We will be verifying the three theorems from Chapter 17 on this homework. See Example 1 in Section 17.1, Example 3 in Section 17.2, and Example 2 in Section 17.3 for examples. Worth noting, please don’t try to convince me that the three theorems we spent three semesters of calculus building up to are wrong.

1. [8] Verify Green’s Theorem for the line integral along the triangle $C$ with vertices $(0,0)$, $(1,0)$ and $(1,1)$, oriented counterclockwise.

\[ \oint_C 3xy\,dx + x^2\,dy \]

2. [14] Verify Stoke’s Theorem for

\[ \vec{F} = < x + y, x, x + y + z > \]

and the part of the sphere of radius 3 in the first octant with outward-pointing normal vectors:

\[ S = \{(x, y, z) : x^2 + y^2 + z^2 = 9, x, y, z \geq 0\}. \]

Note, $\partial S$ has three components, please use $C_1$ for the component in the $xy$-plane, $C_2$ for the component in the $yz$-plane, and $C_3$ for the component in the $xz$-plane.

3. [8] Verify the Divergence Theorem for

\[ \vec{F} = < 2z, x, y^2 > \]

and the solid between the paraboloid $z = 4 - x^2 - y^2$ and the $xy$-plane.