00000	5.17703	12.2961	upper
10	13.2500	4	0.00000	5.17703	0.0000	
11	12.8750	4	0.00000	5.17703	0.0000	
12	14.0750	4	0.00000	5.17703	0.7711	
13	9.3750	4	3.32695	5.17703	0.0000	
14	14.7750	4	1.25390	5.17703	1.4711	
15	7.3500	4	6.60585	5.17703	0.0000	lower
16	11.1667	3	1.48872	5.97792	0.0000	
17	13.0750	4	1.11567	5.17703	0.0000	
18	9.7000	4	4.11762	5.17703	0.0000	

We see upper out-of-control signals for days 6 and 9 at the Firehole location. If we look at the profiles for days 6 and 9 at Firehole (Figure 12), this is not surprising.

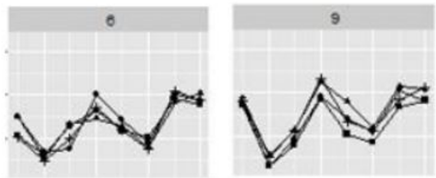

Figure 12: *Road profiles for Day 6 and Day 9 at Firehole.*

We see two distinct "ruts" forming for days 6 and 9. In the next section, we explore potential assignable causes for these out-of-control signals.

5.1.6 Assignable Causes for High Severities at the Firehole Location

Similar to how we investigated potential assignable causes for the Madison location, we do the same for the Firehole location. That is, we might compare the data we observe for days 6 (February 14th) and 9 (February 20th) at Firehole to the other days. Below are bar charts for the number of different types of vehicles for days 6 and 9 at Firehole and histograms showing how the values from days 6 and 9 compare to other observed values.

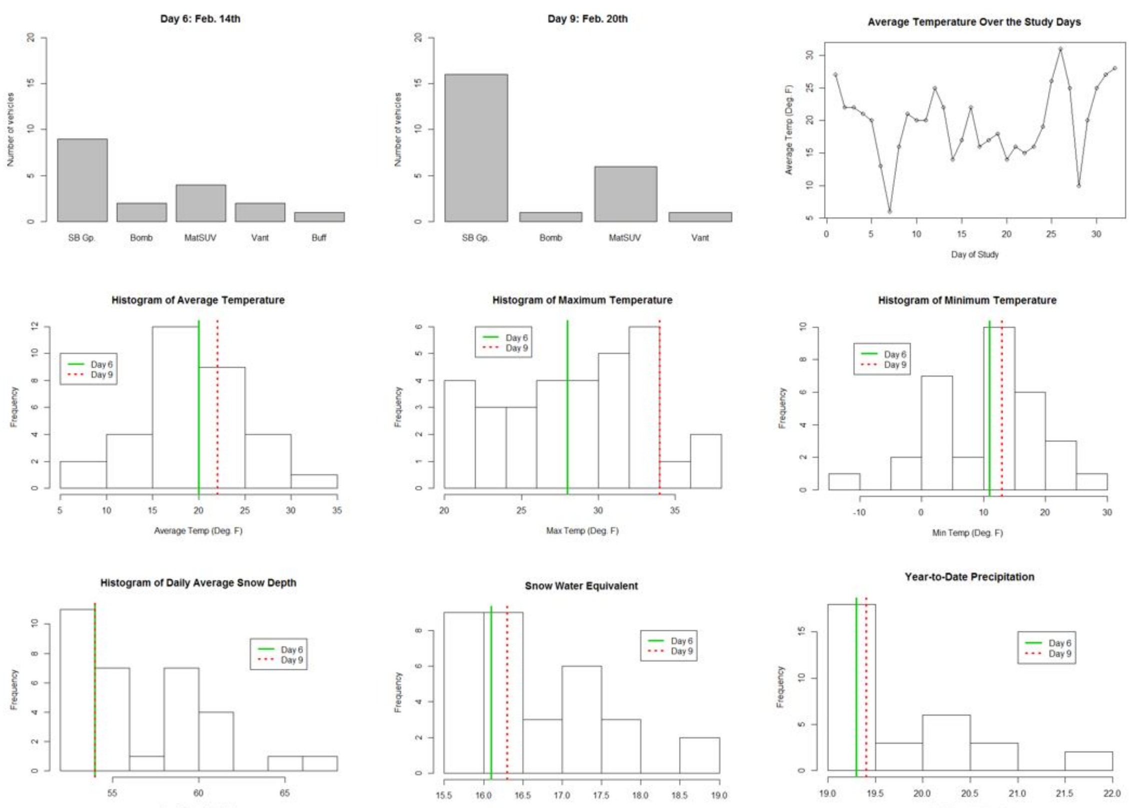

Figure 13: *Comparing Values from Days 6 and 9 at Firehole to other observed values.*

As for the Madison location, the two most frequent vehicle types are groups of snowmobiles and Mattrack SUVs. The values observed on days 6 and 9 at Firehole for the climate variables do not seem particularly unusual, except for maybe the maximum temperature observed on day 9. Again, even if we hypothesize that some condition or combination of conditions is related to high road condition severity, (i.e. many of a certain type of vehicle and high temperature), we are not able to say that these condition(s) *cause* the road condition to be severe, as there was no random assignment of vehicles or climate to the days. We must also consider that there might be other variables that were not accounted for in this study that could be associated with high road condition severity.

5.1.7 Cumulative Sum Chart for the Gibbon Location

Lastly, we consider an upper one-sided CUSUM chart for the Gibbon location using the same parameters: $H = 4.3$, $K = 0.25$, and $\delta = 0.5$. The target value we will use, the average severity observed from the Gibbon location, is $\mu_0 = 24.28983$. Once again, we let SAS estimate Gibbon's process standard deviation, σ. Figure 14 and Table 4 show the upper one-sided CUSUM chart and the tabular CUSUM with re-sets for the Gibbon location.

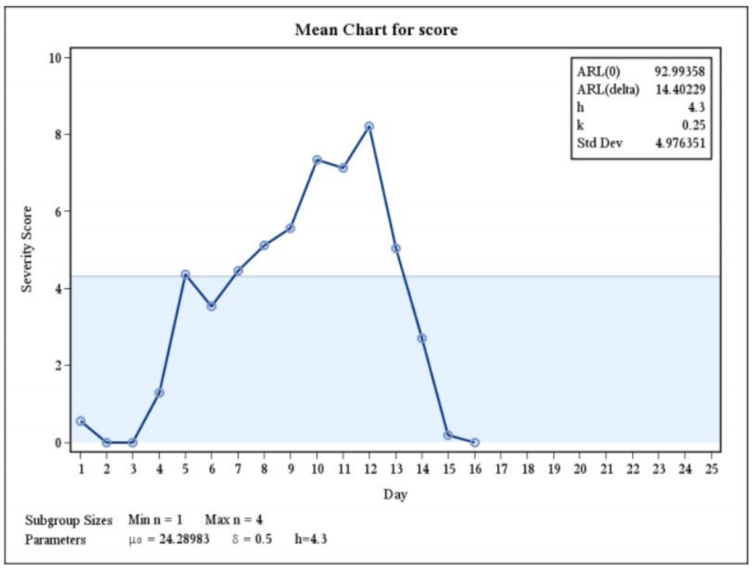

Figure 14: *Upper one-sided CUSUM chart for the Gibbon location.*

Table 4. Tabular CUSUM for Gibbon with re-sets.

day	xbar	n	cusum_l	hsigma	cusum_h	flag
1	26.600	3	0.0000	12.3543	1.5919	
2	18.900	4	4.7678	10.6992	0.0000	
3	20.600	3	7.7393	12.3543	0.0000	
4	28.125	4	3.2821	10.6992	3.2131	
5	32.550	4	0.0000	10.6992	10.8513	upper
6	22.850	4	0.8178	10.6992	0.0000	
7	27.200	4	0.0000	10.6992	2.2881	
8	26.575	4	0.0000	10.6992	3.9513	
9	26.000	4	0.0000	10.6992	5.0394	
10	29.350	4	0.0000	10.6992	9.4775	
11	24.400	4	0.0000	10.6992	8.9656	
12	27.575	4	0.0000	10.6992	11.6288	upper
13	17.050	4	6.6178	10.6992	0.0000	
14	19.075	4	11.2106	10.6992	0.0000	lower
15	18.650	4	5.0178	10.6992	0.0000	
16	18.300	1	9.7635	21.3983	0.0000	

The days at the Gibbon location that were flagged for an out-of-control average severity were days 5 and 12. Next, we explore potential assignable causes for these days.

5.1.8 Assignable Causes for High Severities at the Gibbon Location

Day 5 at Gibbon was February 13th and day 12 was February 27th. Similar to how assignable causes were explored previously, we might check to see if the data we observe from these days at the Gibbon location are unusual. Below are the plots and charts we might use to see how unusual days 5 and 12 are.

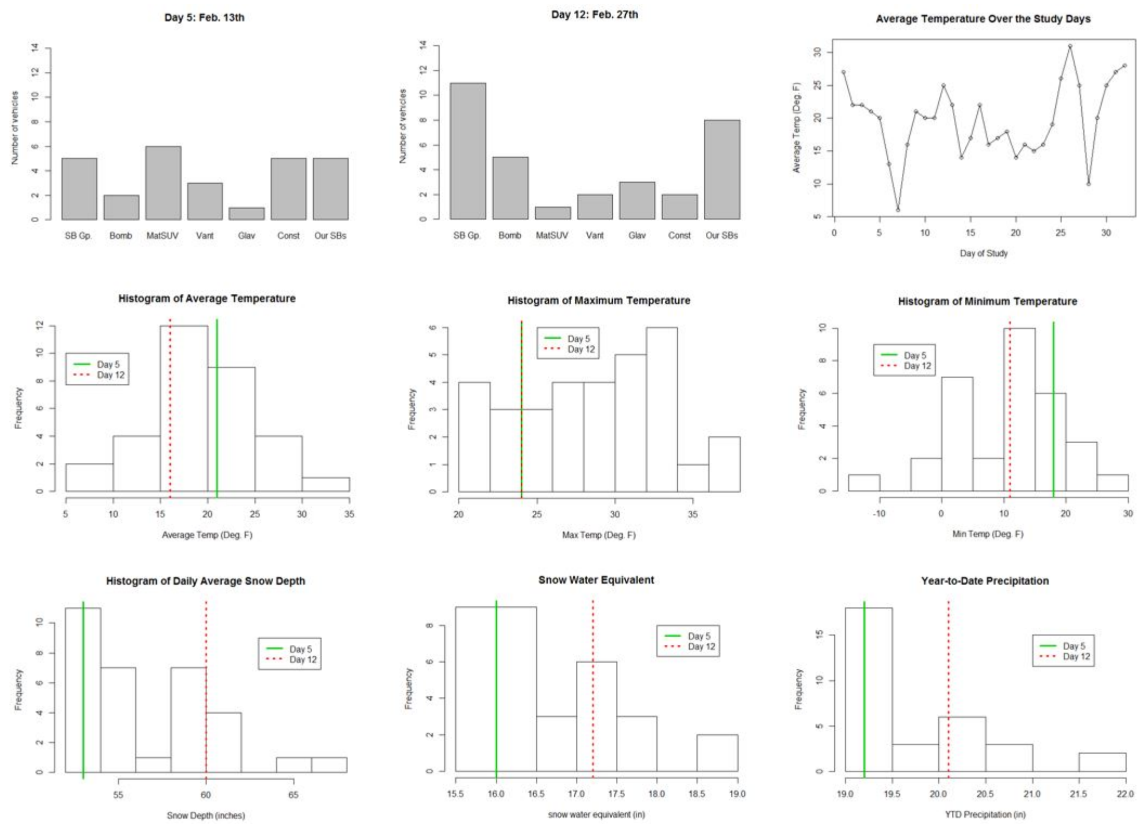

Figure 15: *Comparing Values from Days 5 and 12 at Gibbon to other observed values.*

Unlike the days for which high average severities were identified at the Madison and Firehole locations, we see that the number of Mattrack SUVs for days 5 and 12 at the Gibbon location do not stand out too much. We note, however, that there were more construction vehicles (Const) and Yellowstone snowmobiles (Our SBs) in total on these two days at Gibbon compared to the other locations. We also see that there was a bit of snow accumulation between day 5 and day 12 at Gibbon. We might hypothesize that snow accumulation is associated with high severity.

6 Conclusions

It seems reasonable to use CUSUM charts to begin to investigate the road condition severity at each of the three locations: Madison, Firehole, and Gibbon. From the upper one-sided

CUSUM charts for each location, we found out-of-control signals for days 1 and 3 at Madison, for days 6 and 9 at Firehole, and for days 5 and 12 at Gibbon. When an out-of-control signal is detected, meaning an average severity more than half a standard deviation from the target value was detected, the next step is to find a reason why (an assignable cause). We might check for unusual data observed on days flagged as out-of-control signals to see if there is an assocation between one variable (or a combination of variables) and high road condition severity. For any of the three locations, however, it is very difficult to identify any one particular condition, like vehicle type or temperature, that is related to high severity.

There seems to be many potential reasons why high severities occur. For example, perhaps high road condition severity is associated with a particular combination of temperature, number of vehicles, and snow depth. Or perhaps a high severity is due to some other variable or variables that were not accounted for in the study. Although we cannot determine what *causes* high road condition severity for this study, it is still useful to explore the data using statistical methods, like CUSUM charts.

7 Suggestions for Future Studies

Regarding the use of CUSUM charts to investigate road condition severity for each location, a few suggestions are appropriate. It might be useful in the future to specify a target severity for each location. Management at Yellowstone National Park might consider what severity value they would consider to be "too high" for each location. Additionally, further exploration of the choice of the CUSUM parameters, H, K, and δ is recommended as more data are collected. Perhaps there are more useful in-control and out-of-control average run lengths than the ones chosen for this study that would help obtain more pertinent CUSUM parameter values.

There may also be other ways to develop severity indices. For example, consider the sum of the squared deviations in Equation 4.

$$severity_{SSD} = \sum_{i=1}^{n_{location}-1} (depth_{i+1} - depth_i)^2 \tag{4}$$

It would be interesting to see when, if ever, the flagging of out-of-control signals would differ from those indicated by the severity index based on the sum of the absolute deviations.

A follow-up study of the road conditions is currently being developed to further assess the impacts of different variables. Although it is not clear whether the same general approach will be taken, one recommendation we make for the method of data collection if the new study is similar is to have multiple "cross-sections" (ropes across the road) for each area that are a reasonable distance apart. With multiple locations for measurements for each area it would be possible to get a better understanding of the variability of the road condition for each location for both throughout one given day and throughout the season. It is also recommended that the intervals at which the depth measurements are taken be closer together, say 10 centimeters apart, rather than 60 centimeters. That way, there is less of a chance that a drastic "rut" in the road is missed. It would also be useful to take measurements at the edge of the road to be used as reference points, or measurements of what the snow level would be if no vehicles had passed. It may also be useful to record where the drivable road surface starts so as not to include measurements of the edges of the road where nobody drives.

8 References

Borkowski, J.J. (2013). *Courses Notes for Stat 528: Statistical Quality Control.* Unpublished manuscript, Department of Mathematical Sciences, Montana State University.

Montgomery, D. C. . *Introduction to Statistical Quality Control.* 6th ed. John Wiley Sons, 2009. 399-410. print.

Vagias. (2013). Primer on Snowroad Rutting in Yellowstone National Park (version 2013.08.28).

9 SAS Code

```
DATA csmarl;
Do k = 0.05 to 2 by 0.05;
Do h = 2 to 10 by .1;
arl0 = CUSUMARL('onesided',0,h,k);
arl_pt5 = CUSUMARL('onesided',0.5, h, k);
arl1 = CUSUMARL('onesided',1,h,k);
arl2 = CUSUMARL('onesided',2,h,k);
IF (90 le arl0 le 150) and (0 le arl_pt5 le 15.5) and (0 le arl1 le 7) THEN OUTPUT;
END;
END;

PROC PRINT DATA= csmarl;
RUN;

DATA in;
INPUT loc $ day time score;
LINES;
Madison 1 1 64.2
Madison 1 2 66.8
Madison 2 1 45
Madison 2 2 40.7
Madison 2 3 41.6
Madison 2 4 42
Madison 3 1 44.3
Madison 3 2 42.9
Madison 3 3 60.4
Madison 4 1 25.9
Madison 4 2 39.4
Madison 4 3 41
Madison 4 4 39.3
Madison 5 1 27.9
Madison 5 2 26.8
Madison 5 3 27.7
Madison 5 4 36.6
Madison 6 1 21.6
Madison 6 2 37.7
Madison 6 3 33.1
Madison 6 4 31
Madison 7 1 36.2
Madison 7 2 37.4
Madison 7 3 34
Madison 7 4 29.9
Madison 8 1 20.3
Madison 8 2 25
```

```
Madison 8 3 29.6
Madison 8 4 30.4
Madison 9 1 21.7
Madison 9 2 25.4
Madison 9 3 23.1
Madison 9 4 21.5
Madison 10 1 20
Madison 10 2 21.2
Madison 10 3 27.8
Madison 10 4 24.9
Madison 11 1 20.7
Madison 11 2 50.7
Madison 11 3 32.8
Madison 11 4 49.1
Madison 12 1 23.5
Madison 12 2 45.4
Madison 12 3 36.8
Madison 12 4 39.46
Madison 13 1 22.3
Madison 13 2 38.1
Madison 13 3 45.1
Madison 13 4 41.4
Madison 14 1 26.1
Madison 14 2 31
Madison 14 3 40
Madison 14 4 33.7
Madison 15 1 24.5
Madison 15 2 31.9
Madison 15 3 25.6
Madison 15 4 26.2
Madison 16 1 28
Madison 16 2 19.4
Madison 16 3 18.3
Madison 16 4 21
Madison 17 1 17.1
Madison 17 2 21.7
Madison 17 3 42
Madison 17 4 31.7
Madison 18 1 31
Madison 18 2 28.3
Madison 18 3 36.7
Madison 18 4 71.8
;

PROC CUSUM DATA=in;
XCHART score*day='1'
/ MU0=33.56029 SMETHOD=noweight H=4.3 K=0.3 DELTA=0.5
DATAUNITS HAXIS = 1 TO 18
TABLESUMMARY TABLEOUT OUTTABLE = qsum ;
INSET ARL0 ARLDELTA H K SHIFT / POS = ne;
LABEL score='Severity Score'
day = 'Day';
TITLE 'CUSUM for Severity Score for Madison (sigma unknown)';
RUN;

PROC CUSUM DATA=in;
XCHART score*day='1'
/ MU0=33.56029 SMETHOD=noweight H=4.3 K=0.30 DELTA=0.5
DATAUNITS HAXIS=1 TO 25
SCHEME=onesided TABLESUMMARY TABLEOUT;
```

```
INSET ARL0 ARLDELTA H K SHIFT STDDEV / POS = n;
LABEL score='Severity Score'
day = 'Day';
TITLE 'UPPER ONE-SIDED CUSUM: MADISON';
RUN;

DATA qsum; SET qsum;
h=4.3;
k=0.3;
sigma=7.485;
aim=33.56029; ** enter values **;
xbar=_subx_; n=_subn_;
hsigma=h*sigma/SQRT(_subn_);
ksigma=k*sigma/SQRT(_subn_);
RETAIN cusum_l 0 cusum_h 0;
IF (-hsigma < cusum_l < hsigma) THEN DO;
cusum_l = cusum_l + (aim - ksigma) - xbar;
IF cusum_l < 0 then cusum_l=0; END;
IF (-hsigma < cusum_h < hsigma) THEN DO;
cusum_h = cusum_h + xbar - (aim + ksigma);
IF cusum_h < 0 then cusum_h=0; END;
IF MAX(cusum_l,cusum_h) ge hsigma THEN DO;
IF (cusum_l ge hsigma) THEN DO;
flag='lower'; OUTPUT; END;
IF (cusum_h ge hsigma) THEN DO;
flag='upper'; OUTPUT; END;
cusum_l=0; cusum_h=0; END;
ELSE OUTPUT;

PROC PRINT DATA=qsum;
ID day;
VAR xbar n cusum_l hsigma cusum_h flag;
TITLE 'CUSUM with Reset after Signal (sigma estimated)';
RUN;

DATA in2;
INPUT loc $ day time score;
LINES;
Firehole 1 1 11.8
Firehole 1 2 8.2
Firehole 2 1 6
Firehole 2 2 7.1
Firehole 2 3 8.2
Firehole 2 4 8.8
Firehole 3 1 11.5
Firehole 3 2 11.5
Firehole 3 3 12.5
Firehole 4 1 15.4
Firehole 4 2 15.9
Firehole 4 3 11.6
Firehole 4 4 15.2
Firehole 5 1 10.4
Firehole 5 2 13
Firehole 5 3 12.6
Firehole 5 4 9.2
Firehole 6 1 22.6
Firehole 6 2 18.7
Firehole 6 3 15.6
Firehole 6 4 21.2
Firehole 7 1 11.6
```

```
Firehole 7 2 14.5
Firehole 7 3 15.1
Firehole 7 4 13.5
Firehole 8 1 11.4
Firehole 8 2 14.8
Firehole 8 3 9.6
Firehole 8 4 12.9
Firehole 9 1 25.7
Firehole 9 2 25.4
Firehole 9 3 24.7
Firehole 9 4 26.6
Firehole 10 1 6.9
Firehole 10 2 17.1
Firehole 10 3 15.4
Firehole 10 4 13.6
Firehole 11 1 12.2
Firehole 11 2 12.8
Firehole 11 3 13.1
Firehole 11 4 13.4
Firehole 12 1 15.5
Firehole 12 2 13.3
Firehole 12 3 16
Firehole 12 4 11.5
Firehole 13 1 9.6
Firehole 13 2 7.9
Firehole 13 3 10.7
Firehole 13 4 9.3
Firehole 14 1 13.5
Firehole 14 2 8.4
Firehole 14 3 15.5
Firehole 14 4 21.7
Firehole 15 1 8.8
Firehole 15 2 5.5
Firehole 15 3 8.2
Firehole 15 4 6.9
Firehole 16 1 10.2
Firehole 16 2 15.1
Firehole 16 3 8.2
Firehole 17 1 12.5
Firehole 17 2 14.6
Firehole 17 3 14.7
Firehole 17 4 10.5
Firehole 18 1 5.6
Firehole 18 2 10
Firehole 18 3 12.4
Firehole 18 4 10.8
;


PROC CUSUM DATA=in2;
XCHART score*day='1'
/ MU0=13.00294 SMETHOD=noweight H=4.3 K=0.25 DELTA=0.5
DATAUNITS HAXIS = 1 TO 18
TABLESUMMARY TABLEOUT OUTTABLE = qsum2 ;
INSET ARL0 ARLDELTA H K SHIFT / POS = nw;
LABEL score='Severity Score'
day = 'Day';
TITLE 'CUSUM for Severity Score for Firehole (sigma unknown)';
RUN;
```

```
PROC CUSUM DATA=in2;
XCHART score*day='1'
/ MU0=13.00294 SMETHOD=noweight H=4.3 K=0.25 DELTA=0.5
DATAUNITS HAXIS=1 TO 25
SCHEME=onesided TABLESUMMARY TABLEOUT;
INSET ARL0 ARLDELTA H K SHIFT STDDEV/ POS = ne;
LABEL score='Severity Score'
day = 'Day';
TITLE 'UPPER ONE-SIDED CUSUM: FIREHOLE';
RUN;

DATA qsum2; SET qsum2;
h=4.3;
k=0.25;
sigma=2.407923;
aim=13.00294; ** enter values **;
xbar=_subx_; n=_subn_;
hsigma=h*sigma/SQRT(_subn_);
ksigma=k*sigma/SQRT(_subn_);
RETAIN cusum_l 0 cusum_h 0;
IF (-hsigma < cusum_l < hsigma) THEN DO;
cusum_l = cusum_l + (aim - ksigma) - xbar;
IF cusum_l < 0 then cusum_l=0; END;
IF (-hsigma < cusum_h < hsigma) THEN DO;
cusum_h = cusum_h + xbar - (aim + ksigma);
IF cusum_h < 0 then cusum_h=0; END;
IF MAX(cusum_l,cusum_h) ge hsigma THEN DO;
IF (cusum_l ge hsigma) THEN DO;
flag='lower'; OUTPUT; END;
IF (cusum_h ge hsigma) THEN DO;
flag='upper'; OUTPUT; END;
cusum_l=0; cusum_h=0; END;
ELSE OUTPUT;

PROC PRINT DATA=qsum2;
ID day;
VAR xbar n cusum_l hsigma cusum_h flag;
TITLE 'CUSUM with Reset after Signal (sigma estimated)';
RUN;

DATA in3;
INPUT loc $ day time score;
LINES;
Gibbon 1 1 25.2
Gibbon 1 2 29.8
Gibbon 1 3 24.8
Gibbon 2 1 17
Gibbon 2 2 23.8
Gibbon 2 3 14.4
Gibbon 2 4 20.4
Gibbon 3 1 11.8
Gibbon 3 2 26.1
Gibbon 3 3 23.9
Gibbon 4 1 30.3
Gibbon 4 2 27.7
Gibbon 4 3 35.5
Gibbon 4 4 19
Gibbon 5 1 28.5
Gibbon 5 2 36.2
Gibbon 5 3 34
```

```
Gibbon 5 4 31.5
Gibbon 6 1 17.2
Gibbon 6 2 28.7
Gibbon 6 3 27.5
Gibbon 6 4 18
Gibbon 7 1 31.8
Gibbon 7 2 34.5
Gibbon 7 3 23.7
Gibbon 7 4 18.8
Gibbon 8 1 24.4
Gibbon 8 2 27.1
Gibbon 8 3 27.7
Gibbon 8 4 27.1
Gibbon 9 1 27.6
Gibbon 9 2 23.5
Gibbon 9 3 28.3
Gibbon 9 4 24.6
Gibbon 10 1 37
Gibbon 10 2 30.5
Gibbon 10 3 27
Gibbon 10 4 22.9
Gibbon 11 1 31.1
Gibbon 11 2 21.7
Gibbon 11 3 28.9
Gibbon 11 4 15.9
Gibbon 12 1 28
Gibbon 12 2 29.5
Gibbon 12 3 25.3
Gibbon 12 4 27.5
Gibbon 13 1 15.1
Gibbon 13 2 12.5
Gibbon 13 3 22.6
Gibbon 13 4 18
Gibbon 14 1 20.2
Gibbon 14 2 24.5
Gibbon 14 3 14.4
Gibbon 14 4 17.2
Gibbon 15 1 15.9
Gibbon 15 2 17.7
Gibbon 15 3 23.2
Gibbon 15 4 17.8
Gibbon 16 1 18.3
;

PROC CUSUM DATA=in3;
XCHART score*day='1'
/ MU0=24.28983 SMETHOD=noweight H=4.3 K=0.25 DELTA=0.5
DATAUNITS HAXIS = 1 TO 18
TABLESUMMARY TABLEOUT OUTTABLE = qsum3 ;
INSET ARL0 ARLDELTA H K SHIFT / POS = nw;
LABEL score='Severity Score'
day = 'Day';
TITLE 'CUSUM for Severity Score for Gibbon (sigma unknown)';
RUN;

PROC CUSUM DATA=in3;
XCHART score*day='1'
/ MU0=24.28983 SMETHOD=noweight H=4.3 K=0.25 DELTA=0.5
DATAUNITS HAXIS=1 TO 25
SCHEME=onesided TABLESUMMARY TABLEOUT;
```

```
INSET ARL0 ARLDELTA H K SHIFT STDDEV / POS = ne;
LABEL score='Severity Score'
day = 'Day';
TITLE 'UPPER ONE-SIDED CUSUM: GIBBON';
RUN;

DATA qsum3; SET qsum3;
h=4.3;
k=0.25;
sigma=4.976351;
aim=24.28983; ** enter values **;
xbar=_subx_; n=_subn_;
hsigma=h*sigma/SQRT(_subn_);
ksigma=k*sigma/SQRT(_subn_);
RETAIN cusum_l 0 cusum_h 0;
IF (-hsigma < cusum_l < hsigma) THEN DO;
cusum_l = cusum_l + (aim - ksigma) - xbar;
IF cusum_l < 0 then cusum_l=0; END;
IF (-hsigma < cusum_h < hsigma) THEN DO;
cusum_h = cusum_h + xbar - (aim + ksigma);
IF cusum_h < 0 then cusum_h=0; END;
IF MAX(cusum_l,cusum_h) ge hsigma THEN DO;
IF (cusum_l ge hsigma) THEN DO;
flag='lower'; OUTPUT; END;
IF (cusum_h ge hsigma) THEN DO;
flag='upper'; OUTPUT; END;
cusum_l=0; cusum_h=0; END;
ELSE OUTPUT;

PROC PRINT DATA=qsum3;
ID day;
VAR xbar n cusum_l hsigma cusum_h flag;
TITLE 'CUSUM with Reset after Signal (sigma estimated)';
RUN;
```