Practice Problems for the M221 Final, Fall 2010

1. True or false? Justify your answers.

(a) If a linear system of equations $A\mathbf{x} = \mathbf{b}$ has more than one solution, then it has infinitely many solutions.

(b) If A and B are two invertible n by n matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

(c) Given an n by n matrix A, the matrix $B = A + A^T$ is always symmetric.

(d) The solutions to $A\mathbf{x} = \mathbf{b}$ form a subspace.

(e) If A is a singular n by n matrix, then A^2 is also singular.

(f) If λ is not an eigenvalue of A, then $A - \lambda I$ is invertible.

2. If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are unit vectors such that $\mathbf{u}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$, and the angle between $\mathbf{v}$ and $\mathbf{w}$ is $45°$, find the length of the sum $\mathbf{u} + \mathbf{v} + \mathbf{w}$.

3. What 3 by 3 matrix multiplies (x, y, z) to give $(x + 2z, x + y - z, 2x + z)$?

4. Solve the following system of equations by elimination with matrices.

$$\begin{aligned} x + 2z &= -1 \\ x + y - z &= 2 \\ 2x + z &= 1 \end{aligned}$$

5. Find the determinant and inverse of

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$$

6. Find the rank, and dimensions and bases for all four subspaces of

$$A = \begin{bmatrix} 2 & -2 & -2 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

7. With the same matrix A as in problem 6, find conditions on (b_1, b_2, b_3, b_4) for solvability of $A\mathbf{x} = \mathbf{b}$. Find the complete solution for $\mathbf{b} = (0, 2, 0, 2)$.

8. Let A be an invertible 3 by 3 matrix. Find the four subspaces, rank, and dimension of the block matrices

$$B = \begin{bmatrix} A & A \end{bmatrix} \text{ and } C = \begin{bmatrix} A & A \\ A & 0 \end{bmatrix}.$$

9. If $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions and $\mathbf{c}$ is another vector, how many solutions can $A\mathbf{x} = \mathbf{c}$ possibly have?

10. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$$

11. A 2 by 2 matrix A has eigenvalues 1 and 2 with corresponding eigenvectors $\mathbf{x}_1 = (1, 1)$ and $(-1, 1)$. Find A. (Hint: Diagonalization.)