Final Exam Review Problems, M182, Spring 2013

1. Let $m > 0$ be a positive constant.

(a) Find the area of the region enclosed by the graphs of $y = x^2$ and $y = mx$.

(b) Set up the integrals for, but do not evaluate, the volume and the surface area of the solid obtained by rotating the region in (a) about the x-axis.

2. How much work is done lifting a 12-m chain that has mass density 3 kg/m (initially coiled on the ground) so that its top end is 10 m above the ground?

3. Evaluate the following integrals.

(a) $\displaystyle\int x^2 e^{4x}\, dx$

(b) $\displaystyle\int \frac{2x-1}{x^2-5x+6}\, dx$

(c) $\displaystyle\int \ln(x^2+9)\, dx$

(d) $\displaystyle\int \frac{dx}{\sqrt{9-x^2}}\, dx$

4. Determine for which $p > 0$ the improper integral $\displaystyle\int_0^\infty \frac{x}{\sqrt{x^p+1}}\, dx$ converges.

5. Find the Taylor polynomial $T_4(x)$ centered at $x = 1$, for the function $f(x) = x \ln x$.

6. Solve the differential equation $y' = 1 - y^2$ with initial value $y(0) = 0$.

7. Determine whether the following series converge absolutely, conditionally, or not at all.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{1}{n}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n}$

8. Find the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$. (No need to test the endpoints here.)

9. Find the Taylor series of $f(x) = \tan^{-1}(2x)$ centered at $c = 0$, and determine the interval on which it converges.