1. Let \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) and \( \mathbf{d} \) be arbitrary vectors in \( \mathbb{R}^3 \) and \( \mathbf{r}_1(t) \) and \( \mathbf{r}_2(t) \) be vector valued functions in \( \mathbb{R}^3 \). Determine whether the following are true or false.

(a) \( \text{T \ F} \) If \( \mathbf{a} \) and \( \mathbf{b} \) are unit vectors, then \( \mathbf{a} \cdot \mathbf{b} = 1 \).

(b) \( \text{T \ F} \) \( \mathbf{b} \cdot (\mathbf{a} \times \mathbf{b}) = 0 \)

(c) \( \text{T \ F} \) If \( \mathbf{r}_1(t) \) and \( \mathbf{r}_2(t) \) are parameterizations of perpendicular lines, then \( \mathbf{r}_1(0) \) is perpendicular to \( \mathbf{r}_2(0) \).

(d) \( \text{T \ F} \) \( (\mathbf{a} + \mathbf{b}) \times \mathbf{a} = \mathbf{b} \times \mathbf{a} \)

(e) \( \text{T \ F} \) If the projection of \( \langle 2, 3, 4 \rangle \) onto \( \mathbf{b} \) is \( \langle 2, 3, 0 \rangle \), then \( \langle 0, 0, 4 \rangle \perp \mathbf{b} \).

2. Let \( \mathbf{u} = \langle 1, 1, 2 \rangle \) and \( \mathbf{v} = \langle 1, 1, 3 \rangle \). Find the decomposition of \( \mathbf{u} \) along \( \mathbf{v} \) (which means find \( \mathbf{u}_{\parallel \mathbf{v}} \) and \( \mathbf{u}_{\perp \mathbf{v}} \)) and directly show that one component is parallel to \( \mathbf{v} \) and the other is perpendicular to \( \mathbf{v} \).

3. Which path has higher curvature at \( (0, 0) \): The graph of \( y = x^6 \) or the graph of \( y = x^2 \)?

**Hint:** Sketch (or imagine) these two graphs.
4. Find a parameterization of the line tangent to the path \( r(t) = \langle t^2 + t, \cos^2(t), e^t \rangle \)
when \( t = 0 \).

5. Consider the parametric function \( r(t) = \langle \ln(t), 8t, 8t^2 \rangle \)
   (a) Find the length of this curve between \( t = 1 \) and \( t = 2 \).
       You may leave your answer unsimplified.

   (b) What is the unit tangent vector \( T(t) \) for the curve \( r(t) \)?
6. Find a parameterization of the curve at the intersection of the cylinder $y^2 + z^2 = 16$ and the surface $z^2 + x + y = 4$. 
7. Consider the surface described by the equation

\[ x^2 - y^2 + z^2 = 2 \]

(a) For each of the planes below, state if the trace is a hyperbola, parabola, ellipse or nothing. You do not need to sketch them.

- \( x=0 \)
- \( y=0 \)
- \( z=0 \)

(b) Which of these pictures corresponds to this surface? (Circle the correct one.)
8. Let $R$ be the region described in rectangular coordinates as:

\[
\sqrt{x^2 + y^2} \leq z \leq \sqrt{1 - x^2 - y^2}
\]

\[x \geq 0\]

(a) Describe this region in cylindrical coordinates.

(b) Describe this region in spherical coordinates.
9. At time $t = 0$ two particles depart from the origin along the trajectories $\mathbf{r}_1(t) = \langle 3t, 2t, 3t \rangle$ and $\mathbf{r}_2(t) = \langle t, 3t, 5t \rangle$. Find a formula for the area of the triangle formed by the two particles positions and the origin at any time $t$.

10. Find the equation of a plane which contains the point $P = (2, 3, 2)$ and is perpendicular to the line

$$\mathbf{r}(t) = \langle 2, 1, 1 \rangle t + \langle 3, 1, 1 \rangle$$