1. Let $\mathbf{F}$ be a vector field in 3D and $f$ a scalar function of three variables. Determine whether the following are True or False.

(a) T F If $\nabla \times \mathbf{F} = 0$ then $\oint_{c} \mathbf{F} \cdot d\mathbf{r} = 0$ for any closed loop $c$.

(b) T F If the $x$- and $y$-components of $\mathbf{F}$ are 0 and $S$ is a surface with sides parallel to the $z$-axis, then the flux of $\mathbf{F}$ through $S$ is 0.

(c) T F If $\mathbf{F} = \nabla \times \mathbf{A}$ and $S$ is a closed surface, then the flux of $\mathbf{F}$ through $S$ is 0.

(d) T F In order to find the integral of $f$ around a closed loop $c$, one may apply Stokes’ theorem to transform to a scalar surface integral.

(e) T F If $S$ is a surface with normal $\mathbf{N}$ such that $(\nabla \times \mathbf{F}) \cdot \mathbf{N} = 1$ then the surface area of $S$ can be computed as $\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r}$.

2. Let $\mathbf{F}$ be a vector field in 3D and $f$ a scalar function of three variables. For each of the following, state whether the operations shown produce a Vector field, a Scalar function or whether they cannot be computed, in which case the statement is Nonsense.

(a) Scalar Vector Nonsense $\nabla \cdot \mathbf{F}$

(b) Scalar Vector Nonsense $\nabla(\nabla \cdot \mathbf{F})$

(c) Scalar Vector Nonsense $\nabla \times (\nabla \times \mathbf{F})$

(d) Scalar Vector Nonsense $(\nabla f) \times \mathbf{F}$

(e) Scalar Vector Nonsense $(\nabla \cdot \mathbf{F}) \cdot \mathbf{F}$

3. Find the total charge contained in a straight wire going from (0,1,0) to (2,2,3) with charge density $\delta(x,y,z) = x^2 + y + z^2$ Coulombs/meter.
4. Define the following terms. If they are defined by integrals, try to include an appropriate integrand and domain for the integral. For integral theorems, discuss orientation requirements.

(a) The *flux* of \( \mathbf{F} \) through surface \( S \).
(b) The *circulation* of \( \mathbf{F} \) around closed loop \( c \).
(c) The *mass* of a surface \( S \) with density \( \delta(x, y, z) \).
(d) The *work* done by field \( \mathbf{F} \) moving a particle along path \( c \).
(e) An *irrotational* vector field \( \mathbf{F} \).
(f) A *simply connected* domain \( \mathcal{D} \).

(g) If \( \mathcal{D} \) is a domain, \( \partial \mathcal{D} \) its boundary curve with standard boundary orientation and \( \mathbf{F} = \langle F_1, F_2 \rangle \) is a 2D vector field, **Green’s Theorem** states:

(h) If \( c \) is a curve oriented from \( P \) to \( Q \) and \( \mathbf{F} = \nabla(f) \) is a conservative vector field, the **Fundamental Theorem of Conservative Fields** states:

(i) If \( \mathcal{W} \) is a volume, \( \partial \mathcal{W} \) is its closed surface boundary with outward oriented normal and \( \mathbf{F} \) is a vector field, the **Divergence Theorem** states:

(j) If \( \mathcal{S} \) is a surface, \( \partial \mathcal{S} \) its boundary curve with standard orientation and \( \mathbf{F} \) is a vector field, then **Stoke’s Theorem** states:
5. Let $S$ be a cylinder $x^2 + y^2 = 16$, $0 \leq z \leq 1$, with outward facing normal vector. Let $c_1$ be the circle at the bottom of $S$ and $c_2$ be the circle at the top of $S$, both oriented counterclockwise (viewed from above). Suppose $F$ is a vector field such that $\nabla \times F = (z + 1, 0, 0)$ on $S$ and we know $\oint_{c_1} F \cdot dr = \pi$. What must $\oint_{c_2} F \cdot dr$ be?

6. Let $c_1$ be the perimeter of the square $[-4, 4] \times [-4, 4]$, $c_2$ the perimeter of $[1, 2] \times [-2, 2]$ and $c_3$ the perimeter of $[-2, -1] \times [-2, 2]$, all oriented counterclockwise. Suppose $F$ is conservative everywhere outside $c_2$ and $c_3$, and we know $\oint_{c_1} F \cdot dr = 4\pi$ and $\oint_{c_2} F \cdot dr = -5\pi$. Find $\oint_{c_3} F \cdot dr$. 
7. A torus is parameterized by

\[
\begin{align*}
x(u, v) &= (2 + \cos u) \cos v \\
y(u, v) &= (2 + \cos u) \sin v \\
z(u, v) &= \sin u
\end{align*}
\]

with \(0 \leq u \leq 2\pi\) and \(0 \leq v \leq 2\pi\). Find the surface area of the torus.

8. Let \(\mathbf{F} = (-x, -y, -z/2)\) and \(S\) be the surface of the cone \(z = 2\sqrt{x^2 + y^2}\) with \(0 \leq z \leq 2\) with downward facing normal. Find \(\int_S \mathbf{F} \cdot d\mathbf{S}\).
9. Let $S$ be the surface enclosing a sphere $x^2 + y^2 + z^2 = 9$ with a cylinder $x^2 + y^2 = 1$ removed from the center (so $S$ includes the surface of the cylinder inside the sphere). All normal vectors point out of the enclosed volume. Let $\mathbf{F} = (2, 2y + z, 2z + xe^x)$. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

10. Use Green’s theorem to find $\oint_c \mathbf{F} \cdot d\mathbf{r}$ where $c$ is the perimeter of the triangle with vertices $(0, 1)$, $(2, 0)$ and $(0, -1)$ oriented counterclockwise, and $\mathbf{F} = (-y + x, x^2 - y^2)$. 
11. Let \( c \) be the perimeter of the part of the plane \( z + 2x - 3y = 3 \) sitting over the region \( 1 \leq x \leq 2, \ 0 \leq y \leq 1 \). The orientation of \( c \) is counterclockwise when viewed from above. Let \( \mathbf{F} = \langle x y^2, y z^2, z x^2 \rangle \). Find \( \oint_c \mathbf{F} \cdot d\mathbf{r} \).

12. Let \( \mathbf{F} = \langle 2x + y, 2y + x, 3z^2 \rangle \).

   (a) Show that \( \nabla \times \mathbf{F} = \mathbf{0} \) and use this to justify that \( \mathbf{F} \) is conservative.

   (b) Find a potential function for \( \mathbf{F} \).

   (c) Use the Fundamental Theorem of Conservative Fields to compute \( \int_c \mathbf{F} \cdot d\mathbf{r} \) where \( c \) is the quarter of the circle \( x^2 + y^2 = 4 \) with \( x, y \geq 0 \) and \( z = 3 \), oriented counterclockwise.