Where I give a numerical approximation for an integral, if you want to test your answer, use Wolfram Alpha or some similar tool to evaluate the integral you were able to set up.

1. (a) T, (b) F, (c) F, (d) T, (e) T

2. \[ \int_{\pi/2}^{\pi} \int_0^\pi \int_2^3 \rho^4 \sin^3 \phi \, d\rho \, d\theta \, d\phi \approx 88.4 \]

3. (a) I accidentally made this rely on a quadratic that isn’t the nicest ... but really we should all be able to handle any quadratic equation life throws at us.

   Lets do this in cylindrical coordinates. The lower and upper surfaces become \( r^2 \leq z \leq \sqrt{4 - r^2} \). The intersection, which forms the boundary for the projection into the xy-plane, occurs where \( z + z^2 = 4 \).

   The \( z \)-coordinate where the intersection occurs is positive, so by the quadratic equation \( z = \frac{1 + \sqrt{17}}{2} \). Substituting this back into one of the equations gives \( r^2 = x^2 + y^2 = \frac{1 + \sqrt{17}}{2} \). Lets call the square root of that \( a = \sqrt{\frac{1 + \sqrt{17}}{2}} \). The \( a \) is for apologies that it’s ugly.

   \[
   \text{vol} = \int_0^{2\pi} \int_0^a \int_{\sqrt{4-r^2}}^\sqrt{4-r^2} 1 \cdot r \, dz \, dr \, d\theta \approx 2.83
   \]

   (b) By symmetry of the region (which is enough for the centroid/uniform density) \( \bar{x} = \bar{y} = 0 \).

   The great thing about centroid is that all the integrals have the same bounds. So

   \[
   \bar{z} = \frac{1}{\text{vol}} \int_0^{2\pi} \int_0^a \int_{\sqrt{4-r^2}}^\sqrt{4-r^2} z \cdot r \, dz \, dr \, d\theta \approx 0.75
   \]

4. A is (i), B is a typo and would be \( \int_{-\pi/2}^{\pi/2} f(r, \theta) r \, dr \, d\theta \), C is (iii).
5. (a) The four sides \( u = 0, \ u = 2, \ v = 0, \ v = 2 \) become the lines \( y = -2x, \ y = 10 - 2x, \ y = \frac{1}{2}x, \ y = \frac{1}{2}x + 5. \)

(b) \( \int_0^2 \int_0^2 -3ve^{5u} \ du \ dv \approx -26430 \)

6. A little sketch reveals that this domain of integration is the tetrahedron with \( x, y \) and \( z \) all \( \geq 0 \), sitting under the plane \( 6 = 3x + 2y + z \). To find the projection into any plane, set the relevant variable equal to zero. For example in the \( yz \)-plane this is the triangle under the line \( 6 = 2y + z \).

(a)
\[
\int_0^3 \left( \int_0^{2 - \frac{5}{2}y} \left( \int_0^{6 - 2y - 3x} f(x, y, z) \ dz \right) \ dx \right) \ dy
\]

(b)
\[
\int_0^3 \left( \int_0^{2 - \frac{5}{2}y + \frac{1}{2}z} \left( \int_0^{6 - 2y} f(x, y, z) \ dx \right) \ dz \right) \ dy
\]

(c)
\[
\int_0^6 \left( \int_0^{3 - \frac{3}{2}y + \frac{1}{2}z} \left( \int_0^{2 - \frac{1}{2}z} f(x, y, z) \ dx \right) \ dz \right) \ dx
\]

7. Mass = \( \int_0^{\pi/2} \int_0^{\cos \theta \sin \theta} \cos \theta \ dr \ d\theta \)

8. (a) Rectangular
\[
\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{100-x^2-y^2}} (x^2 + y^2 + z^2) \ dz \ dy \ dx
\]
(b) Cylindrical
\[ \int_{0}^{2\pi} \int_{0}^{2} \int_{0}^{\sqrt{100-r^2}} (r^2 + z^2)r \, dz \, dr \, d\theta \]

(c) Spherical: Must be done in two pieces!
\[ \left( \int_{0}^{2\pi} \int_{0}^{\sin^{-1} 1/5} \int_{0}^{10} \rho^4 \sin \phi \, d\rho \, d\phi \, d\theta \right) + \left( \int_{0}^{2\pi} \int_{\sin^{-1} 2 \csc \phi}^{\pi/2} \int_{0}^{\rho^4 \sin \phi} \rho \, d\rho \, d\phi \, d\theta \right) \]