Third Practice Test Key, M221-01, Fall 2010

1. True or false? Justify your answers.

(a) $\det(2I) = 2$

(b) If an n by n matrix A has n different eigenvalues, then A is diagonalizable.

(c) If an n by n matrix A has n different eigenvalues, then A is invertible.

(d) If a matrix A has only real eigenvalues, then $A^2 + I$ is invertible.

2. Find the determinant of

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 0 & 3 & 0 \\ 5 & 1 & 2 & 3 \\ 1 & 6 & 4 & 3 \end{bmatrix}.$$

3. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$

4. (a) For which parameters a is the following matrix diagonalizable?

$$A = \begin{bmatrix} 1 & 0 \\ 1+a & a^2 \end{bmatrix}.$$

(b) Find the diagonalization $A = S\Lambda S^{-1}$ for the case $a = 2$.

5. (a) The matrix

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & -2 \\ 3 & -3 & 1 \end{bmatrix}$$

has an eigenvector $\mathbf{x}_1 = (1, 1, 0)$. Find the corresponding eigenvalue λ_1.

(b) Knowing that another eigenvalue is $\lambda_2 = 4$ (you don't have to verify this), what is the third eigenvalue λ_3?

6. Let R be a 3 by 3 matrix corresponding to a 120° rotation about the line $x = y = z$ in space.

(a) Find one eigenvalue and corresponding eigenvector of R. (Hint: Try the direction of the axis of rotation.)

(b) What are the other two eigenvalues? (Hint: If you apply R three times you get a rotation by 360° which is the same as not rotating at all, so $R^3 = I$.)