1. Let $C$ by the semicircular path from $(0,0)$ to $(2,0)$ with $y \geq 0$. Let $\vec{F}(x,y) = <y^3 + 1, 3xy^2 + 1>$. We are interested in evaluating $\int_C \vec{F} \cdot d\vec{s}$.

(a) [4] Parametrize $C$, then express $\int_C \vec{F} \cdot d\vec{s}$ as an integral in terms of your parametrization. In other words, substitute into the vector field, compute the appropriate dot product, and include appropriate limits. This integral should be a mess, don’t evaluate it it.

(b) [4] Find a potential function and use the Fundamental Theorem of Line Integrals to evaluate $\int_C \vec{F} \cdot d\vec{s}$.

(c) [4] Since $\vec{F}$ is conservative, we know that the integral is independent of path. In particular, we can evaluate the along any path with the same endpoints. Let $C_2$ be the line segment from $(0,0)$ to $(2,0)$. Parametrize the line segment and compute $\int_C \vec{F} \cdot d\vec{s} = \int_{C_2} \vec{F} \cdot d\vec{s}$. Do not apply the Fundamental Theorem.

2. Let $S$ be the hemispherical shell $x^2 + y^2 + z^2 = R^2$ with $z \geq 0$. The surface has a mass density proportional to the distance from the $xy$-plane, i.e. $\delta(x,y,z) = kz$ for some constant $k$.

(a) [6] Compute the mass $M$.

$$M = \int_S \delta(x,y,z) dS$$

(b) [6] Find the center of mass. Specifically, compute

$$\bar{z} = \frac{1}{M} \int_S z\delta(x,y,z) dS.$$ 

3. [6] Let $\vec{F} = <x, y, z>$ and $S$ be the part of $z = 9 - x^2 - y^2$ with $z \geq 0$ oriented with an upward normal vector, compute

$$\iint_S \vec{F} \cdot d\vec{S}.$$