

Statistical Analysis for Financial Data: A Case Study of Four Stocks

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Abstract

This paper investigates different models for predicting stock returns through a case study involving four stocks from 2013 to 2018. By examining these methods, the predictive ability of several common, and some less common, financial models are compared. This paper uses returns from four stocks, Harmony Gold Mining Co., Hewlett-Packard, Target Corporation, and Tiffany and Co., from 2013 to 2017 to train models and compares the predictions made for the 2018 data to the actual returns. These models include the Capital Asset Pricing Model, the Fama-French 3- and 5-Factor Models, as well as variants of these models that include an autoregressive moving average temporal structure and generalized additive models. This paper concludes that the autoregressive moving average and generalized additive models improve violations with more classical linear models; however, many of these models had difficulty predicting for 2018 data. Only a few models provide remarkable increases in predictive ability beyond the mean-only model, likely due to macroeconomic differences in 2018 compared to 2013 to 2017.

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Introduction

Over the years, financial methods for analyzing stock returns have vastly improved, but, within an academic setting, the most common method that is taught to estimate returns is the Capital Asset Pricing Model (CAPM). Although the CAPM, a simple linear regression model for the monthly returns on the stock versus the Expected Market Risk Premium (defined as the market return minus the risk-free rate), has been proven to be a poor descriptor and predictor of returns, it continues to be taught extensively in academic settings (Fama & French, 2004).

Predicting returns is important because stockholders are interested in whether investments will generate a return that is high enough. In many cases, returns will include payments from dividends as well as fees and transaction costs; however, the scope of this paper will be limited to capital gains and losses from increases and decreases in the stock price. This paper aims to compare several different methods for predicting returns with the CAPM to see whether using more complex models helps the accuracy of predictions, focusing on four stocks of varying volatility to the market. The methods being evaluated in this paper are the CAPM, the Fama-French 3- and 5-Factor Models (Fama & French, 2015), Autoregressive Moving Average Models with and without covariates (Cryer & Chan, 2008), and Generalized Additive Models (Wood, 2017). All models and plots were generated using R (R Core Team, 2018).

All models are trained on data from 2013 to 2017 and predictions are made for 2018. The returns from 2018 for all four stocks are compared to the predictions from every model using mean absolute deviation (MAD) and mean squared error (MSE) to identify the models with the best predictions. The models with the best predictive ability were the Fama-French 3-Factor Model for both HMY and HPQ, the Capital Asset Pricing Model with an ARMA structure for TGT, and the Fama-French 5-Factor Generalized Additive Model for TIF. These models offered little

improvement in prediction over the mean-only model. In contrast to the previous five years, 2018 was particularly different economically. The method used in this paper makes the underlying assumption that the training set will not differ too much from the testing set; these macroeconomic differences may have changed the relationship between the returns and covariates of interest so that the models estimated using data from 2013-2017 do not accurately represent the 2018 data. This shows the dangers of extrapolation in situations where the process being studied may be changing over time.

Data

The dataset includes six years (2013-2018) of monthly returns from four stocks with different variabilities: Tiffany and Company (TIF), Hewlett-Packard (HPQ), Target (TGT), and Harmony Gold Mining (HMY). The monthly returns are calculated from the daily closing prices at the end of each month, such that $R_{i,t} = \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}}$ where $S_{i,t}$ is the stock price of stock i at time t and $S_{i,t-1}$ is the stock price at the end of the previous month (FactSet Research Systems, n.d.).

Figure 1 displays the time series of returns for the selected stocks.

Further, the dataset includes the five Fama-French factors which can be used as potential explanatory variables. These are the Expected Market Risk Premium (EMRP), the return of the "market" (S&P 500) minus the risk-free rate; Small Minus Big (SMB), the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios; High Minus Low (HML), the average return on the two value portfolios minus the average return on the two growth portfolios; Robust Minus Weak (RMW), the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios; and Conservative Minus Aggressive (CMA), the average return on the two conservative investment portfolios minus the average return on the two aggressive portfolios (Kenneth R.

French Data Library, n.d.). Figure 2 shows scatterplots of the returns vs. each of the five Fama-French factors.

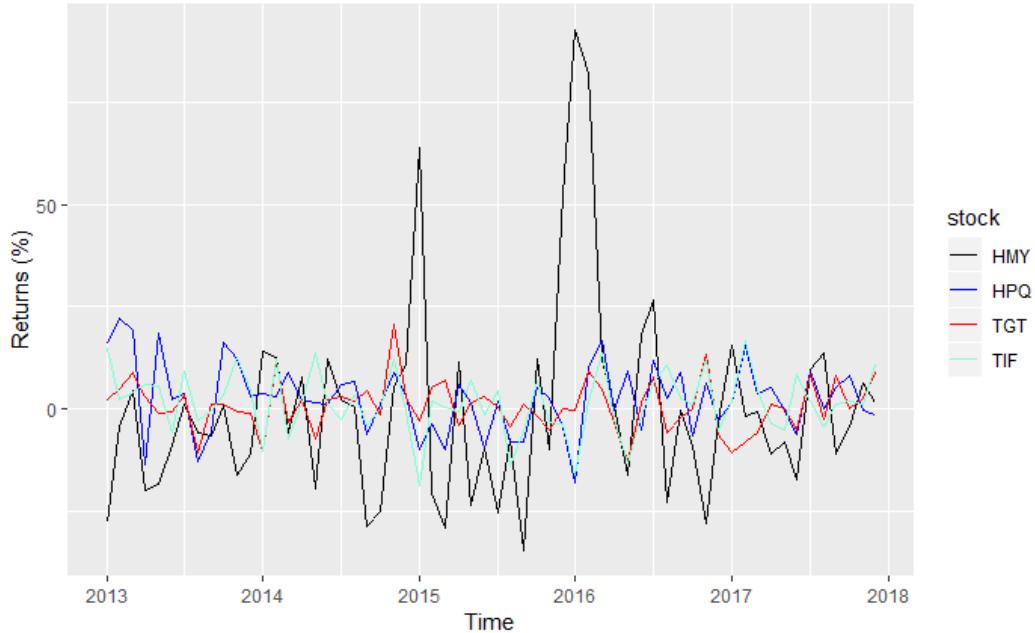


Figure 1: Monthly Returns (2013-2017) for Four Selected Stocks.

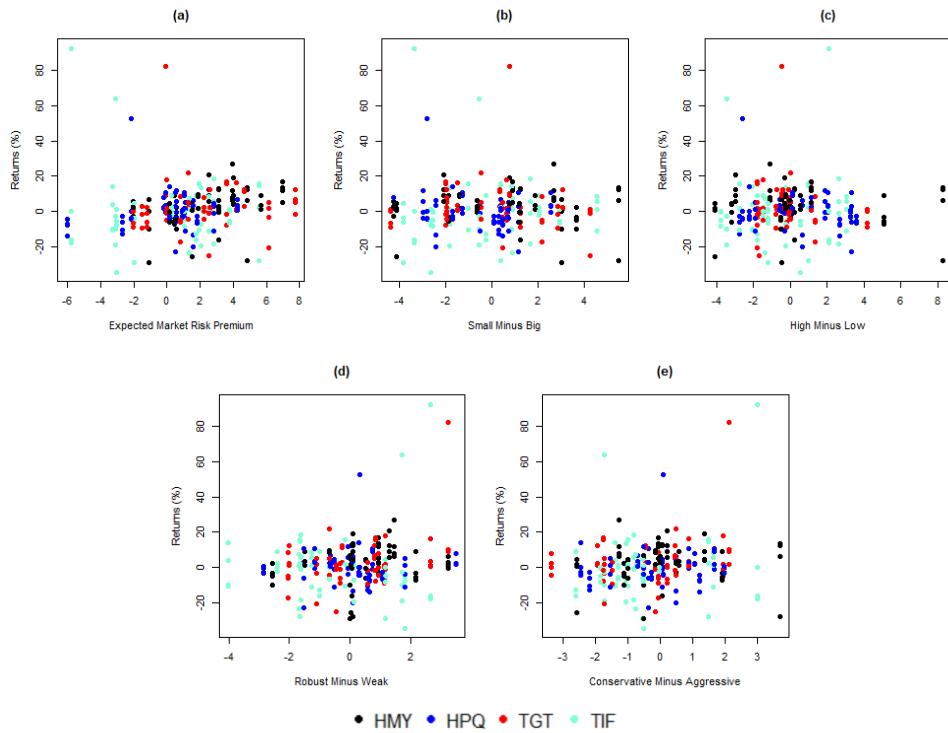


Figure 2: Returns vs. Fama-French Factors by Stock w/ (a) EMRP, (b) SMB, (c) HML, (d) RMW, (e) CMA.

Analysis

Capital Asset Pricing Model

In the 1960s, financial theory was making relatively fast headway and several researchers converged on a common theory-based model now known as the Capital Asset Pricing Model. Most of the late 1960s and early 1970s were spent refining the models developed by Sharpe, Lintner, Mossin, and Treynor into a single model, which became the most common model in finance for several decades before Eugene Fama and Kenneth French introduced the Fama-French 3-Factor Model (French, 2003).

The Capital Asset Pricing Model (CAPM) is the most basic method for modeling stock returns. It is a simple linear regression model that regresses the return at time t , R_t , against the expected market risk premium for each stock, separately. In theory, it is assumed that

$$R_{i,t} = \alpha_i + \beta_i [E(R_{M,t}) - R_{f,t}] + \epsilon_{i,t}$$

where $R_{i,t}$ is the estimated return of stock i at time t , α_i is the intercept of stock i , $R_{f,t}$ is the risk-free rate at time t , $E(R_{M,t})$ is the expected market return at time t , and β_i is the “beta” of stock i , and $\epsilon_{i,t}$ is the error term of stock i at time t that is assumed to be *i.i.d.* $N(0, \sigma_i^2)$. In finance, “beta” is defined as a measure of the volatility of the stock compared to the market; statistically, it is the slope coefficient. In practice, the expected market return is replaced with the return on the S&P 500.

The intercept minus the risk-free rate is Jensen's alpha ($\hat{\alpha}_{i,t} = \hat{\alpha}_i - R_{f,t}$), a measure of excess returns. If Jensen's alpha is negative, this means the stock underperformed expectations and if it is positive, the stock outperformed expectations. On average, Jensen's alpha tends toward zero over time. As one time period is supposedly not indicative of another when running this type of

analysis, Jensen's alpha will be set to zero when making predictions for 2018. It may compromise predictive performance to not use the estimated intercept from the model.

Table 1: Table of Parameter Estimates for the Capital Asset Pricing Model for Four Stocks from 2013-2017.

	Intercept	EMRP
<i>HMY</i>	2.633	-2.144
<i>HPQ</i>	0.362	1.742
<i>TGT</i>	-0.188	0.629
<i>TIF</i>	-0.922	1.873

The assumptions for the CAPM are both statistical and financial in nature. Statistically, the errors are assumed to be normally distributed, independent, and to have equal variance over t . The EMRP is also assumed to be linearly related to the returns. The financial assumptions are also relatively stringent. The model assumes that asset quantities are given, fixed, and infinitely divisible. It also assumes that all investors aim to maximize economic utility, are rational, are risk-averse, hold diversified portfolios, are price takers, can lend and borrow using the risk-free rate, trade without transaction costs, have the same market expectations, and have all information (Singal, 2018).

To visualize the estimated relationships between different predictors and the responses in these models, term plots made using the effects package are used to visualize the predicted mean response and 95% confidence intervals over the range of predictors (Fox & Weisberg, 2018). In more complex models, these plots are made by holding the other predictors at the mean values. To aid in assessing model assumptions, partial residuals and smoothing lines based on those residuals are added to plots to visualize potential missed curvature in the residuals based on each individual predictor. These plots also generalize to visualizing the results from the Generalized Additive Models that relax the linearity assumption (more details below).

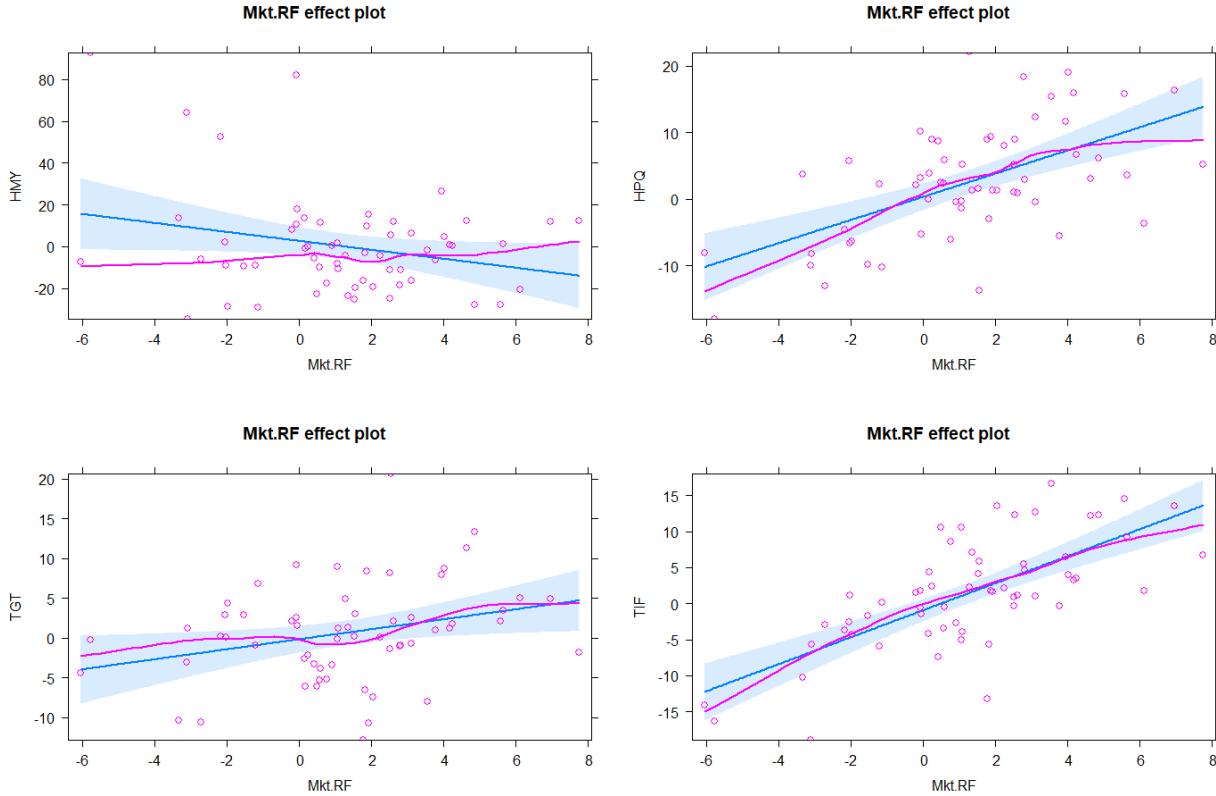


Figure 3: Term Plots for Capital Asset Pricing Models for Four Stocks.

Based on the term plots (Figure 3) for each of the Capital Asset Pricing Models, there is evidence of curvature in the residuals, suggesting that the linearity assumption may be violated. The diagnostic plots (Figures A-1, A-2, A-3, A-4) suggest potential violations of normality for HMY and violations of both linearity and constant variance for all stocks. While it is widely acknowledged that the financial assumptions of the CAPM are violated, statistical assumptions are often overlooked (Fama & French, 2004). These violations may be helped by using models with fewer statistical assumptions, such as Generalized Additive Models or that attempt to account for another violation in these models, autocorrelation, which is addressed by using ARMA models.

Fama-French 3-Factor Model

In the 1990s, Eugene Fama and Kenneth French expanded on the Capital Asset Pricing Model by adding two more factors: Small market capitalization Minus Big market capitalization

and High book-to-market ratio Minus Low book-to-market ratio. While this model was an improvement in terms of potential predictive ability, very little explanation of any theory-based reasoning was offered until 2013 (Anon., 2013). The model remains under discussion due to its country- and time-specific results (Petkova, 2006). In particular, Griffin has shown that local factors do a better job of explaining returns than the Fama-French Models (Griffin, 2002).

The Fama-French 3-Factor Model is an expansion on the CAPM that includes the Small Minus Big (SMB) and High Minus Low (HML) factors, such that

$$R_{i,t} = \alpha_i + \beta_{1,i}[E(R_{M,t}) - R_{f,t}] + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \epsilon_{i,t}.$$

Table 2: Table of Parameter Estimates for the Fama-French 3-Factor Model for Four Stocks from 2013-2017.

	Intercept	EMRP	SMB	HML
<i>HMY</i>	2.183	-1.867	-1.037	-1.304
<i>HPQ</i>	0.333	1.767	-0.157	0.215
<i>TGT</i>	-0.149	0.610	0.031	0.306
<i>TIF</i>	-0.952	1.900	-0.181	0.272

This model suffers from many of the same constraints as the Capital Asset Pricing Model. As can be seen from the term plots (Figures A-5, A-6, A-7, A-8) and diagnostic plots (Figures A-9, A-10, A-11, A-12), there continue to be substantial issues with constant variance and possible nonlinearity. The variance inflation factors (Table 3) show no evidence of multicollinearity between the three variables of interest; these were calculated using the car package (Fox & Weisberg, 2011). Because the VIFs are all near one, the multicollinearity of these predictors is not impacting estimation of these models. Another issue with these models is the lack of consideration of interactions. While this paper will not fit models with interactions present, it is certainly possible their inclusion would improve predictive ability.

Table 3: Table of Variance Inflation Factors for the Fama-French 3-Factor Models (same for all models).

Factor	EMRP	SMB	HML
VIF	1.072	1.012	1.030

Fama-French 5-Factor Model

In 2015, Fama and French introduced the Fama-French 5-Factor Model which also included a profitability factor of Robust (high) Minus Weak (low) operating profitability and an investment factor of Conservative Minus Aggressive investment. Generally, this model was shown to improve return predictions, although it is considered relatively redundant in the United States due to issues with multicollinearity (Fama & French, 2015). The Fama-French 5-Factor Model is similar to the Fama-French 3-Factor Model, but it also includes the Robust Minus Weak (RMW) and Conservative Minus Aggressive (CMA) factors. The model is

$$R_{i,t} = \alpha_i + \beta_{1,i}[E(R_{M,t}) - R_{f,t}] + \beta_{2,i}SMB_t + \beta_{3,i}HML_t + \beta_{4,i}RMW_t + \beta_{5,i}CMA_t + \epsilon_{i,t}.$$

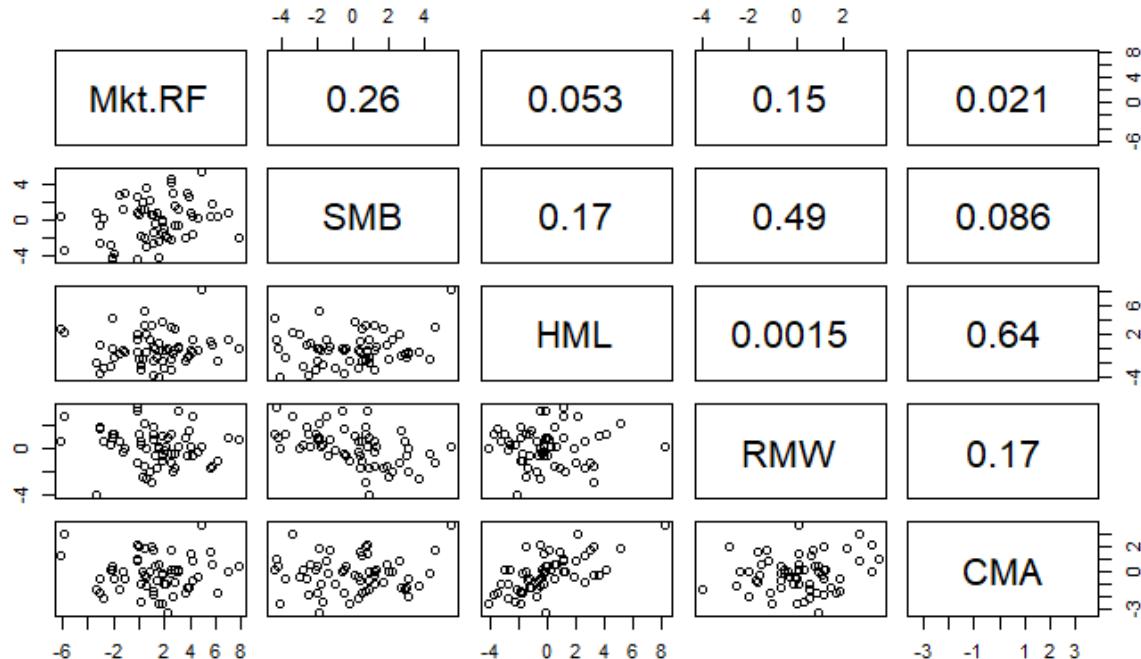


Figure 4: Pairs Plot of Fama-French Factors for 2013-2017 with Pearson correlations on upper diagonal.

Table 4: Table of Parameter Estimates for the Fama-French 5-Factor Models for Four Stocks from 2013-2017.

	Intercept	EMRP	SMB	HML	RMW	CMA
<i>HMY</i>	2.696	-1.611	0.388	-4.514	4.195	7.424
<i>HPQ</i>	0.464	1.773	-0.315	0.036	-0.491	0.518

<i>TGT</i>	0.020	0.681	0.389	-0.612	1.048	2.145
<i>TIF</i>	-0.778	1.913	-0.350	-0.017	-0.529	0.796

These models appear to have more issues with normality, linearity, and non-constant variance than the Fama-French 3-Factor Models (Figures A-17, A-18, A-19, A-20), which may suggest some degree of overfitting. While the VIFs do not show evidence of multicollinearity (Table 5), since all were less than 1.9, based on the pairs plot (Figure 4), there appears to be a relationship between the HML and CMA factors, with a Pearson's correlation coefficient of 0.64. While this appears to be in direct contradiction to what was reported by Fama and French, it is important to note that their analysis focused on monthly returns from 1963 to 2013 (Fama & French, 2015). In this shorter and newer time series, the previous multicollinearity did not manifest itself.

Table 5: Table of Variance Inflation Factors for the Fama-French 5-Factor Models (same for all models).

Factor	EMRP	SMB	HML	RMW	CMA
VIF	1.077	1.443	1.757	1.406	1.813

Autoregressive Moving Average Model

Autoregressive Moving Average (ARMA) models are used for time series data, such as the monthly returns from the four stocks of interest. For an ARMA(p,q) model, there are p autoregressive terms and q moving average terms, such that $R_{i,t} = c_i + \epsilon_{i,t} + \sum_{j=1}^p \phi_j R_{i,t-j} + \sum_{k=1}^q \theta_k \epsilon_{i,t-k}$ where c_i is the intercept, $R_{i,t-j}$ is the return at j months before time t , $\epsilon_{i,t-k}$ is the error k months before time t . This models the returns at time t as a function of the previous returns from p previous periods and a function of the errors from q previous periods (Cryer & Chan, 2008). ARIMA models were not considered due to the detrending that occurs when calculating returns rather than using stock price.

For each ARMA model, the returns or some residual value from the linear models used previously (CAPM, Fama-French 3-Factor and 5-Factor Models) were explored in the armasubsets

function in the TSA package in R (Chan & Ripley, 2018). The armasubsets function considers subset ARMA models from the residuals of the data fitted to a “long” AR model. Subset ARMA models involve considering all possible lags up to a predetermined number of both prior observations and errors and then exploring models that set all combinations of those lag coefficients to be estimated or set to zero. The ARMA model is approximated by a regression model for the residuals where the covariates are lags of the time series and error process.

The ARMA model with lowest Bayesian Information Criterion (BIC) from armasubsets was selected for each and estimated using the arima function. BIC is a penalized likelihood used to compare models using the same dataset; the model with the lowest BIC is considered the “best” model from the models being considered. The autocorrelation function (ACF) plots (Figures A-22, A-25, A-28, A-30) and partial autocorrelation function (PACF) plots (A-23, A-26, A-29, A-32) were checked for their agreement with the results from armasubsets (Figures A-21, A-25, A-27, A-30); no major issues with the selections were found and the optimal BIC ARMA subset structure was estimated for prediction error assessment.

Figure 5 shows an example of this selection process. The analyses are being run on the residuals for the Fama-French 5-Factor model for TIF. The ACF and PACF plots are not strongly suggestive of a time structure, although there are clearly spikes at lag 2, around lag 6, and after lag 12 in both the ACF and PACF plots. The maximum lag structure that is allowed in this analysis is 12, due to the returns being monthly and the overall length of the time series being only four years. In this case, the lowest BIC model has an autoregressive term at lag 6 and no error lags.

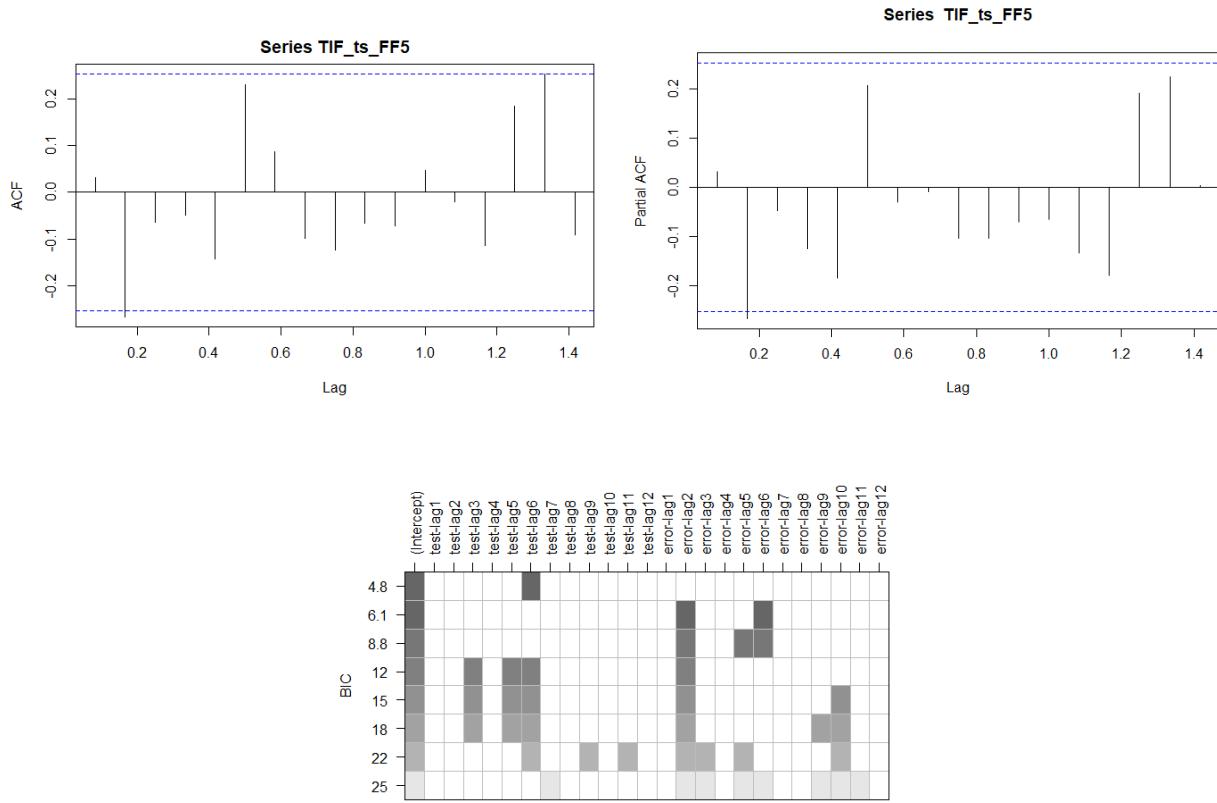


Figure 5: ACF, PACF, and armasubsets Plots for TIF Fama-French 5-Factor Residuals.

It is important to note that there are several limitations to this method. First, it is not precise, particularly because the structure is being fit to the residuals in the exploratory analysis. Second, the BIC between the two lowest BIC models is often less than two units apart, suggesting that the two models have similar support. In practice, it would be best to use armasubsets as well as the ACF and PACF plots iteratively to fit the model. This paper uses the ARMA structure associated with the lowest armasubsets BIC model and assesses these models' predictive performance.

Generalized Autoregressive Conditional Heteroskedasticity Model

In 1982, Robert Engle introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model. The model incorporated a time-varying structure on volatilities (Bera & Higgins, 1993). In the latter part of the 1980s, Bollerslev, Chou, and Kroner created the Generalized Autoregressive Conditional Heteroskedasticity model, described as an ARMA model on error

terms. This allowed for the ability to deal with issues of non-normality and non-constant variance in errors (Andersen et al., 2010).

While ARMA models are primarily concerned with modeling time series effects on returns, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are focused on modeling changes in volatility over time. The GARCH model follows the structure of $R_{i,t} = \mu_i + \sigma_{i,t}\epsilon_{i,t}$ where $\sigma_{i,t}^2 = \omega_i + \beta_i\sigma_{i,t-1}^2 + \alpha_i(R_{i,t-1} - \mu_i)^2$. It is typically used for predicting future volatilities and is not considered particularly useful for generating return predictions, because the structure is fit to the errors rather than the mean terms (Andersen et al., 2010). While it is possible to combine ARMA and GARCH models, which may improve the predictive ability of models included here, this is typically done for larger datasets, such as monthly returns over longer time periods or daily data. It is difficult to identify and fit ARMA and GARCH patterns with only 60 observations (Lai & Xing, 2008). Further, this typically requires some pattern in volatility over the years, which is not apparent in any of the time series (Figure 6).

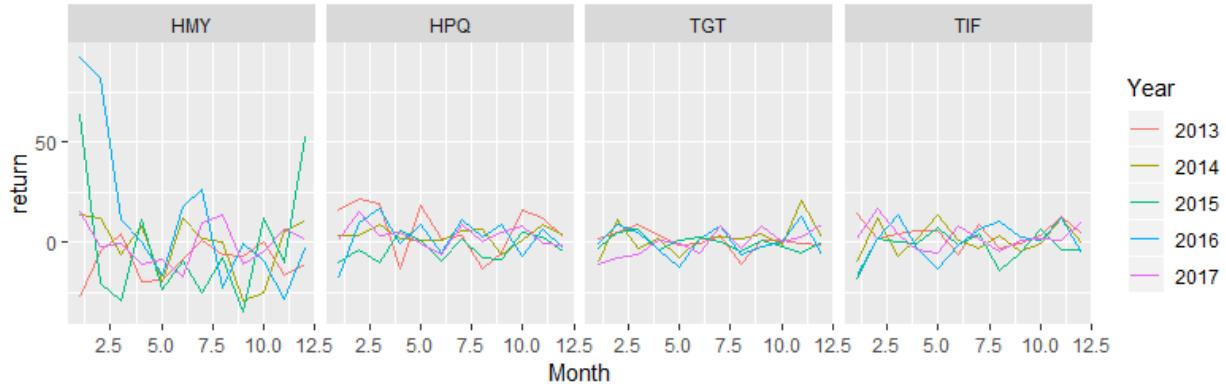


Figure 6: Returns by Month (one line per year).

Generalized Additive Model

Generalized Additive Models (GAMs) are generalized linear models where the response is fit with a sum of smoothing functions of covariates, such that $g(\mathbf{x}_i) = f_1(x_{1i}) + f_2(x_{2i}) + \dots$ such that

$R_{i,t} = g(\mathbf{x}_i) + \epsilon_{i,t}$ (Wood, 2017). In general, this allows for the ability to fit models with predictors that are non-linearly related to the response but retains the parametric assumptions about the errors mentioned before (constant variance, independence, and normality). The GAMs in this section were fit using the mgcv package in R (Wood, 2003).

The models included in this paper make use of thin-plate splines, which generate estimates of $g(\mathbf{x}_i)$ by finding the function \hat{f} which minimizes $\|y - f\|^2 + \lambda J_{md}(f)$ where $J_{md}(f)$ is a penalty function which measures the “wiggleness” of f and λ is a smoothing parameter. Thin plate splines can be computationally intensive; however, with the data included in this paper, this does not appear to be an issue, with all models converging quickly. These splines can be defined to include shrinkage and the term plots (Figures A-33, A-38 to A-41, A-46 to A-49, A-54, A-59 to A-62, A-67 to A-70, A-75 to A-78) illustrate this property, as many of the factors have been “zeroed out.” Thin plate splines have advantages over some other basis functions such as B-splines by avoiding knots that join piecewise local polynomials which can often provide better estimates of smoothed terms (Wood, 2017).

A common measurement of the “wiggleness” in GAMs is effective degrees of freedom (EDF). When using thin plate splines, a maximum number of basis functions is selected. This is based on both the number of distinct values of predictors and the sample size. GAMs are made up of smoothing functions where $f(x_i) = \sum_{j=1}^k \beta_j * b_j(x_i)$ where $b_j(x_i)$ are the set of k basis functions for a particular covariate x_i and β_j are the spline coefficients. EDF, then, is a measurement of the effective amount of information actually being used to fit the response with this predictor, x_i (Wood, 2017). Figure 7 demonstrates how increases in EDF change the fit of the GAM. As EDF increases, the fit becomes more “wiggly.” A few models had low p-values for a diagnostic related to choosing too few initial basis functions; however, upon further inspection,

the EDF was well below the maximum constraint. It is important to note that all terms with this potential issue were time factors and this issue is commonly encountered when using time as a predictor in time series with GAMs.

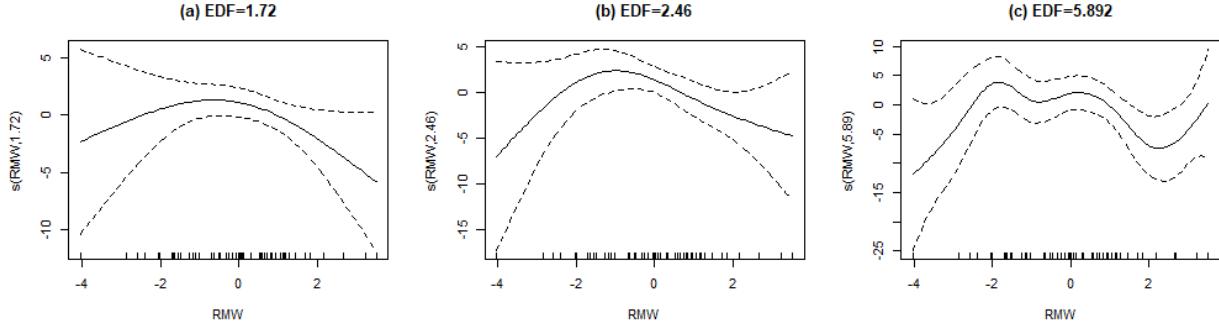


Figure 7: TIF Returns by RMW for 3 EDFs w/ (a) EDF = 1.72, (b) EDF = 2.46, (c) EDF = 5.892.

Based on the residuals vs. linear predictor plots (Figures A-34 to A-37, A-42 to A-45, A-50 to A-53, A-55 to A-58, A-63 to A-66, A-71 to A-74, A-79 to A-82), the GAM models do not appear to have any major violations of homogeneity of variance, suggesting that previous violations of this assumption in linear models may have been due in part to missed curvature. The Normal Q-Q plots suggest no major issues with normality. From an assumption standpoint, the GAMs appear to provide fewer assumption issues than the linear models. The fits of these models will also be assessed using prediction.

Predictions

Relatively few models offered a large improvement in predictive ability relative to the mean-only models as seen in Tables 6 and 7. For HMY and HPQ, the model with the lowest MAD/MSE was the Fama-French 3-Factor Model. For TGT, the model with the lowest MAD/MSE was the Capital Asset Pricing Model with an ARMA structure. For TIF, the lowest MAD/MSE model was the Fama-French 5-Factor Model.

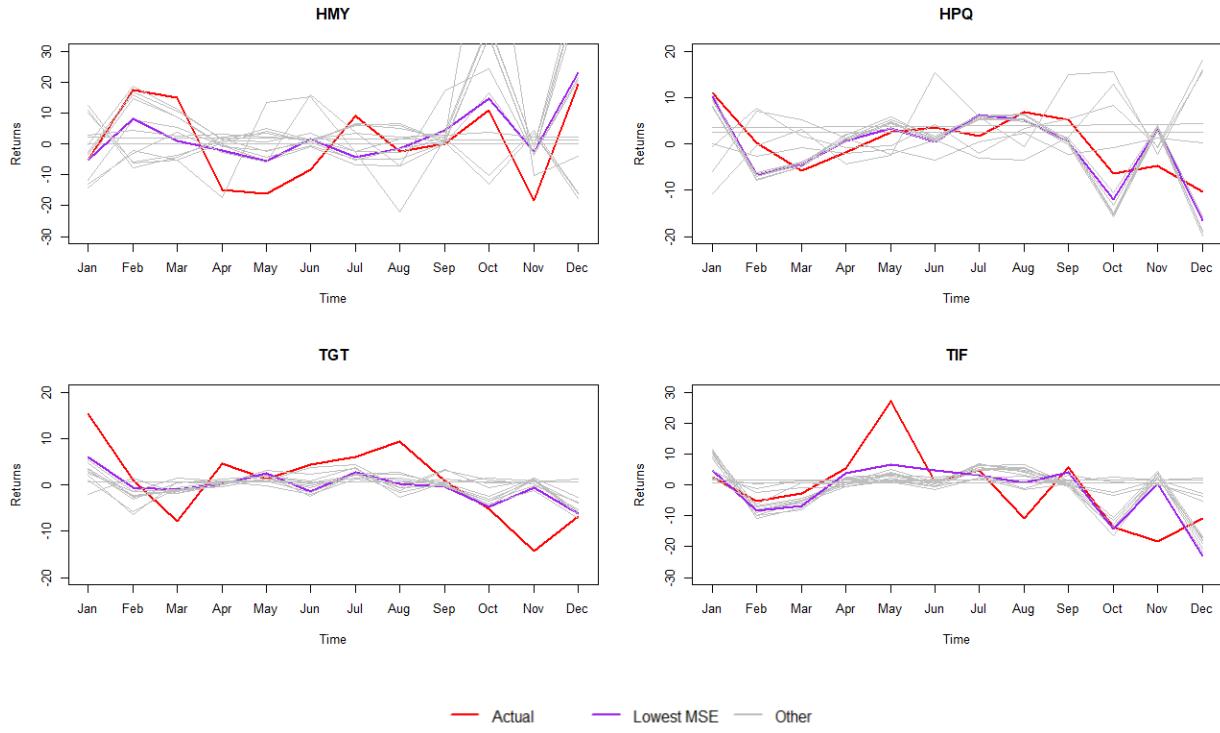


Figure 8: Actual and Predicted Returns in 2018.

Plots of the actual vs. predicted returns from all models (Figures A-83 to A-96) offer insight into how predictions diverge from the actual returns. For HMY and HPQ, the ARMA models fit substantially worse than those that did not include a time structure. This appears to be due to attempts of the model to exploit a pattern that did not exist in 2018. Further, the GAMs predicted particularly poorly for HMY, potentially due to issues with overfitting—there was far more volatility in returns for HMY in 2013-2017 than 2018. For TGT and TIF, most models had around the same predictive ability. Based on Figure 8, all models appear to make similar predictions for 2018 and they are relatively close to the actual returns. This is likely due to the lower volatility in returns for these companies over 2018. Improved predictions from ARMA models or GAMs may also be due to using the risk-free rate as an intercept in the Capital Asset Pricing Model as well as the Fama-French 3- and 5-Factor models, rather than the estimated intercept that is used in those models.

Table 6: Table of MAD from Actual vs. Predicted Returns.

	Mean Only	CAPM	FF3	FF5	ARMA	ARMA CAPM	ARMA FF3	ARMA FF5
HMY	11.44	8.43	8.23	11.98	11.50	16.47	16.69	16.47
HPQ	5.12	3.93	3.85	3.79	5.43	9.40	7.12	10.18
TGT	6.22	4.91	5.23	5.68	6.56	4.82	4.98	5.73
TIF	9.07	7.57	7.92	7.91	9.11	7.76	8.07	8.26

	GAM CAPM	GAM FF3	GAM FF5	GAM Time	GAM Time CAPM	GAM Time FF3	GAM Time FF5
HMY	10.36	10.90	20.32	11.73	11.31	12.85	20.32
HPQ	4.40	4.40	4.60	5.61	4.44	4.44	4.71
TGT	5.33	6.05	6.29	6.21	5.15	6.05	6.29
TIF	8.10	8.01	6.74	9.00	8.10	8.14	7.28

Table 7: Table of MSE from Actual vs. Predicted Returns.

	Mean Only	CAPM	FF3	FF5	ARMA	ARMA CAPM	ARMA FF3	ARMA FF5
HMY	169.66	95.03	95.02	183.35	165.67	360.72	375.81	357.98
HPQ	41.03	20.92	20.59	21.00	39.35	140.45	110.42	171.97
TGT	61.41	44.39	48.19	57.49	68.78	39.42	41.45	56.27
TIF	140.78	118.68	123.79	123.65	146.37	116.66	123.82	128.72

	GAM CAPM	GAM FF3	GAM FF5	GAM Time	GAM Time CAPM	GAM Time FF3	GAM Time FF5
HMY	174.09	198.05	759.76	171.06	218.86	266.59	759.76
HPQ	27.48	27.48	29.48	50.42	27.30	27.30	30.80
TGT	48.24	59.78	59.05	61.41	41.53	59.78	59.05
TIF	120.81	117.02	91.60	136.88	119.17	121.06	108.80

The top models do often offer predictive performance improvements over the mean-only model. The predictions from the top models are 3.21, 1.27, 1.40, and 2.33 percentage points closer to the actual returns in MAD than the mean-only models for HMY, HPQ, TGT, and TIF, respectively. The monthly return on the top 3000 largest stocks in the market was 1.09% from 2013 to 2017, which shows that these predictions were closer to the actual returns by more than the average monthly market return. The prediction loss in the top models is decreased by 44%, 50%, 36%, and 35% for HMY, HPQ, TGT, and TIF, respectively, compared to the mean-only models.

Conclusions and Discussion

Despite well-documented issues with the Capital Asset Pricing Model, it is still used ubiquitously throughout the finance community, particularly in an academic setting. This paper aimed to investigate the predictive ability of various alternative models to the Capital Asset Pricing Model for four stocks of interest. This was done by training the models on data from 2013 to 2017, then testing the predictions on data from 2018. This is particularly interesting because 2018 was different than previous years on a macroeconomic scale (Isidore, 2018).

Likely as a result of these differences, many models had difficulty predicting for 2018 data; in fact, some models only modestly outperformed the mean-only model. Despite apparent issues with statistical, as well as financial, assumptions in the linear models (Capital Asset Pricing Model, Fama-French 3-Factor Model, and Fama-French 5-Factor Model), attempts to fix these, such as including a time structure with ARMA models or dispensing with the linearity assumptions with Generalized Additive Models, offered somewhat limited improvement in predictive efficacy.

Models designed to fix some of these violations offered a better fit to the training data but did not demonstrate better predictive ability than the Capital Asset Pricing Model. Both HMY and HPQ, for instance, demonstrated autocorrelation shown in ACF and PACF plots. However, models fit with ARMA to solve this issue surprisingly did not demonstrate improved predictive performance. Perhaps more surprisingly, HMY had much worse predictions for 2018 when utilizing smooth GAM estimation, although the training data appeared to have major linearity concerns. This could indicate that the more complex models are somewhat overfit to the training data, or that 2018 differed so substantively that good models fit on the training data are not necessarily good models for predicting to 2018.

Future work may focus on modeling techniques with more lenient assumptions, such as random forests or LASSO. These methods have often been shown to offer improvement in predictive ability; however, they often lack interpretability. For this particular problem, this might not be an issue; it is important to note that many of the models considered here already lack interpretability due to the use of proxy factors and are generally only used for predictions. Moreover, since the data often violate assumptions of traditional methods, relaxing as many assumptions as possible and considering models that can allow interactions and nonlinearity to generate predictions may produce a more ideal solution.

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Plot Appendix

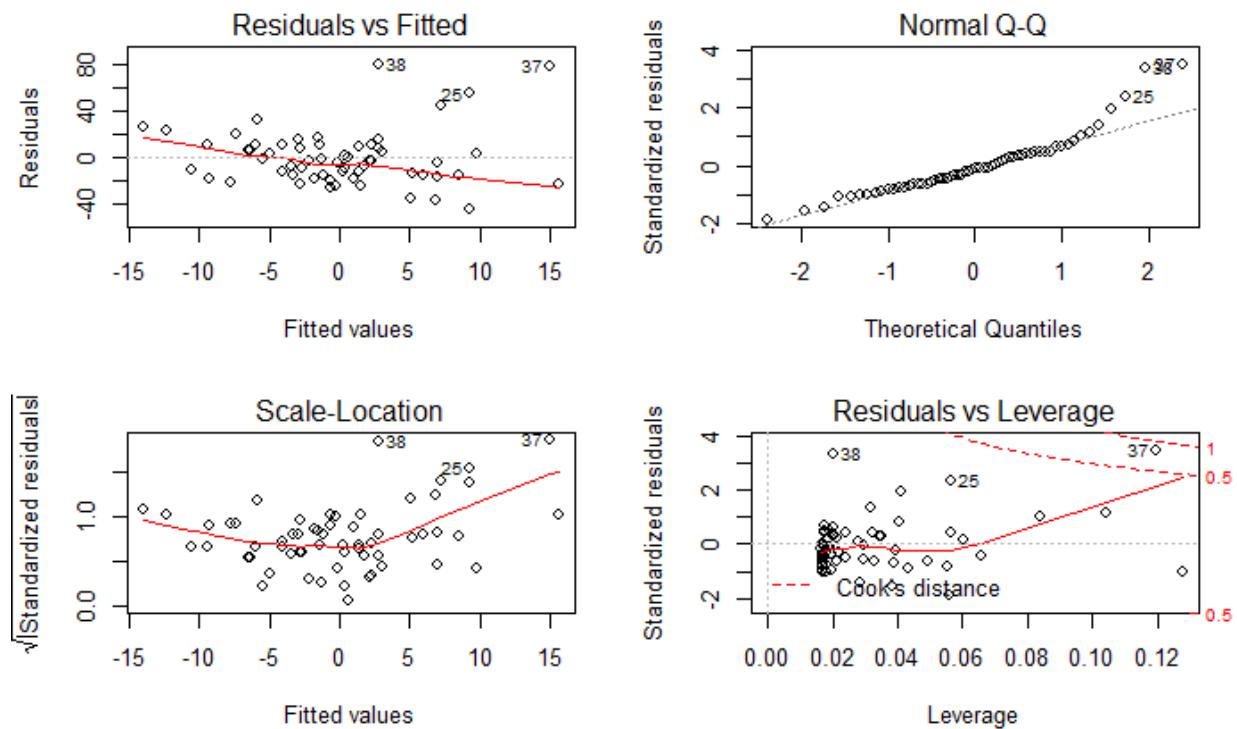


Figure A-1: Diagnostic Plots for HMY Capital Asset Pricing Model.

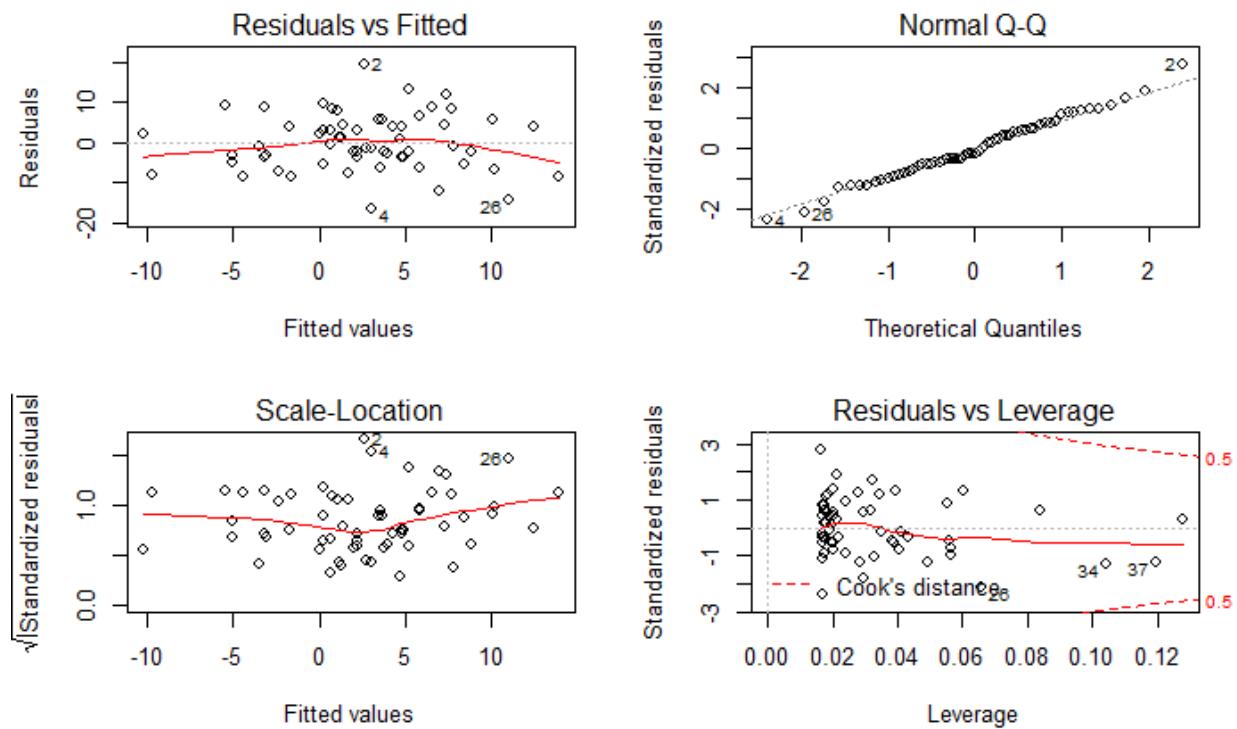


Figure A-2: Diagnostic Plots for HPQ Capital Asset Pricing Model.

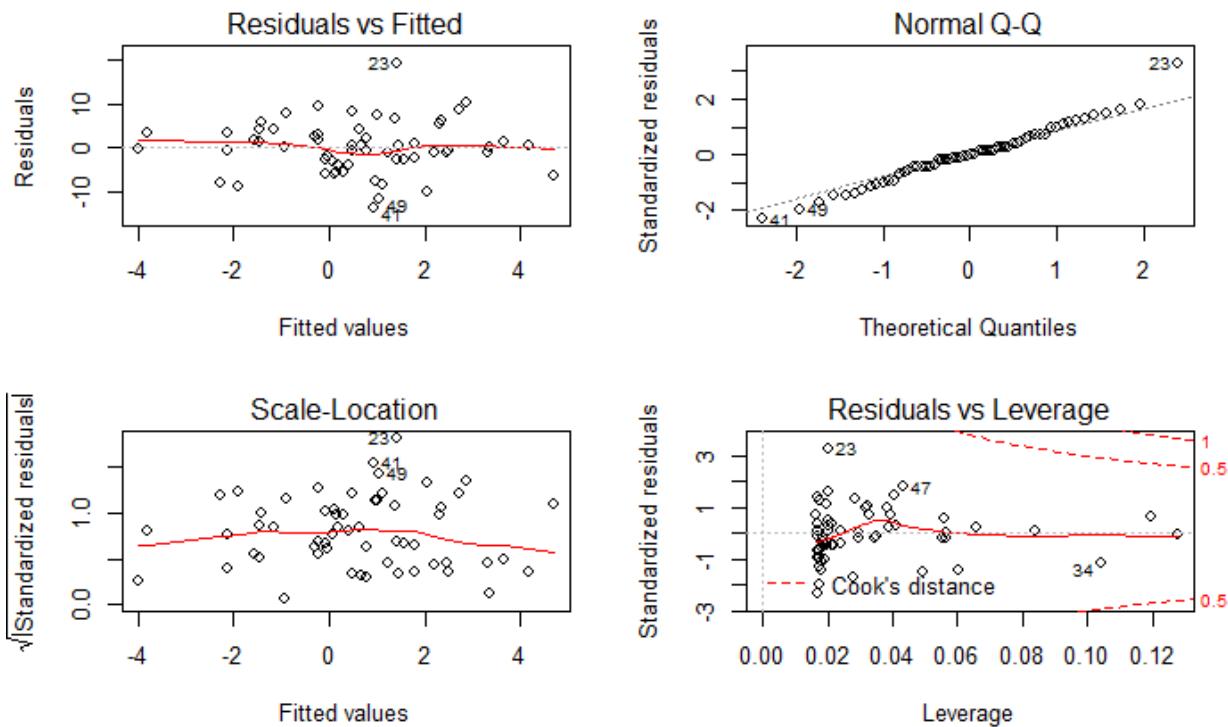


Figure A-3: Diagnostic Plots for TGT Capital Asset Pricing Model.

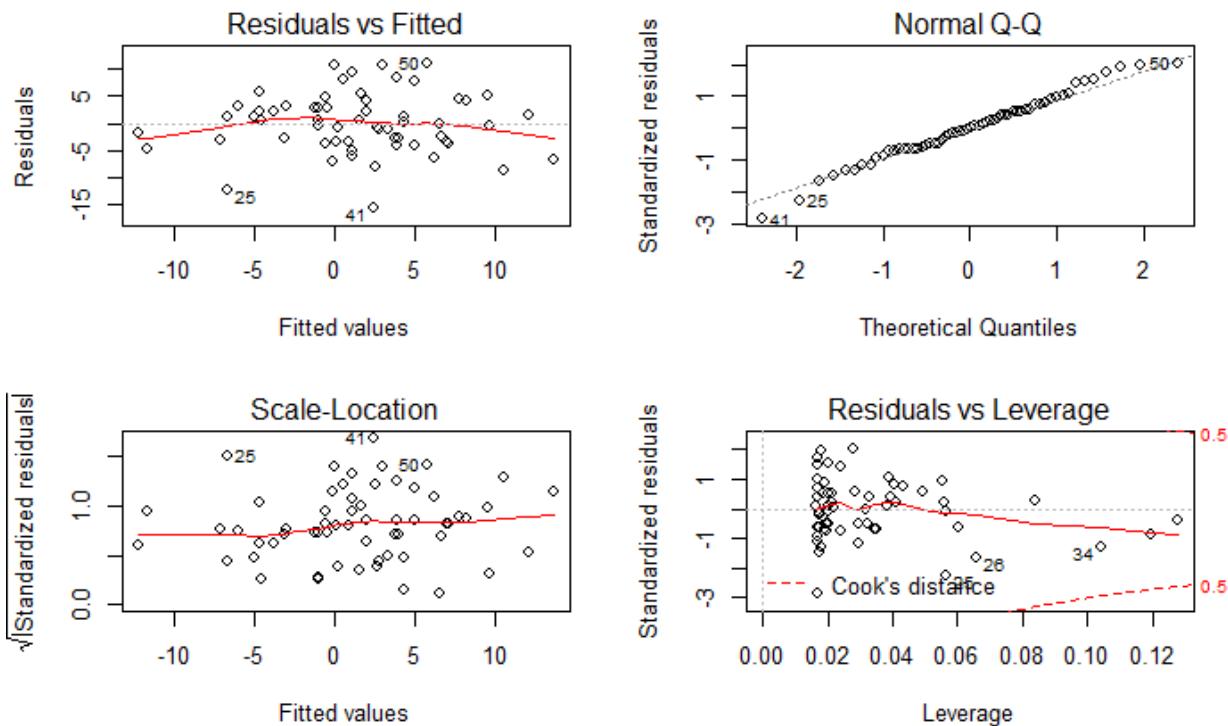


Figure A-4: Diagnostic Plots for TIF Capital Asset Pricing Model.

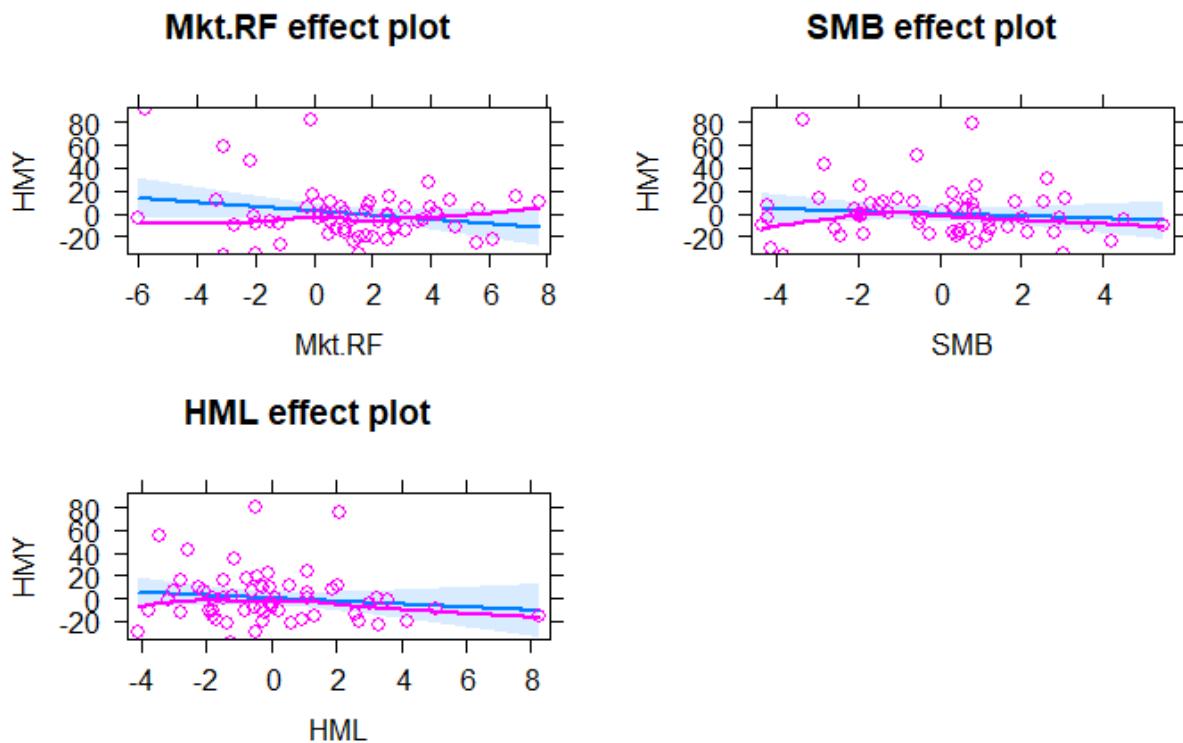


Figure A-5: Term Plots for HMY Fama-French 3-Factor Model.

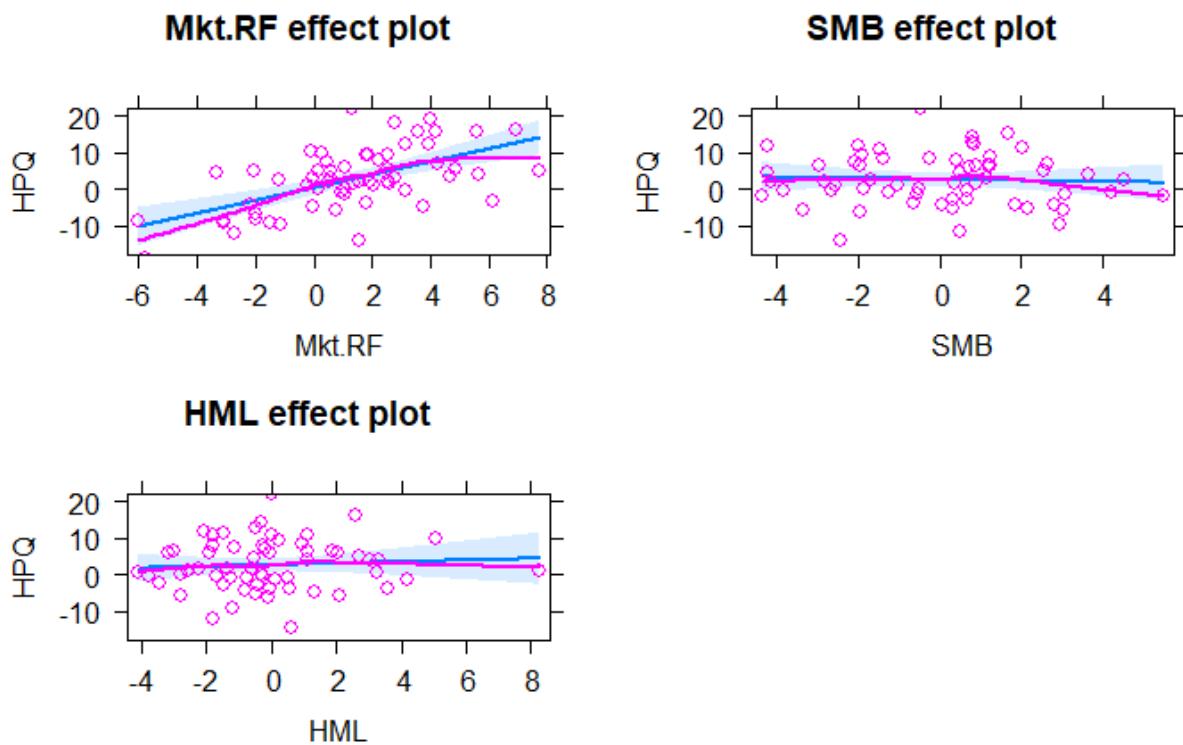


Figure A-6: Term Plots for HPQ Fama-French 3-Factor Model.

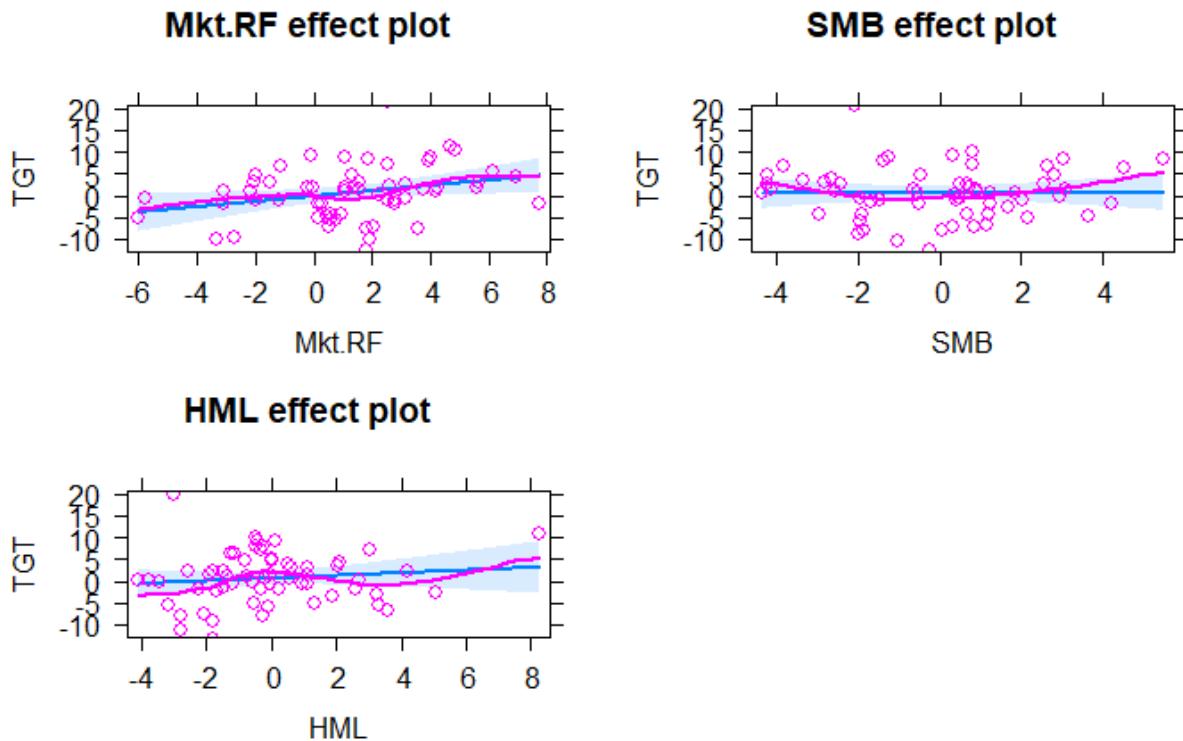


Figure A-7: Term Plots for TGT Fama-French 3-Factor Model.

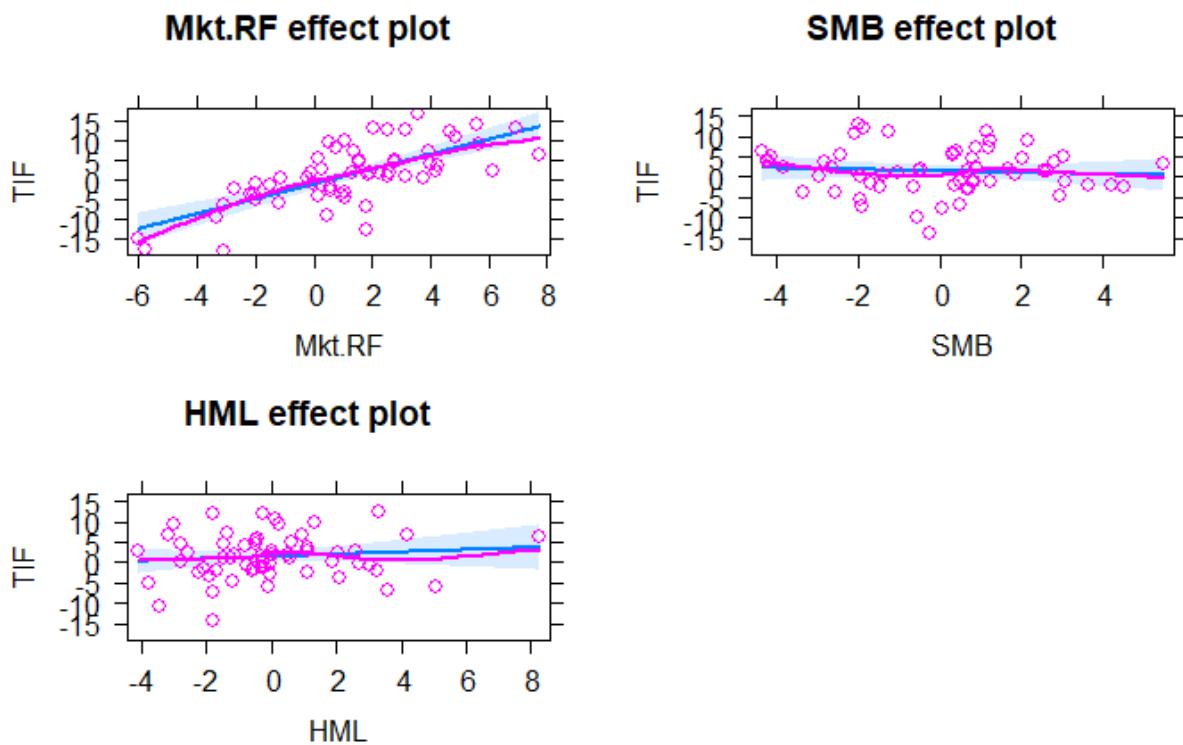


Figure A-8: Term Plots for TIF Fama-French 3-Factor Model.

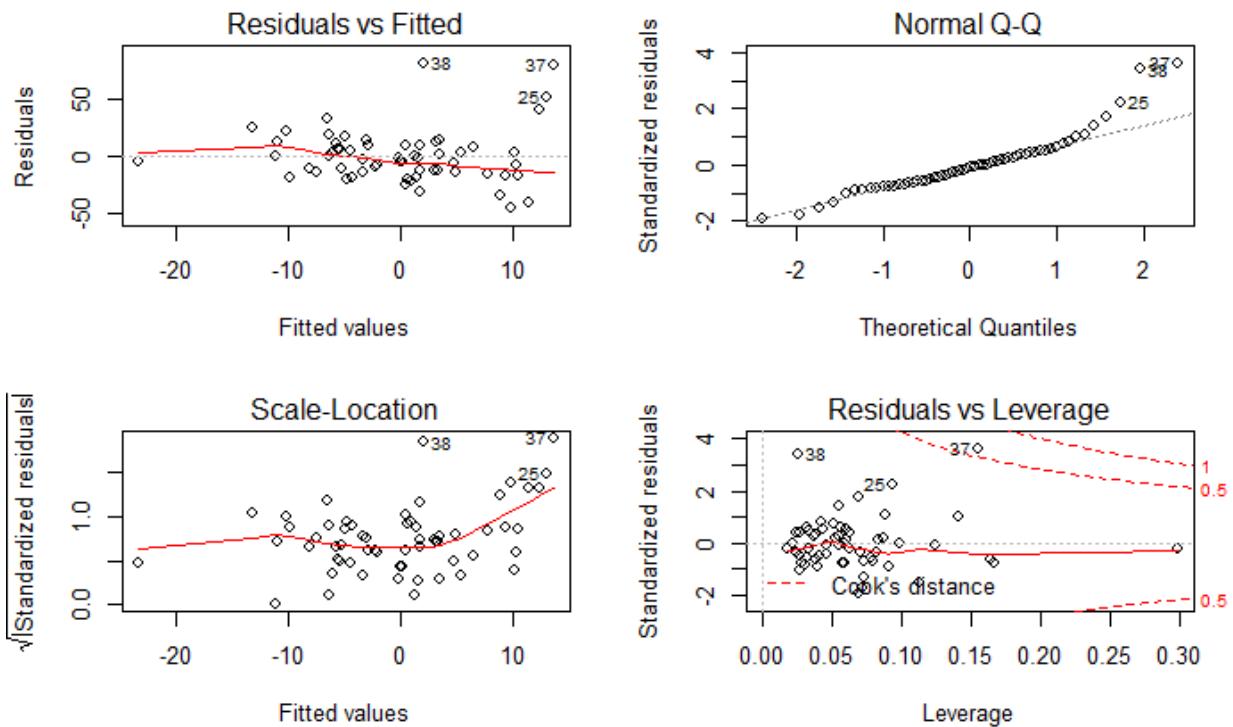


Figure A-9: Diagnostic Plots for HMY Fama-French 3-Factor Model.

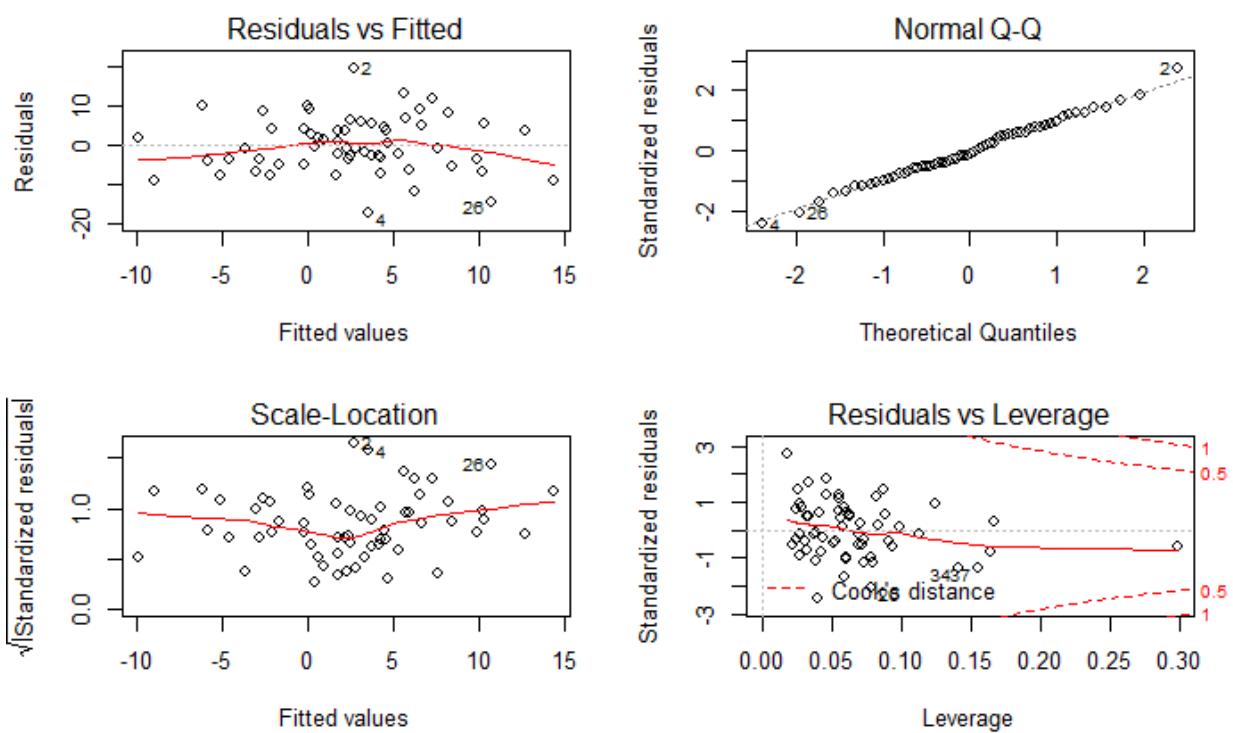


Figure A-10: Diagnostic Plots for HPQ Fama-French 3-Factor Model.

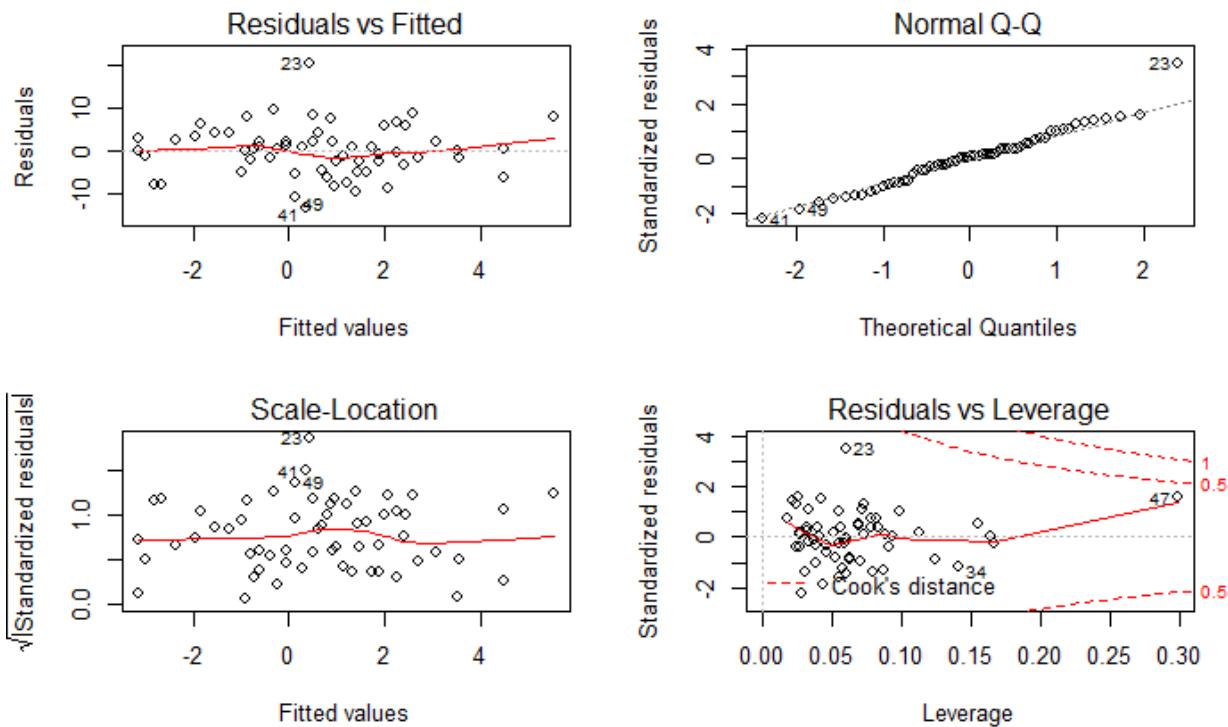


Figure A-11: Diagnostic Plots for TGT Fama-French 3-Factor Model.

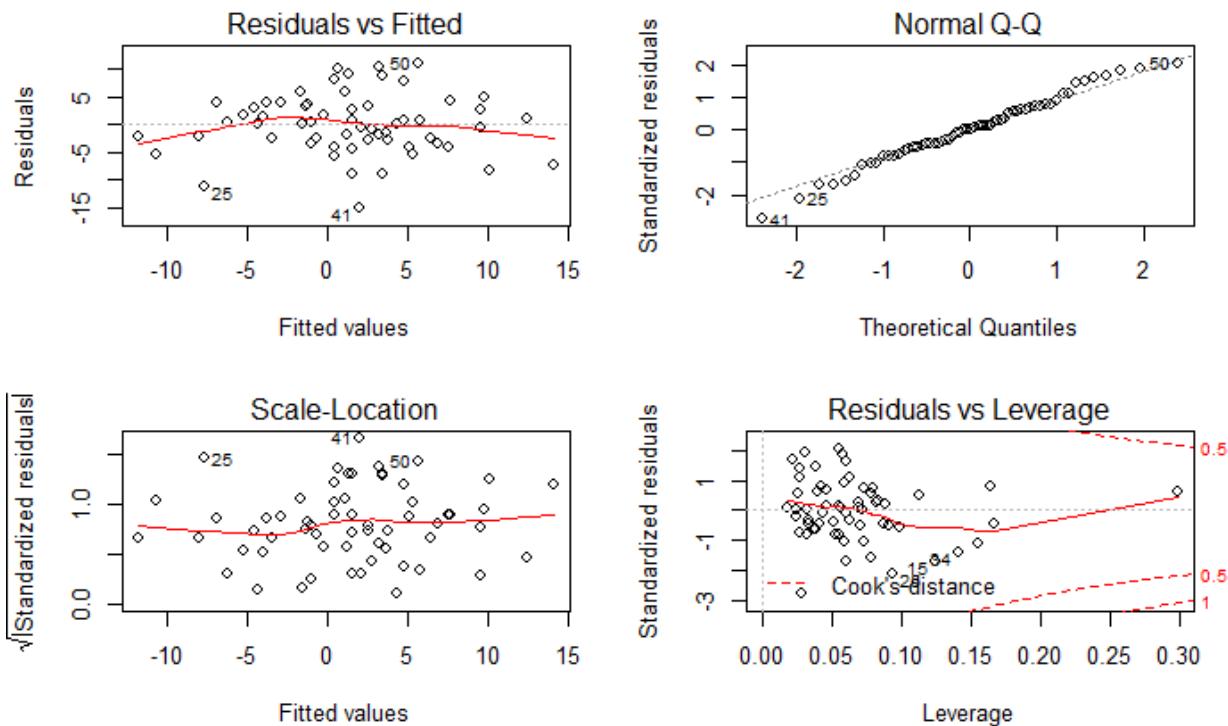


Figure A-12: Diagnostic Plots for TIF Fama-French 3-Factor Model.

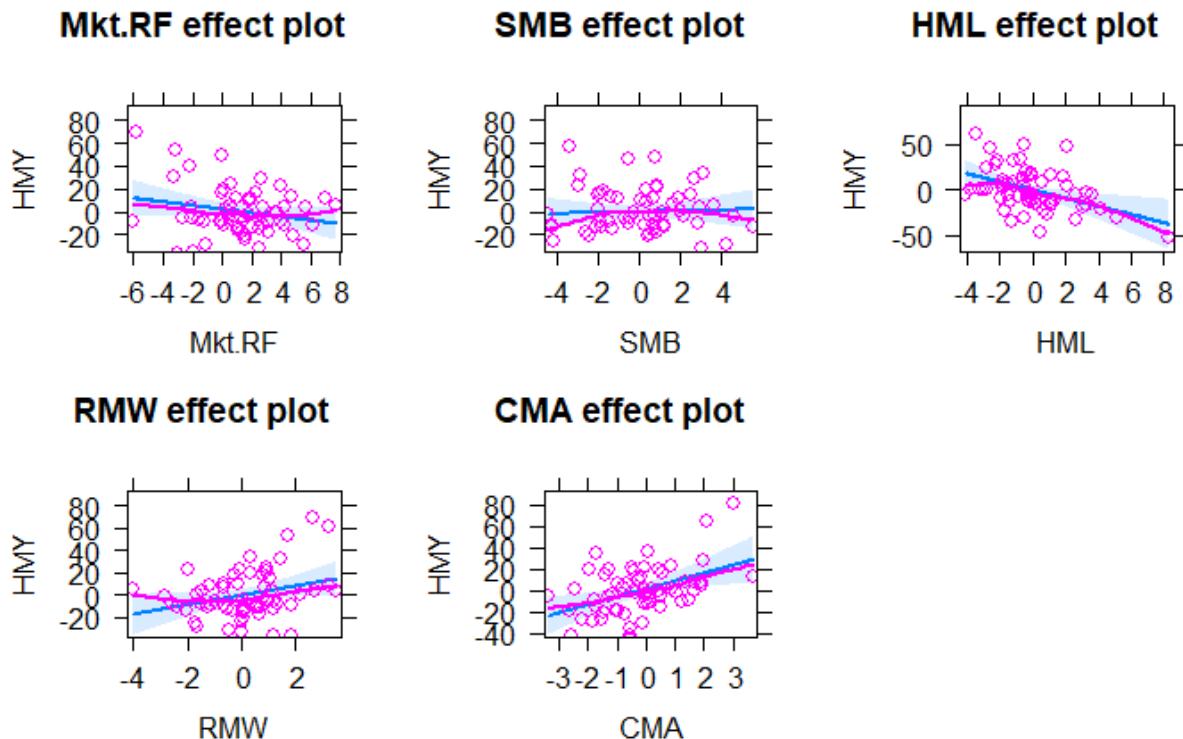


Figure A-13: Term Plots for HMY Fama-French 5-Factor Model.

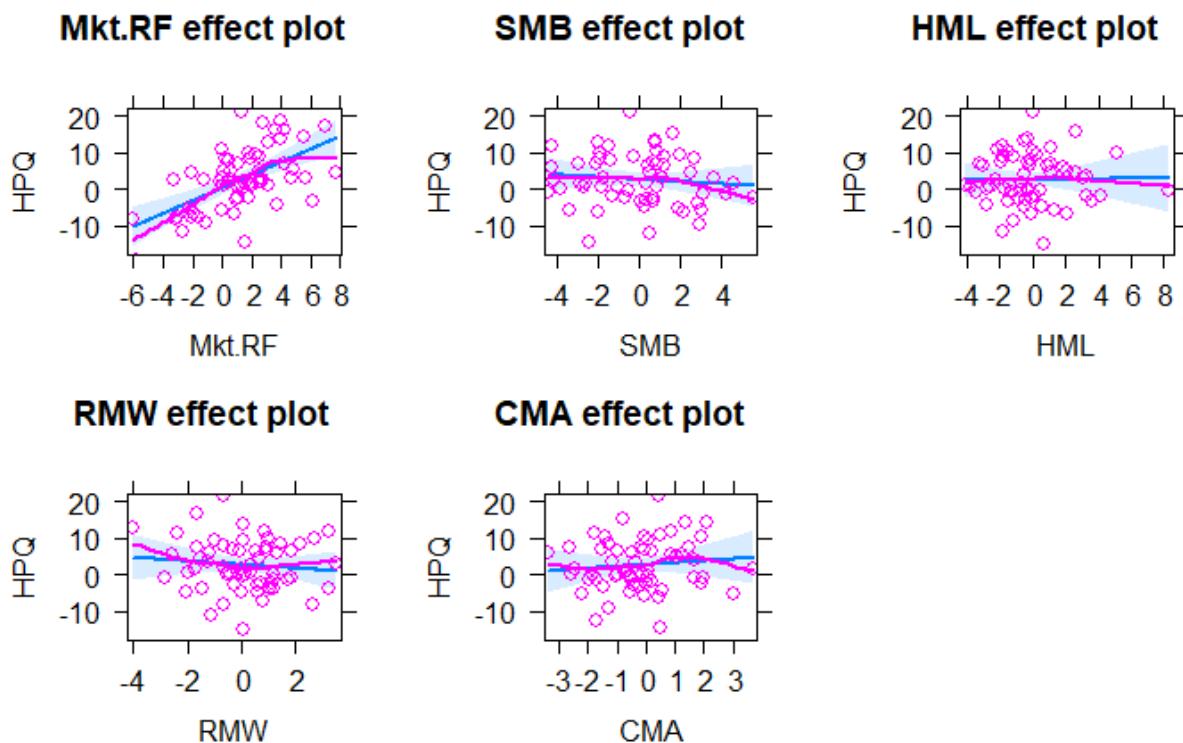


Figure A-14: Term Plots for HPQ Fama-French 5-Factor Model.

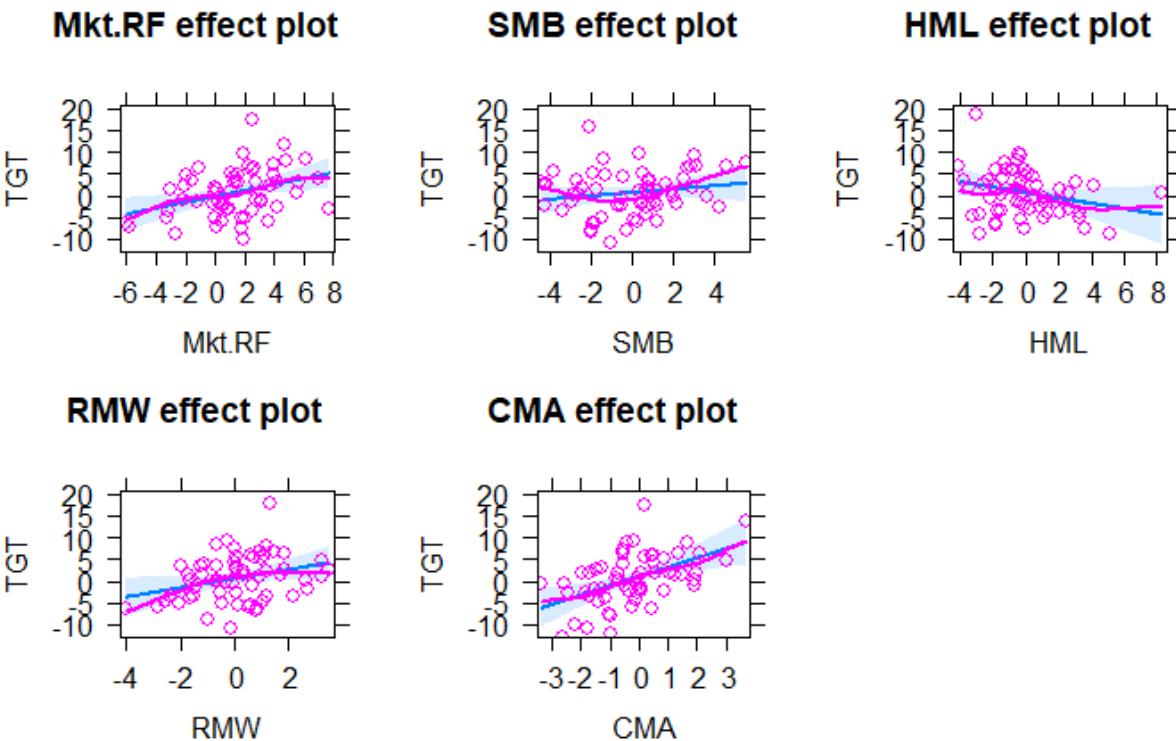


Figure A-15: Term Plots for TGT Fama-French 5-Factor Model.

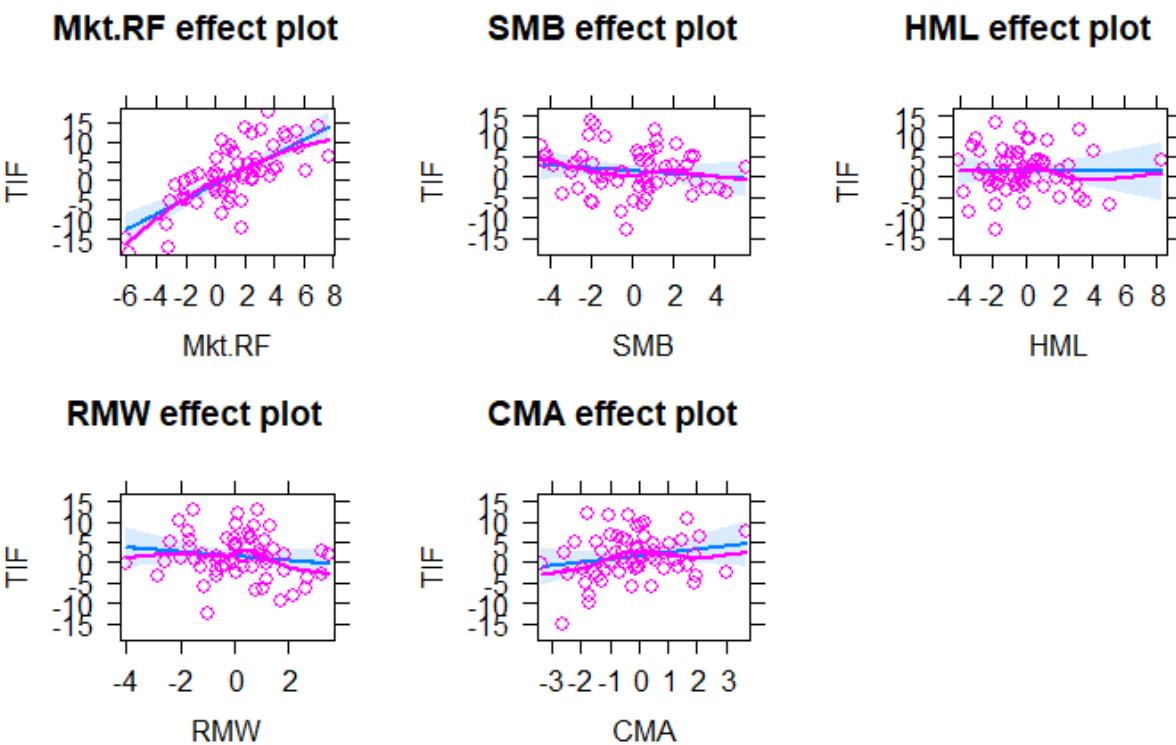


Figure A-16: Term Plots for TIF Fama-French 5-Factor Model.

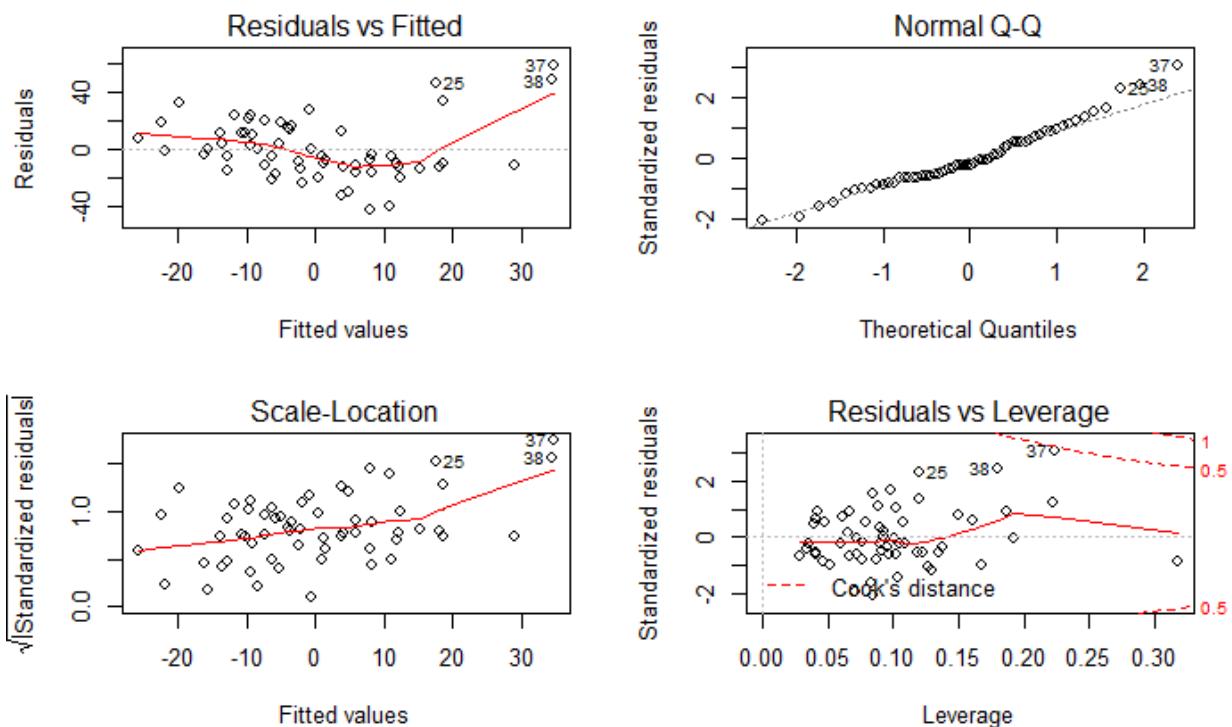


Figure A-17: Diagnostic Plots for HMY Fama-French 5-Factor Model.

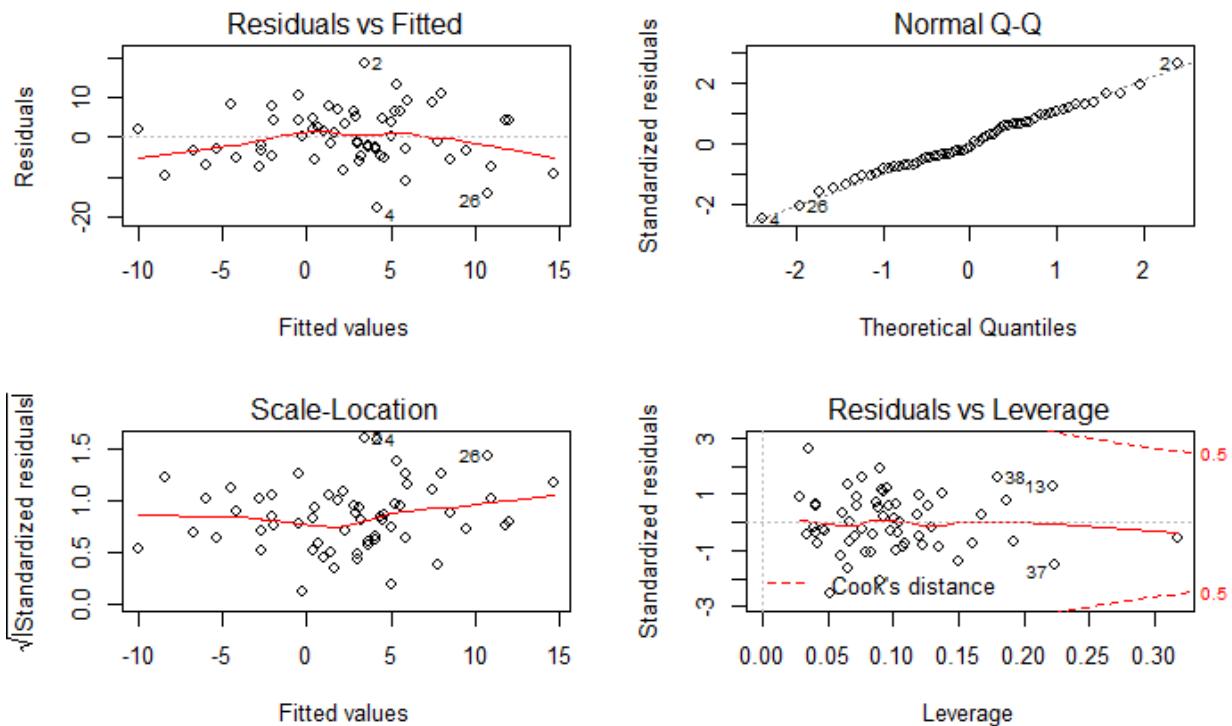


Figure A-18: Diagnostic Plots for HPQ Fama-French 5-Factor Model.

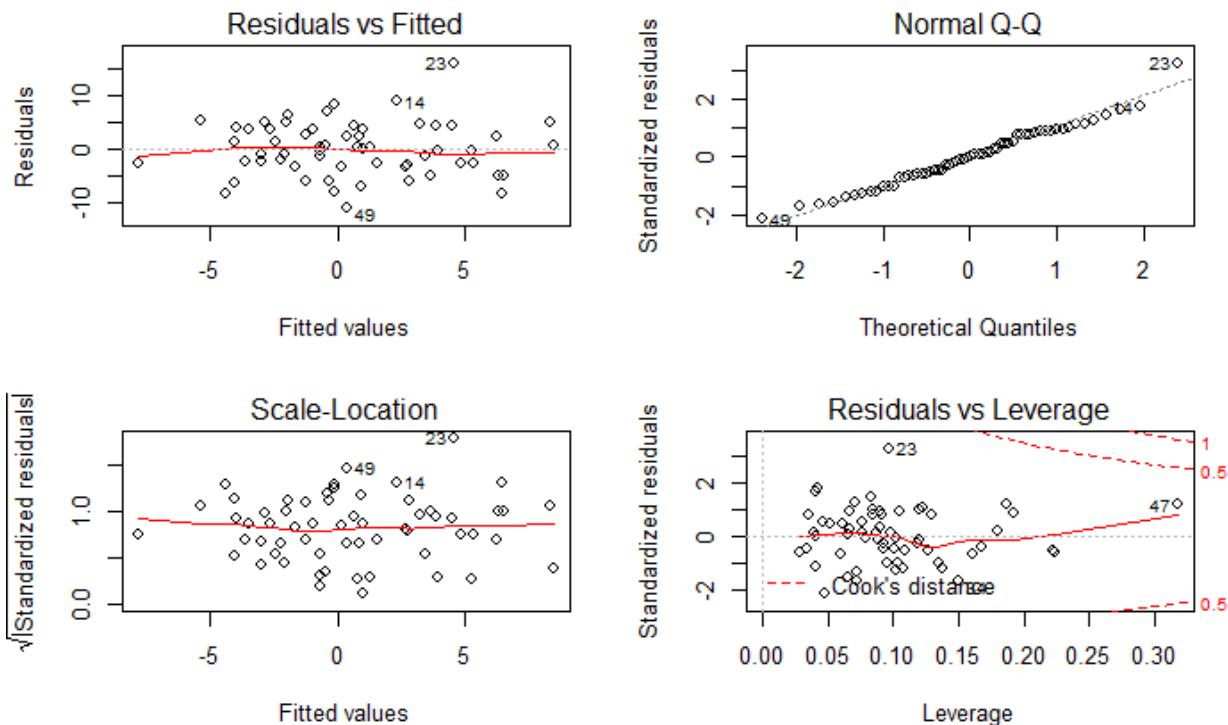


Figure A-19: Diagnostic Plots for TGT Fama-French 5-Factor Model.

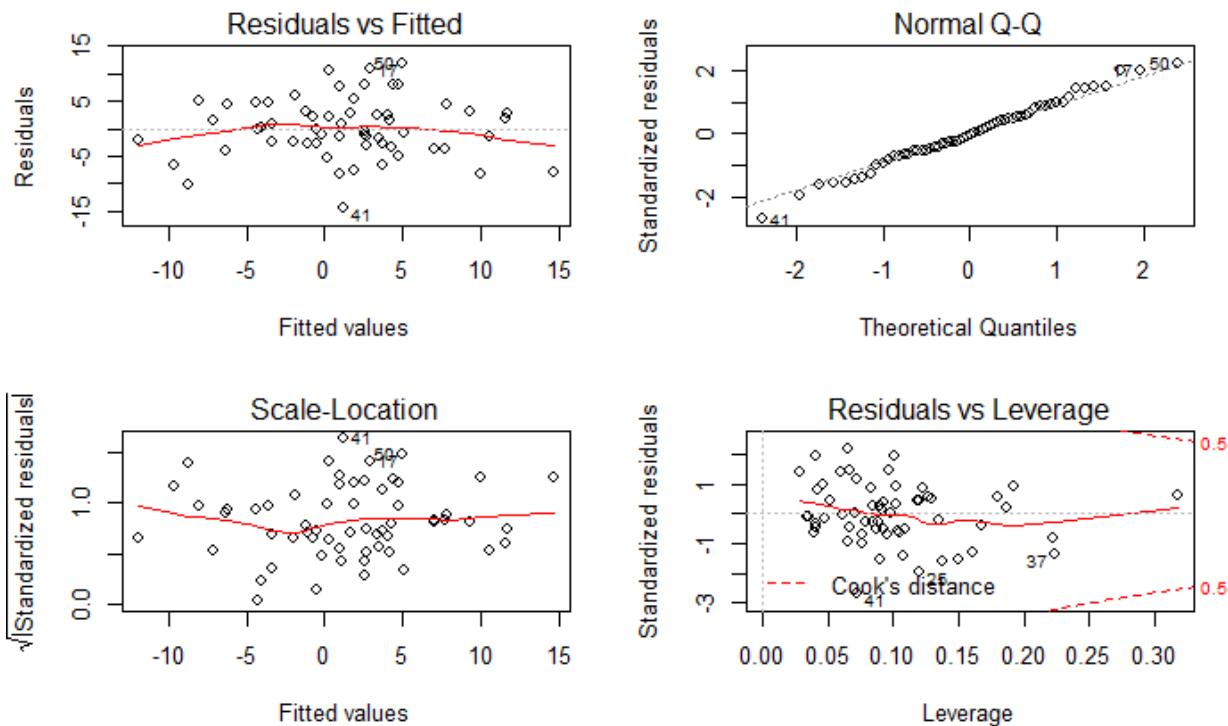


Figure A-20: Diagnostic Plots for TIF Fama-French 5-Factor Model.

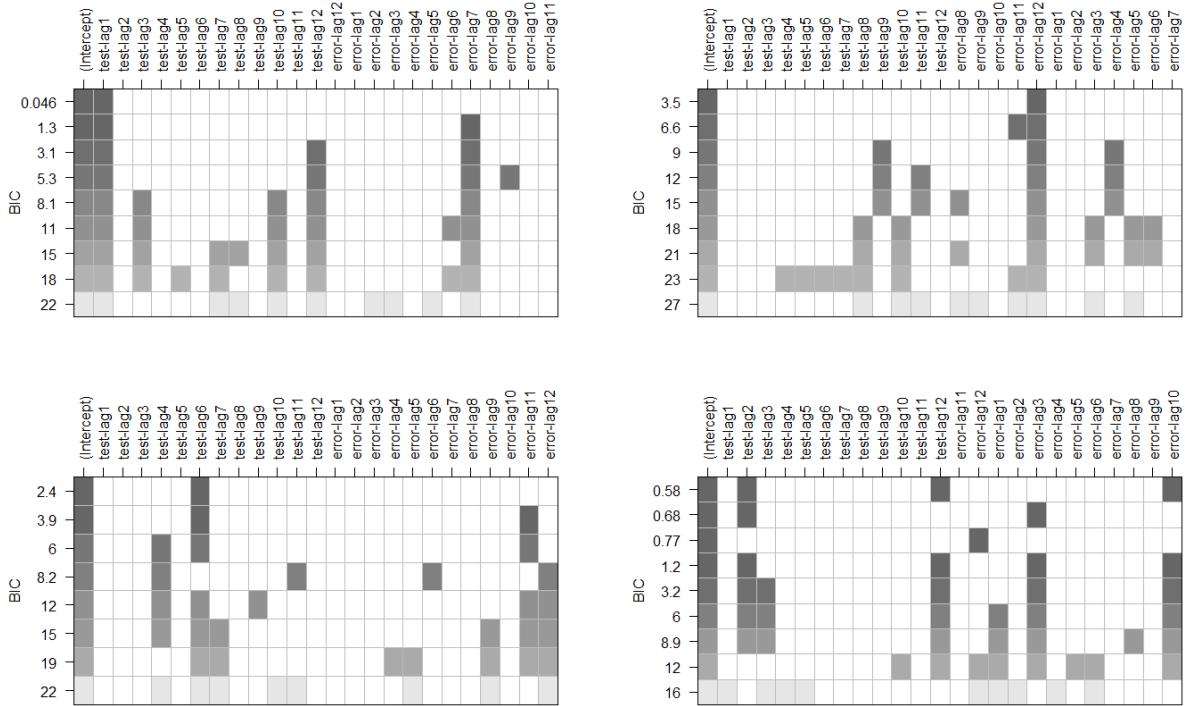


Figure A-21: Lowest BIC ARMA Models.

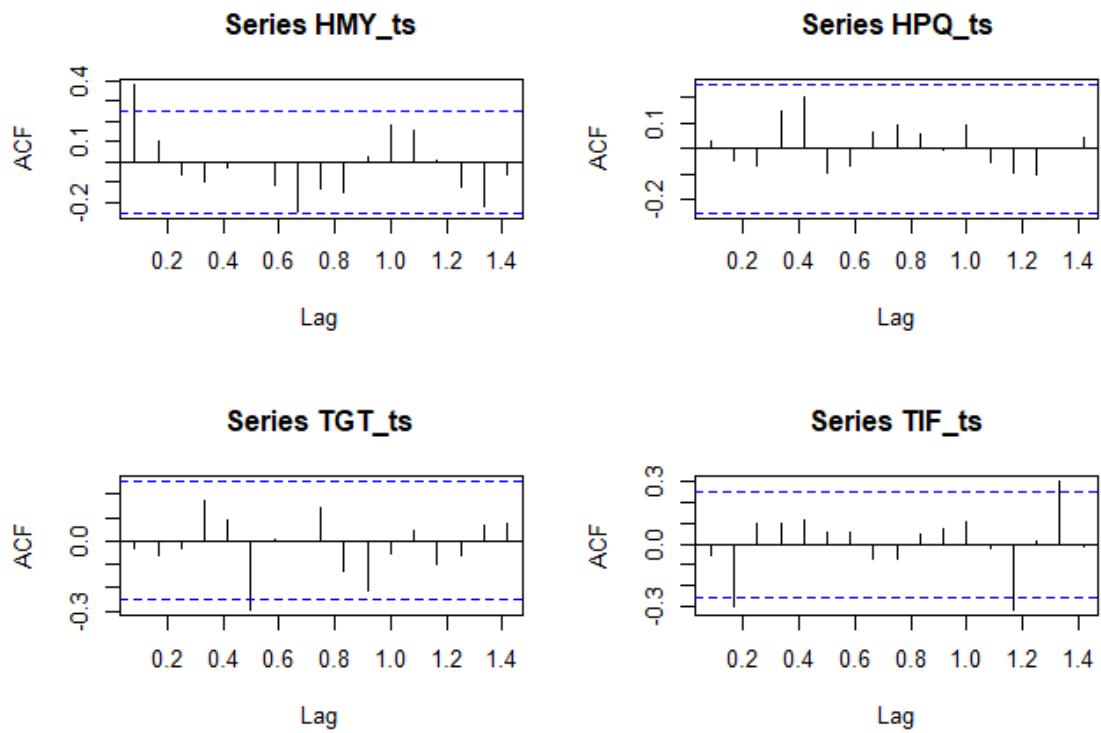


Figure A-22: ACF Plots for Returns.

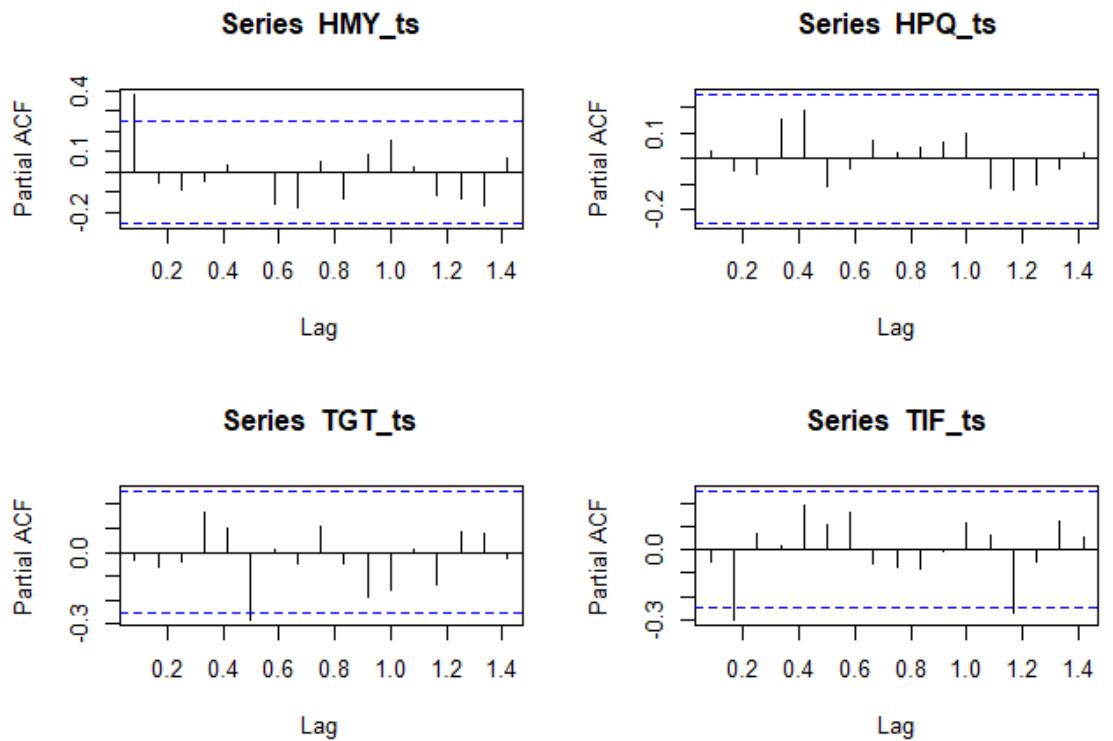


Figure A-23: Partial ACF Plots for Returns.

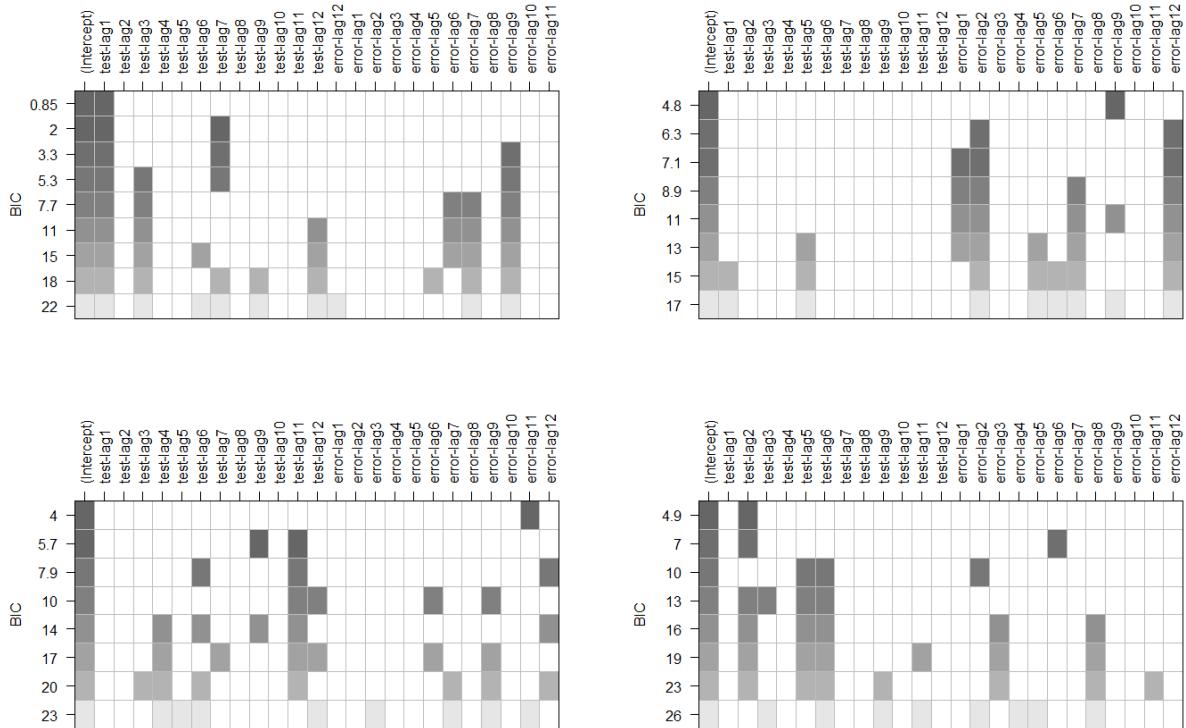


Figure A-24: Lowest BIC ARMA Models from CAPM Residuals.

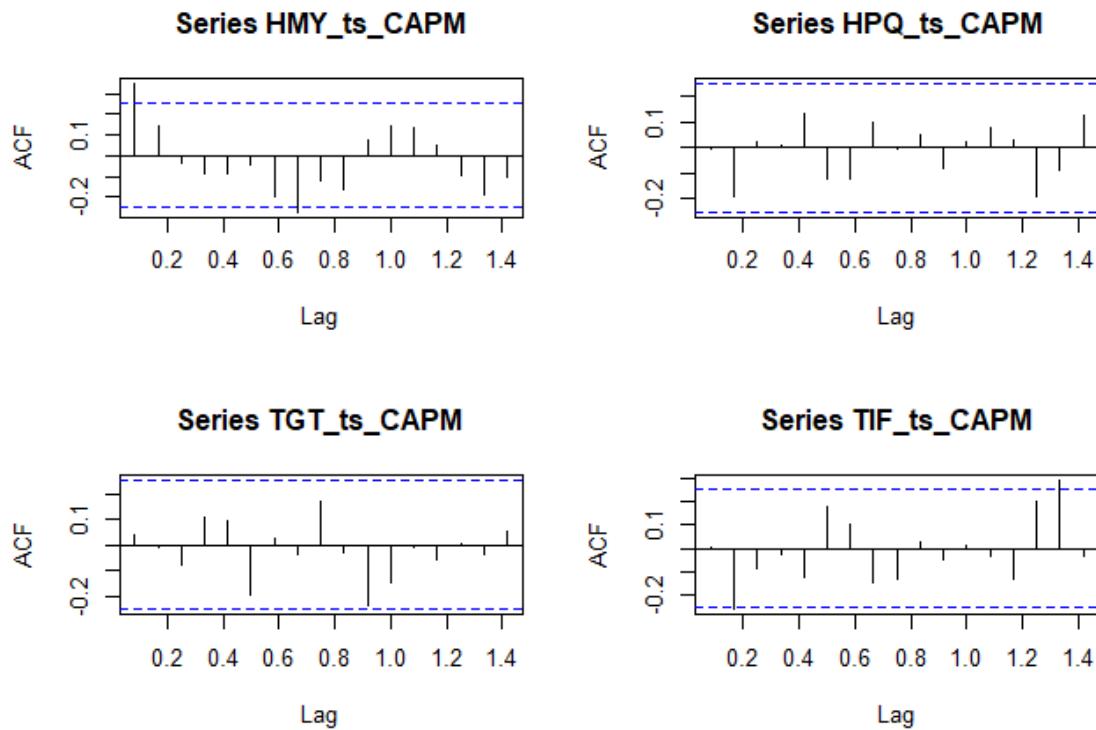


Figure A-25: ACF Plots for CAPM Residuals.

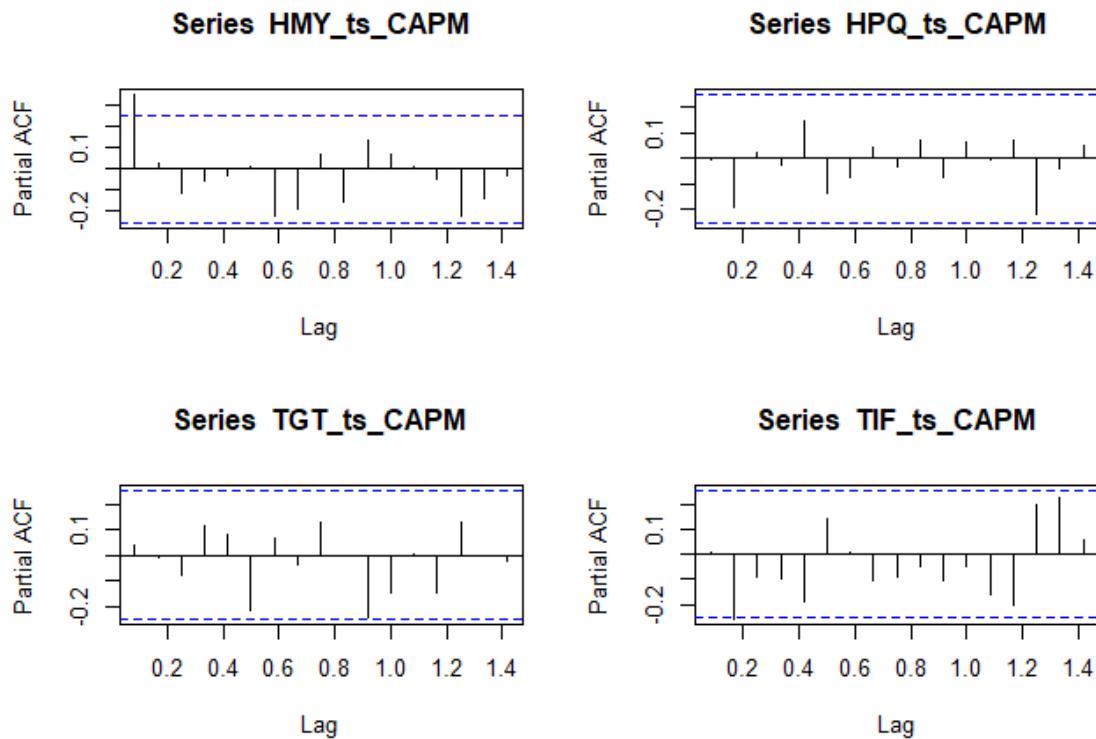


Figure A-26: Partial ACF Plots for CAPM Residuals.

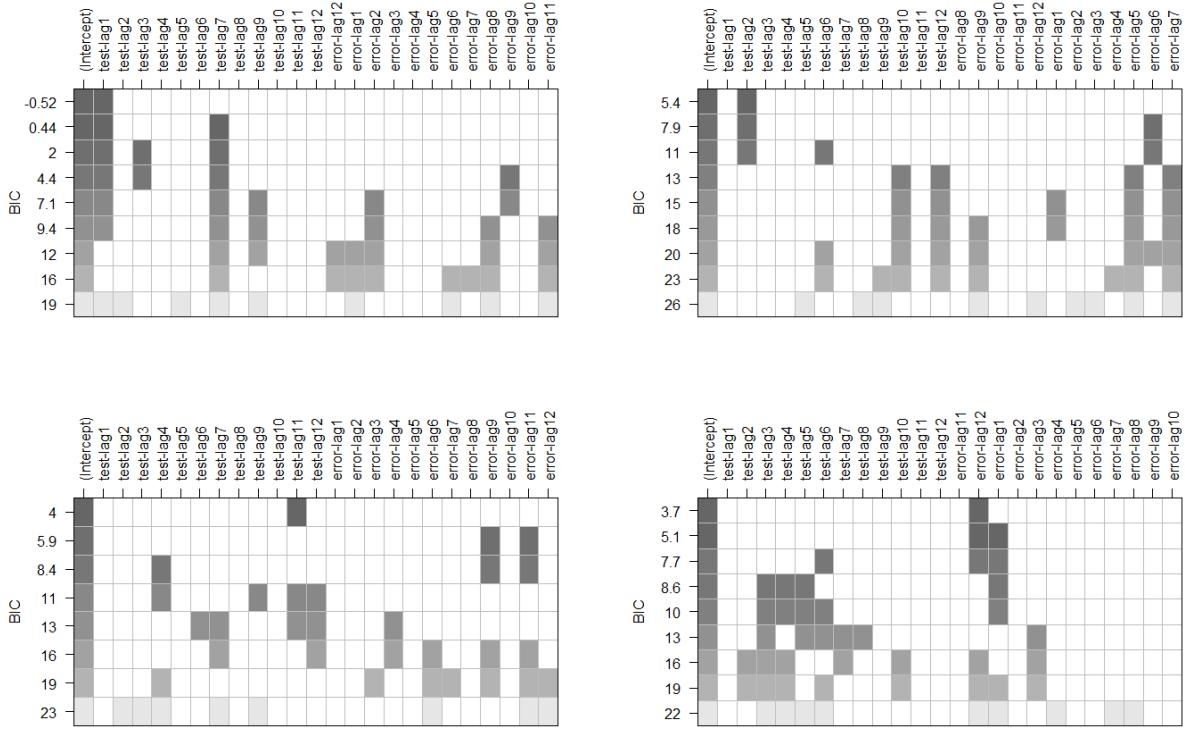


Figure A-27: Lowest BIC ARMA Models from Fama-French 3-Factor Residuals.

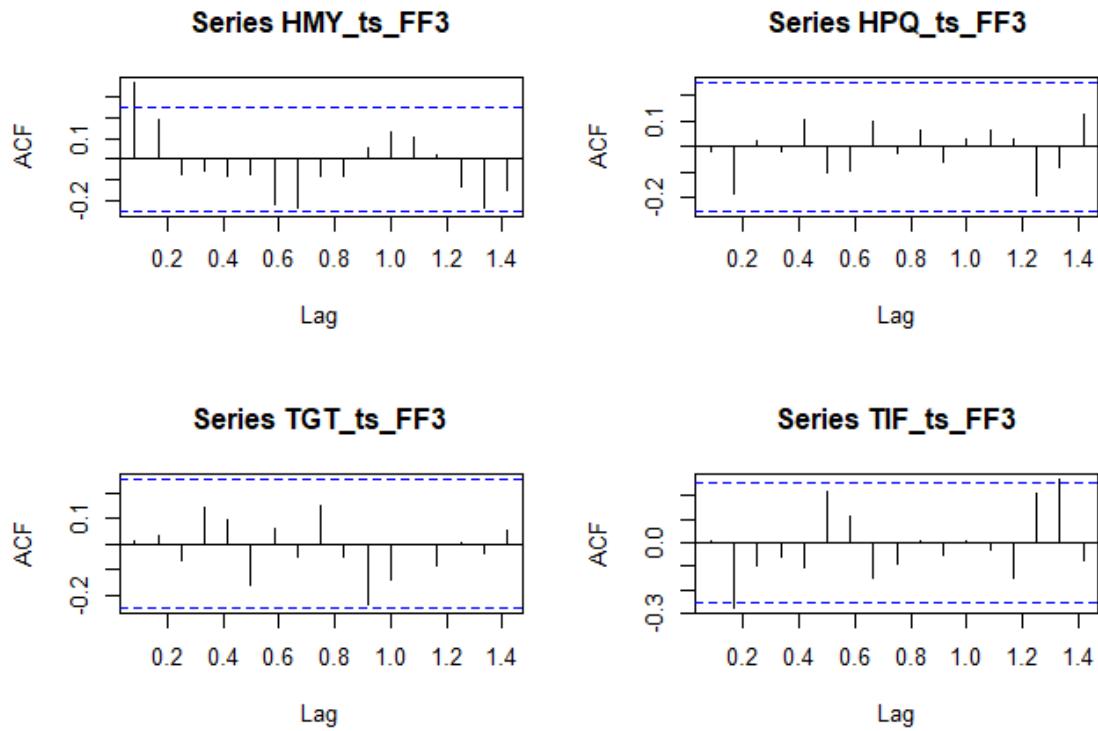


Figure A-28: ACF Plots for Fama-French 3-Factor Residuals.

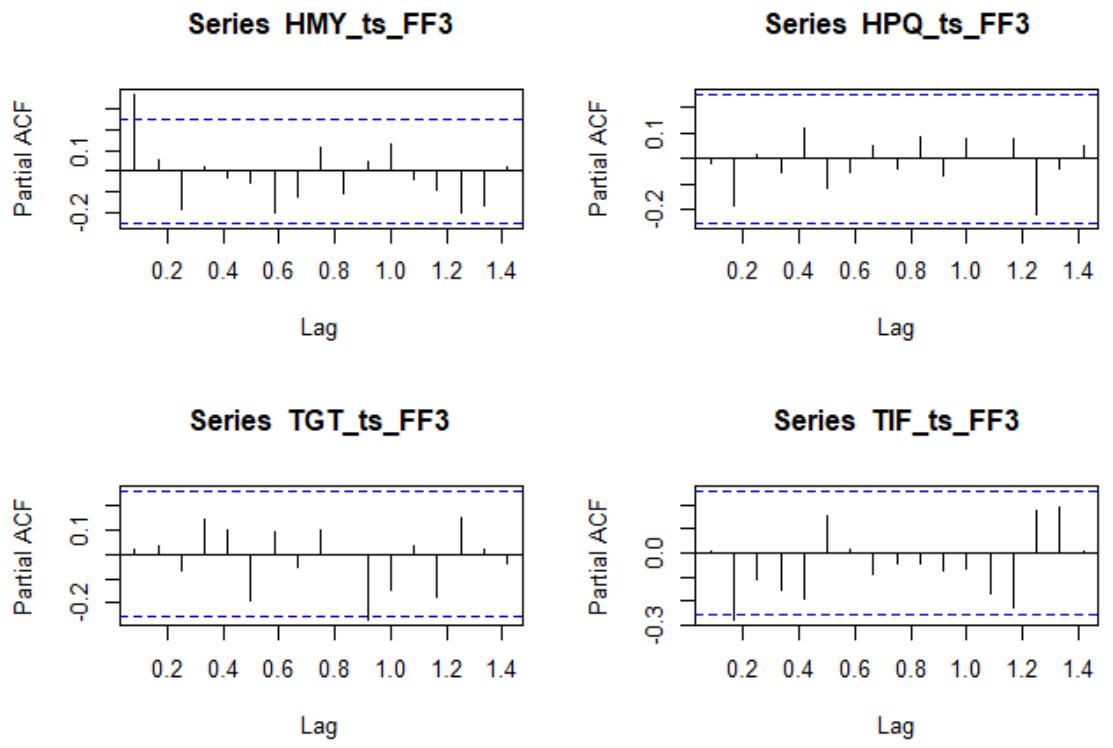


Figure A-29: Partial ACF Plots for Fama-French 3-Factor Residuals.

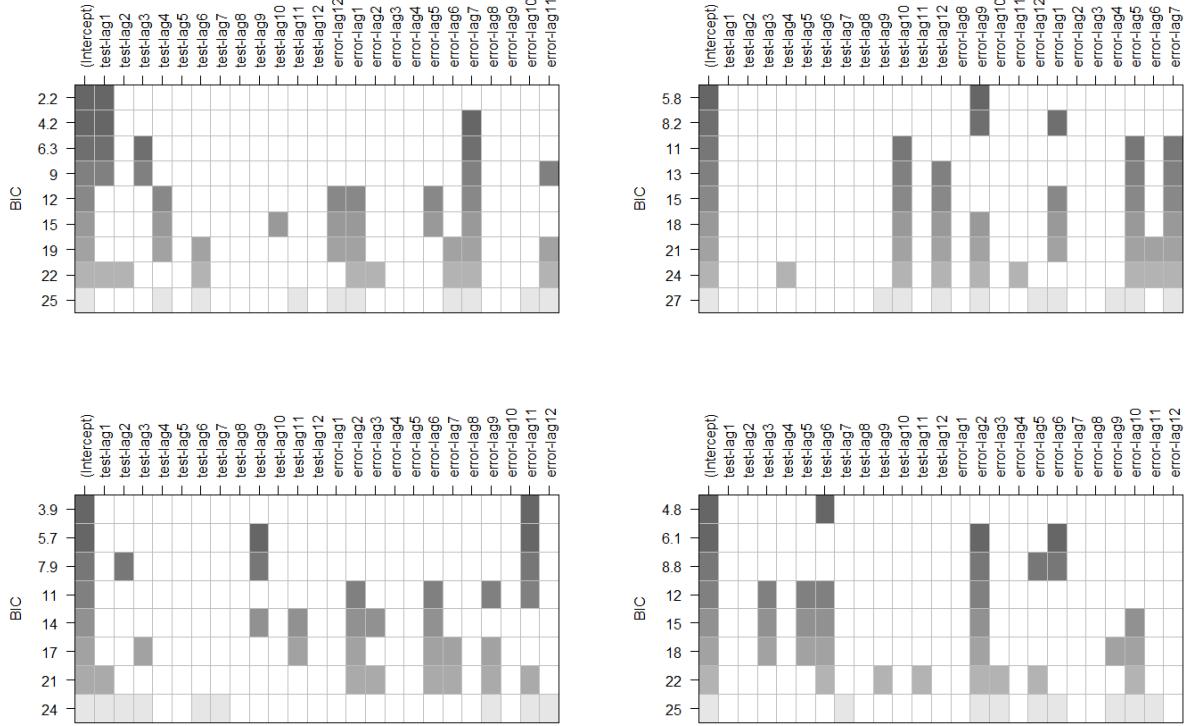


Figure A-30: Lowest BIC ARMA Models from Fama-French 5-Factor Residuals.

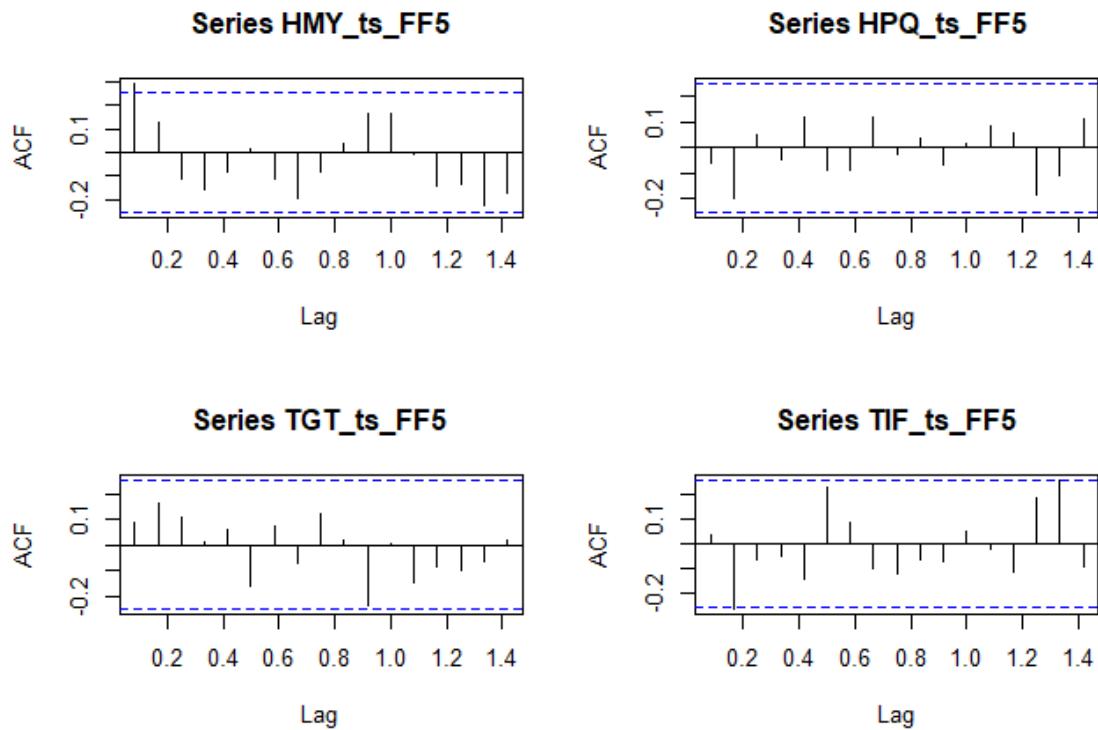


Figure A-31: ACF Plots for Fama-French 5-Factor Residuals.

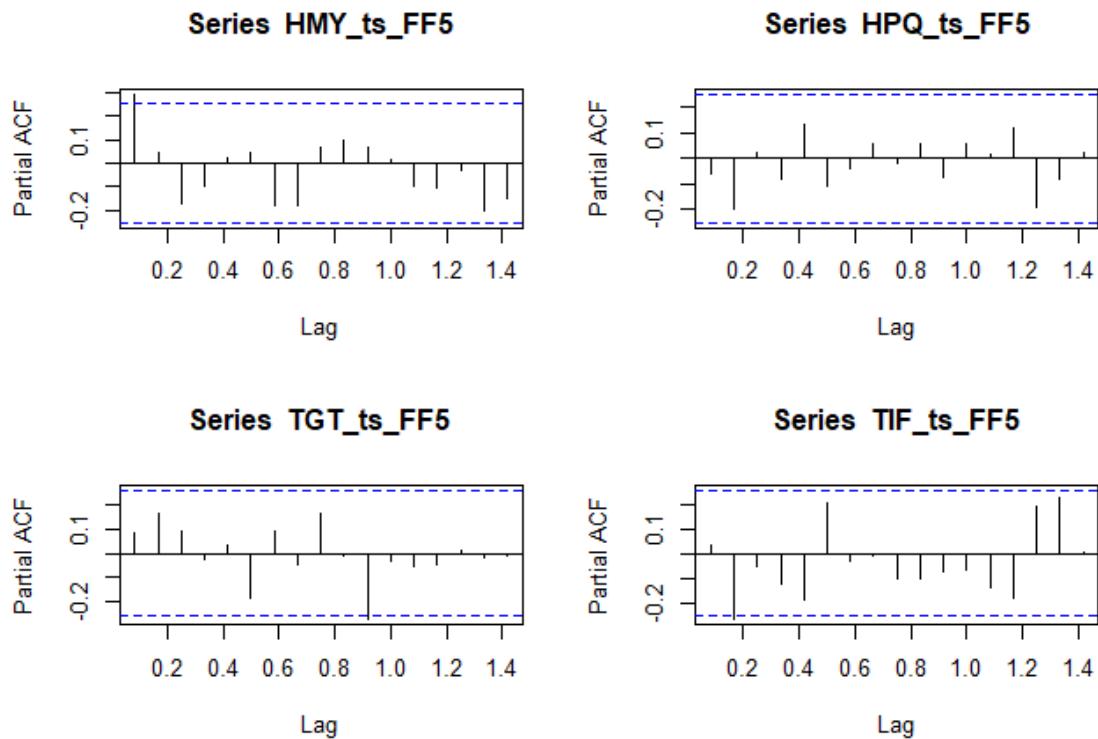


Figure A-32: Partial ACF Plots for Fama-French 5-Factor Residuals.

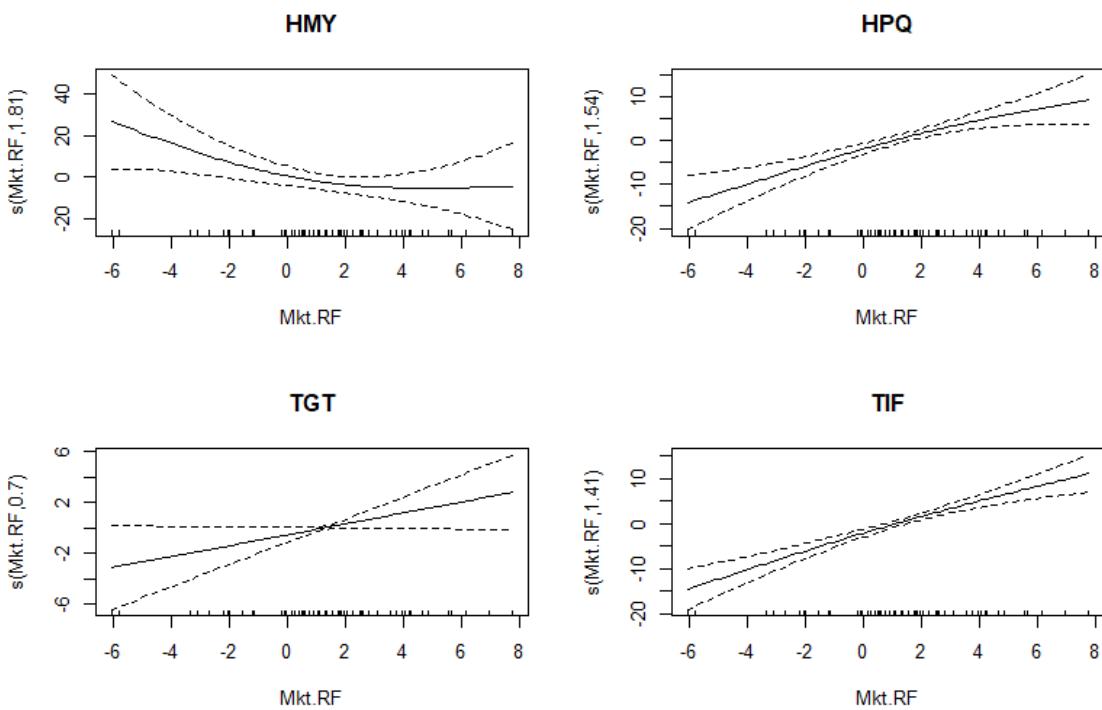


Figure A-33: Term Plots for CAPM GAMs.

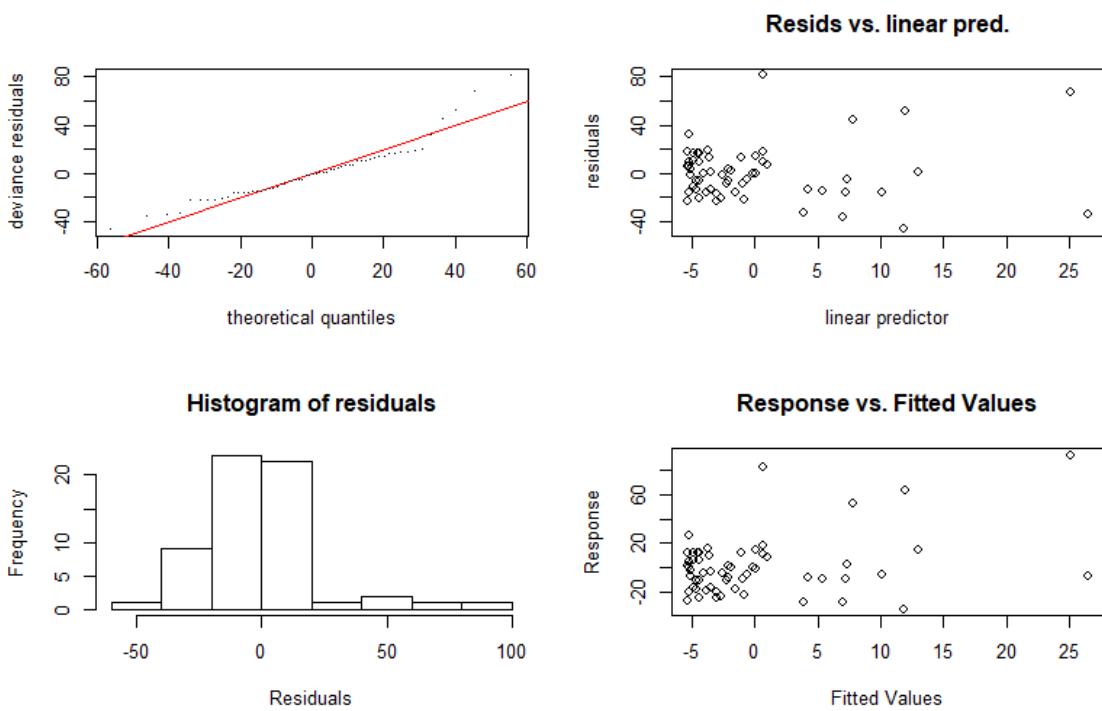


Figure A-34: Diagnostic Plots for HMY CAPM GAM.

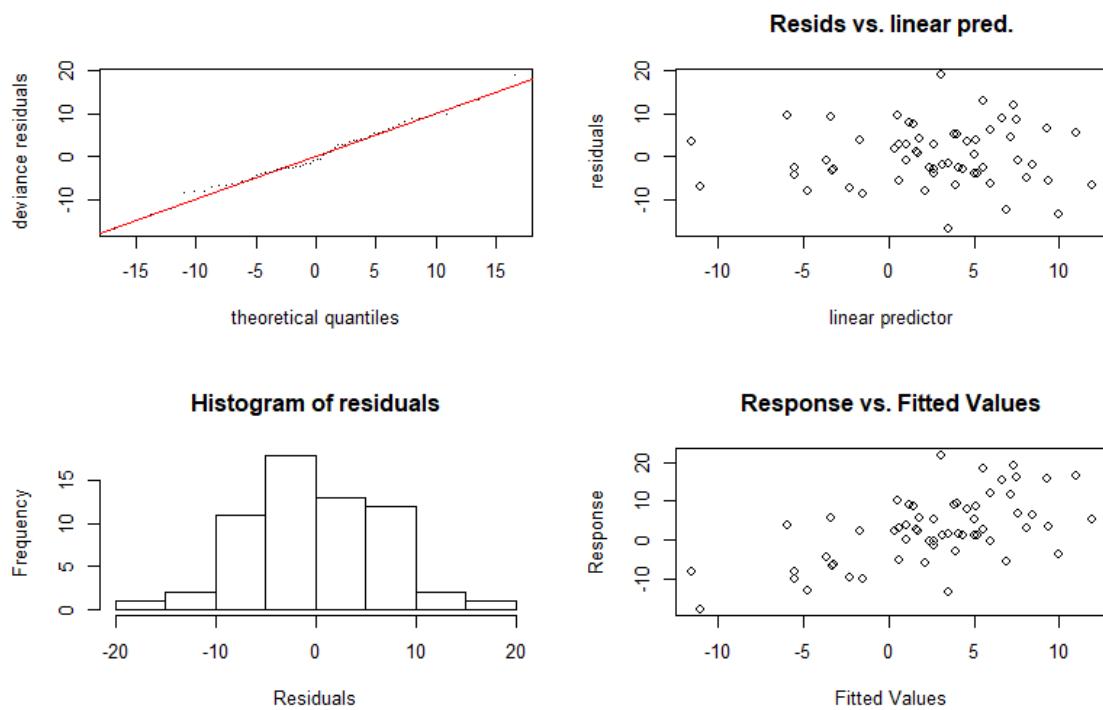


Figure A-35: Diagnostic Plots for HPQ CAPM GAM.

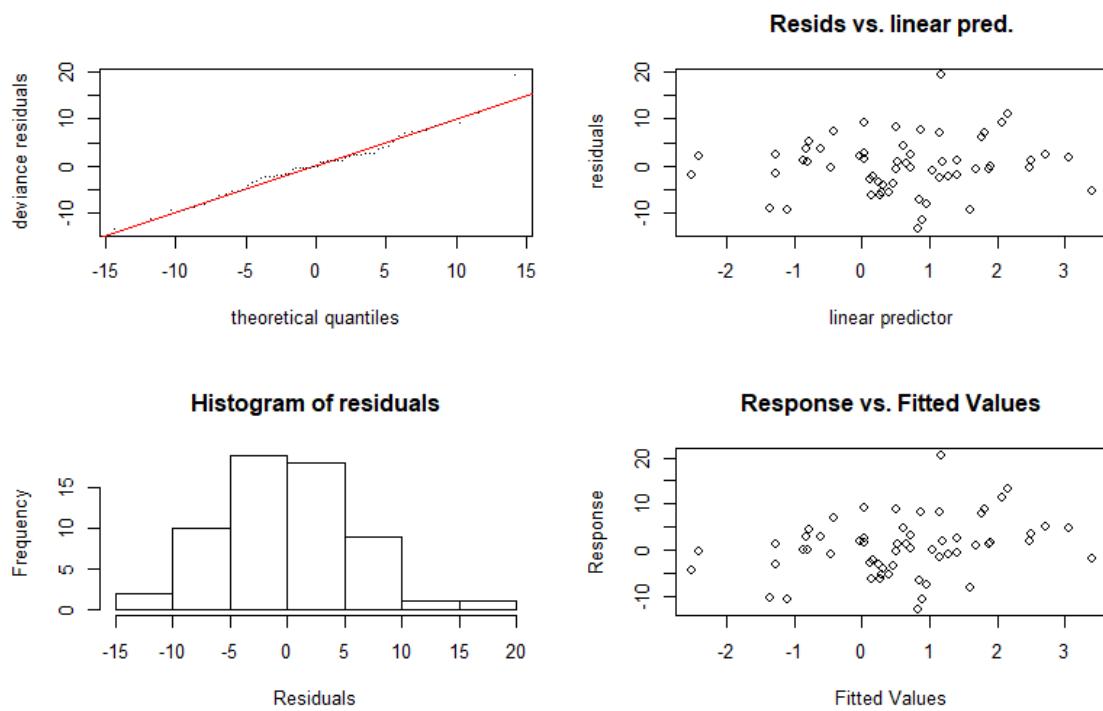


Figure A-36: Diagnostic Plots for TGT CAPM GAM.

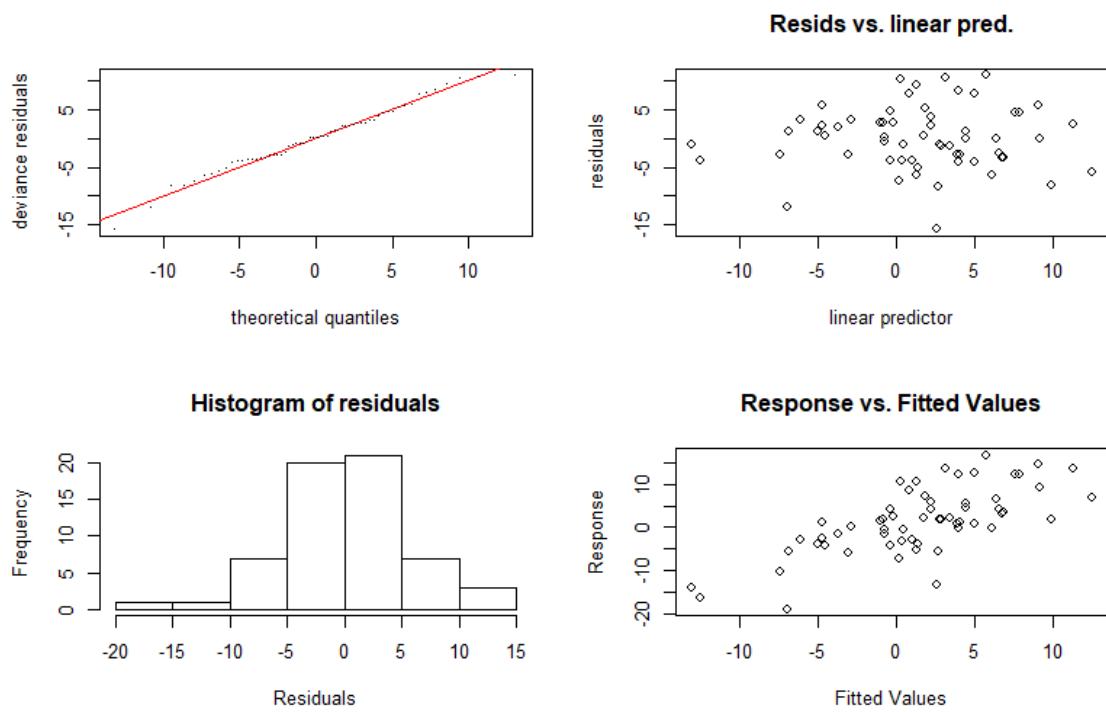


Figure A-37: Diagnostic Plots for TIF CAPM GAM.

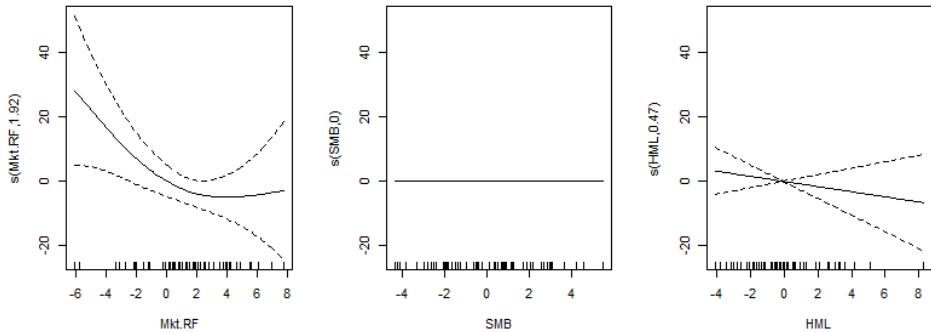


Figure A-38: Term Plots for HMY Fama-French 3-Factor GAM.

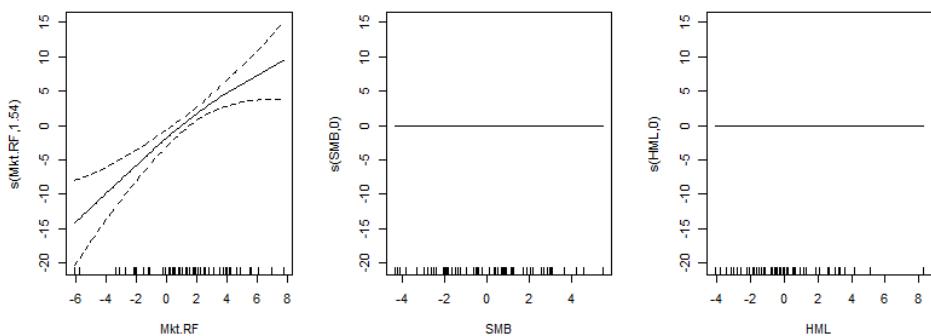


Figure A-39: Term Plots for HPQ Fama-French 3-Factor GAM.

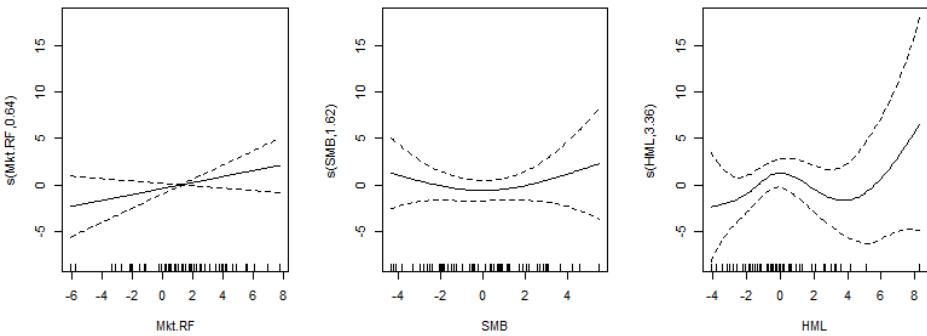


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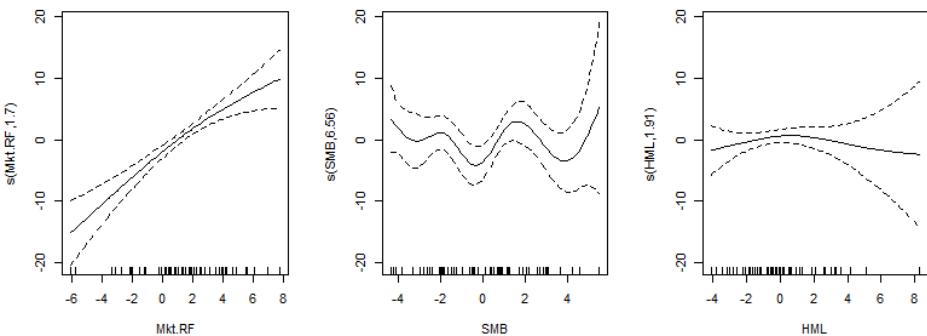


Figure A-41: Term Plots for TIF Fama-French 3-Factor GAM.

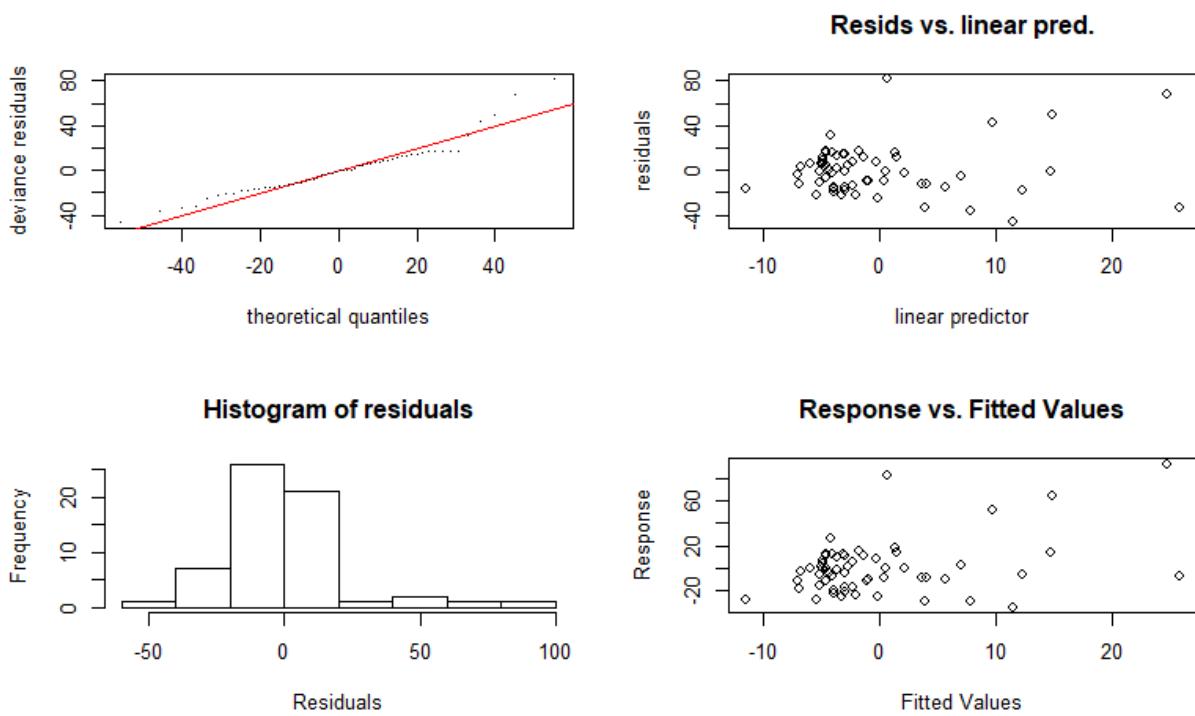


Figure A-42: Diagnostic Plots for HMY Fama-French 3-Factor GAM.

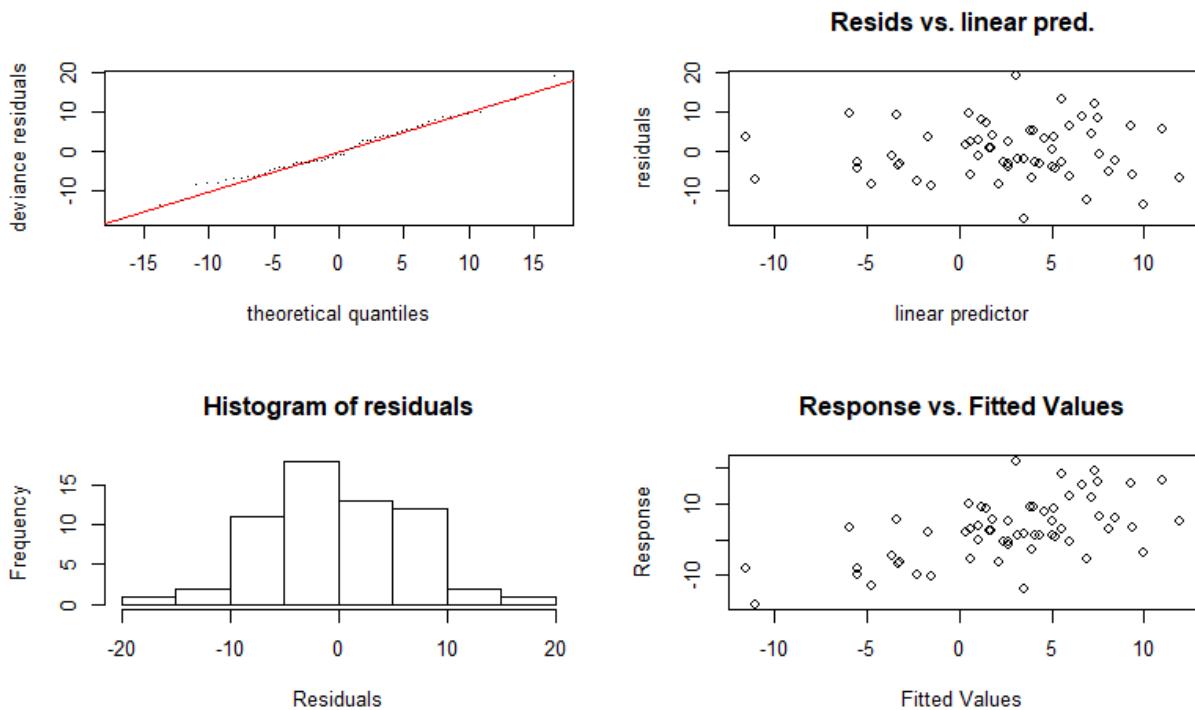


Figure A-43: Diagnostic Plots for HPQ Fama-French 3-Factor GAM.

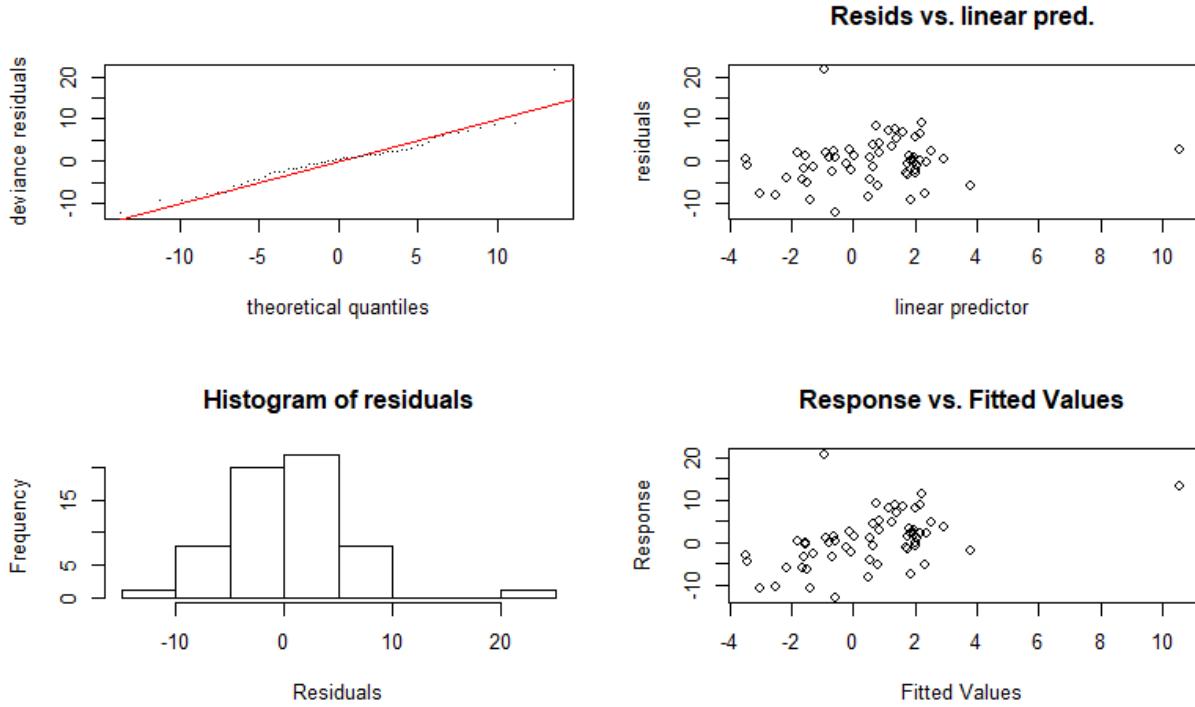


Figure A-44: Diagnostic Plots for TGT Fama-French 3-Factor GAM.

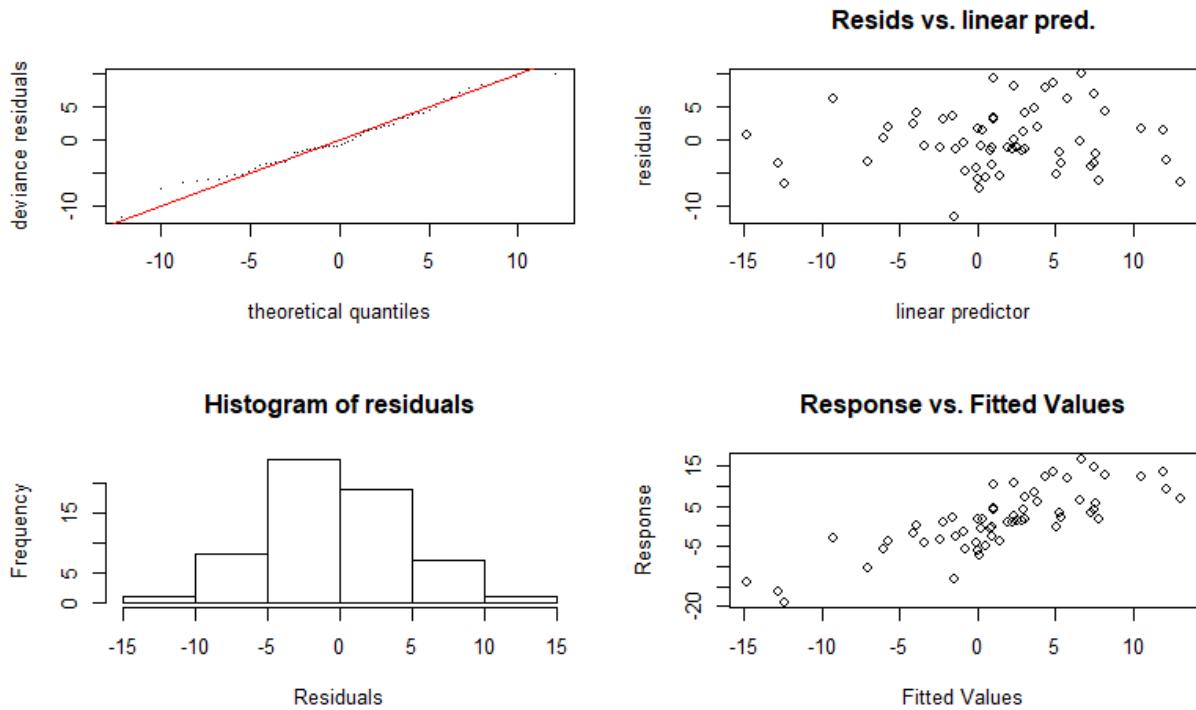


Figure A-45: Diagnostic Plots for TIF Fama-French 3-Factor GAM.

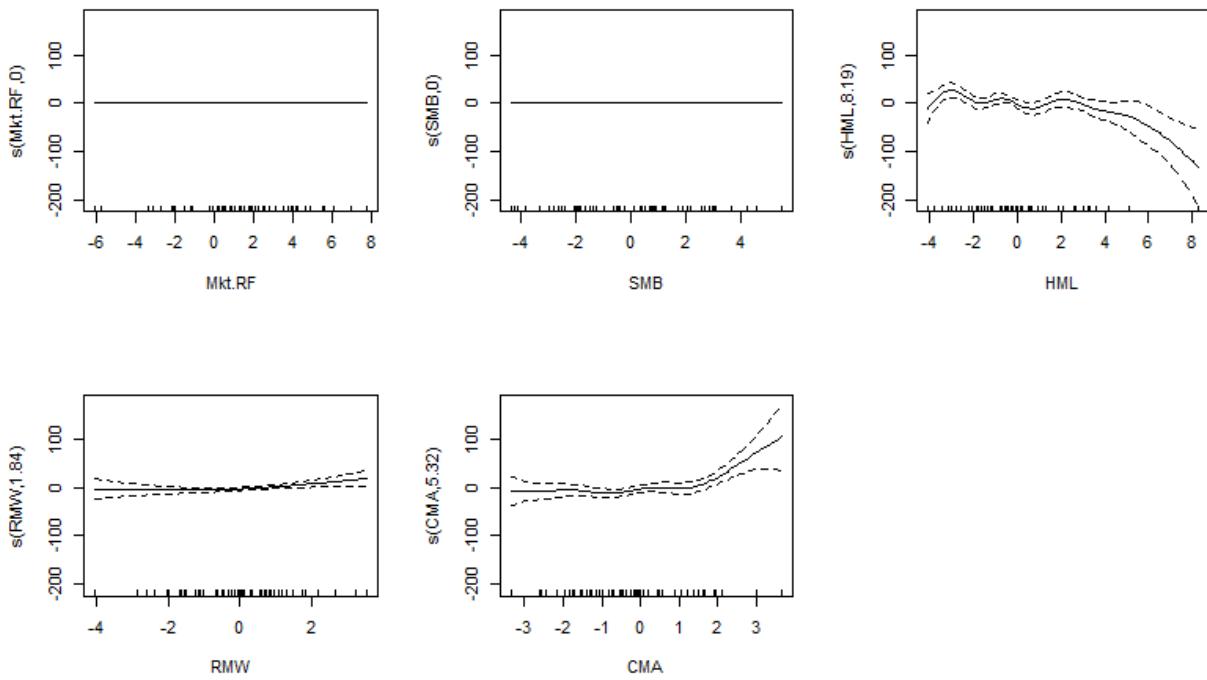


Figure A-46: Term Plots for HMY Fama-French 5-Factor GAM.

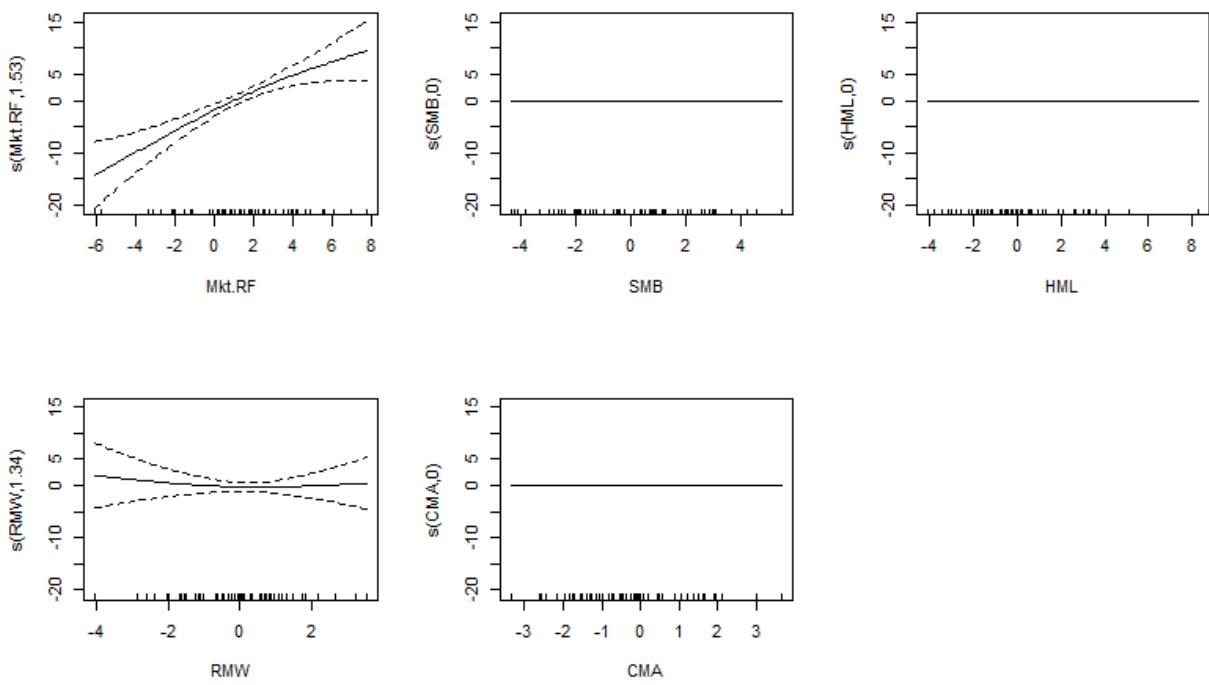


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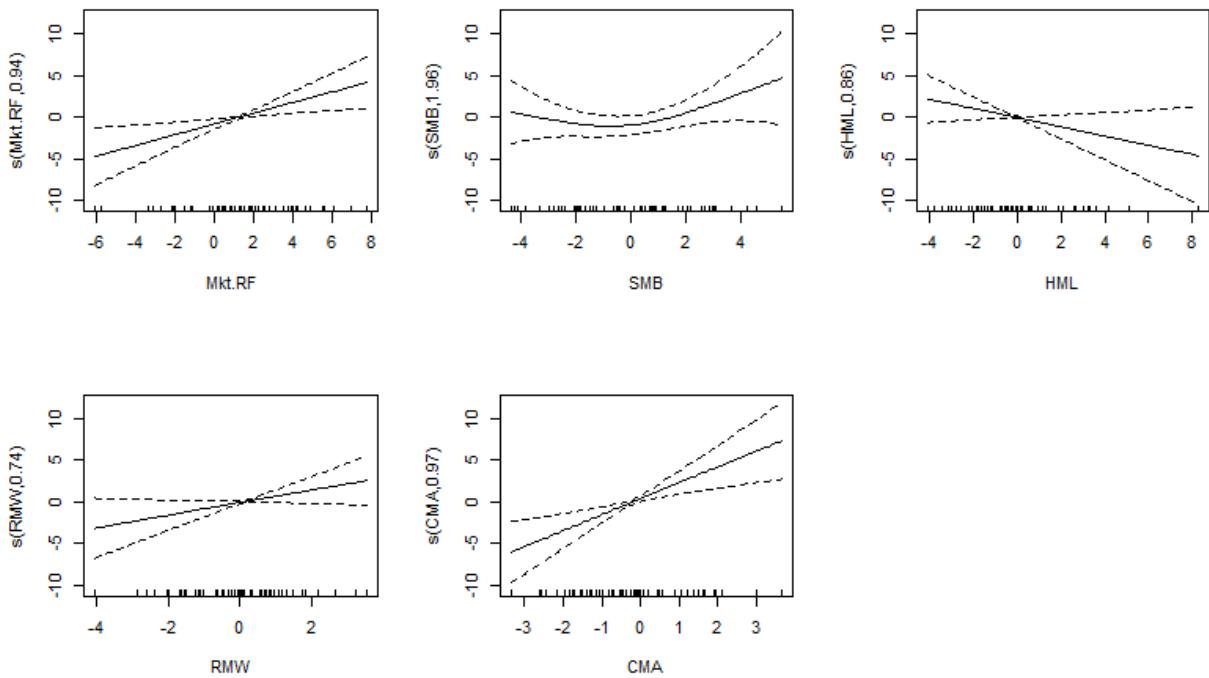


Figure A-48: Term Plots for TGT Fama-French 5-Factor GAM.

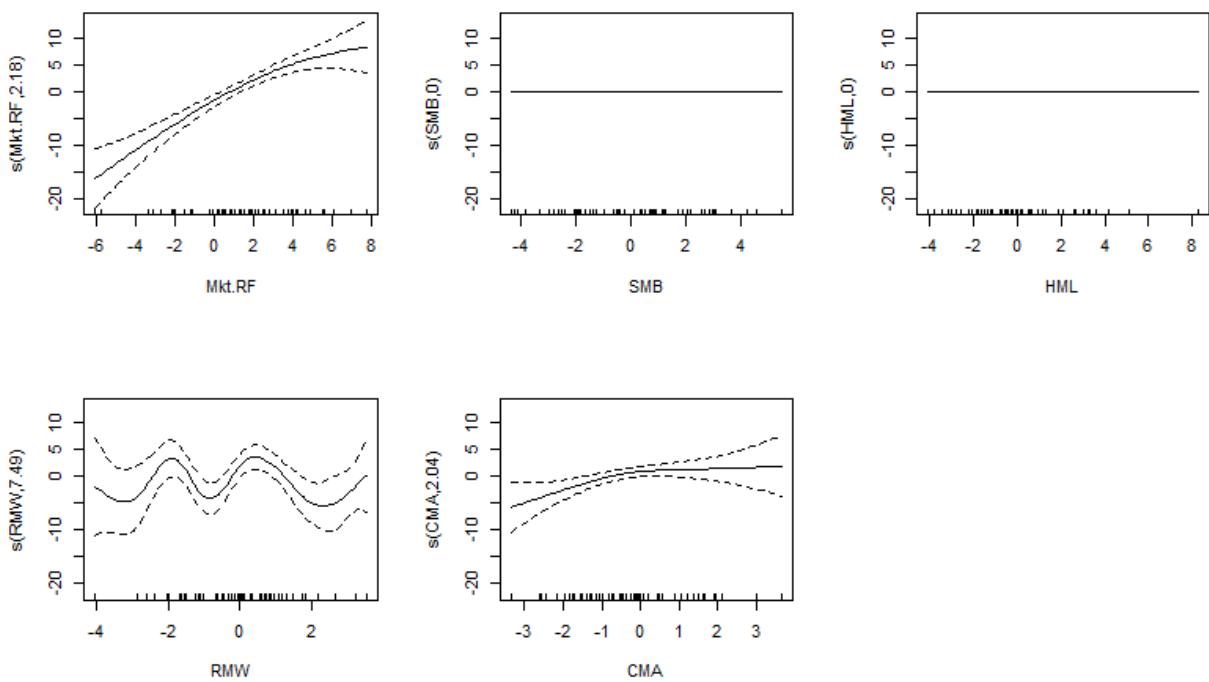


Figure A-49: Term Plots for TIF Fama-French 5-Factor GAM.

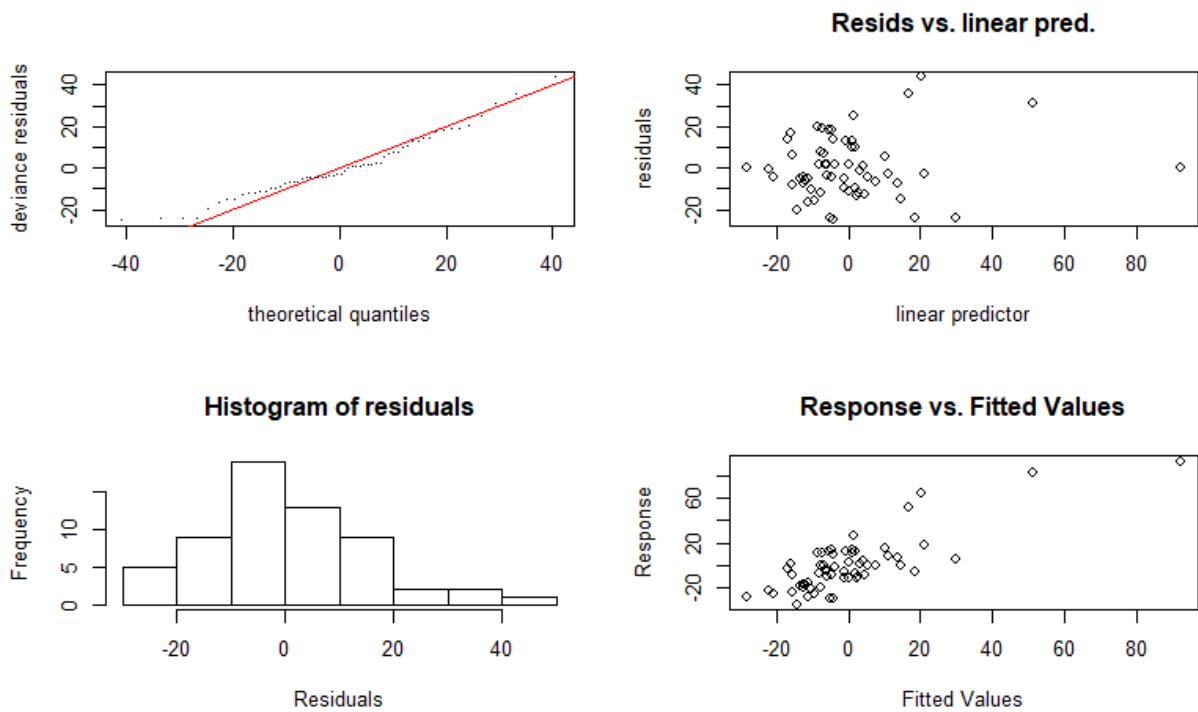


Figure A-50: Diagnostic Plots for HMY Fama-French 5-Factor GAM.

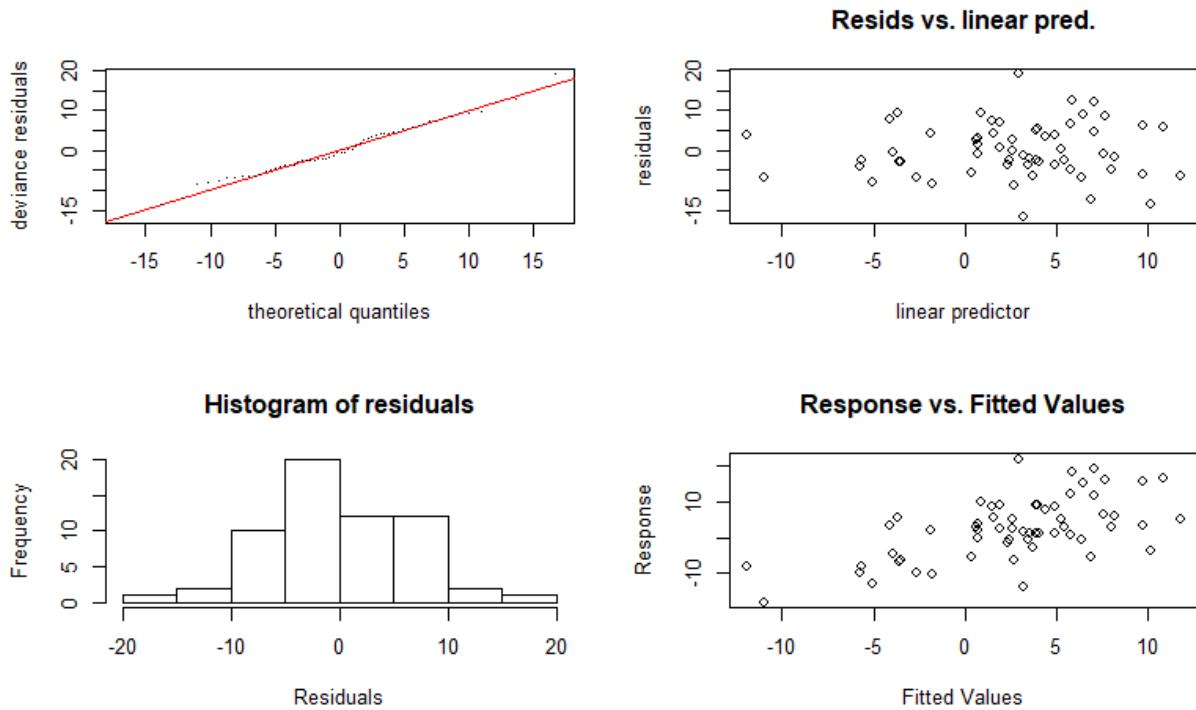


Figure A-51: Diagnostic Plots for HPQ Fama-French 5-Factor GAM.

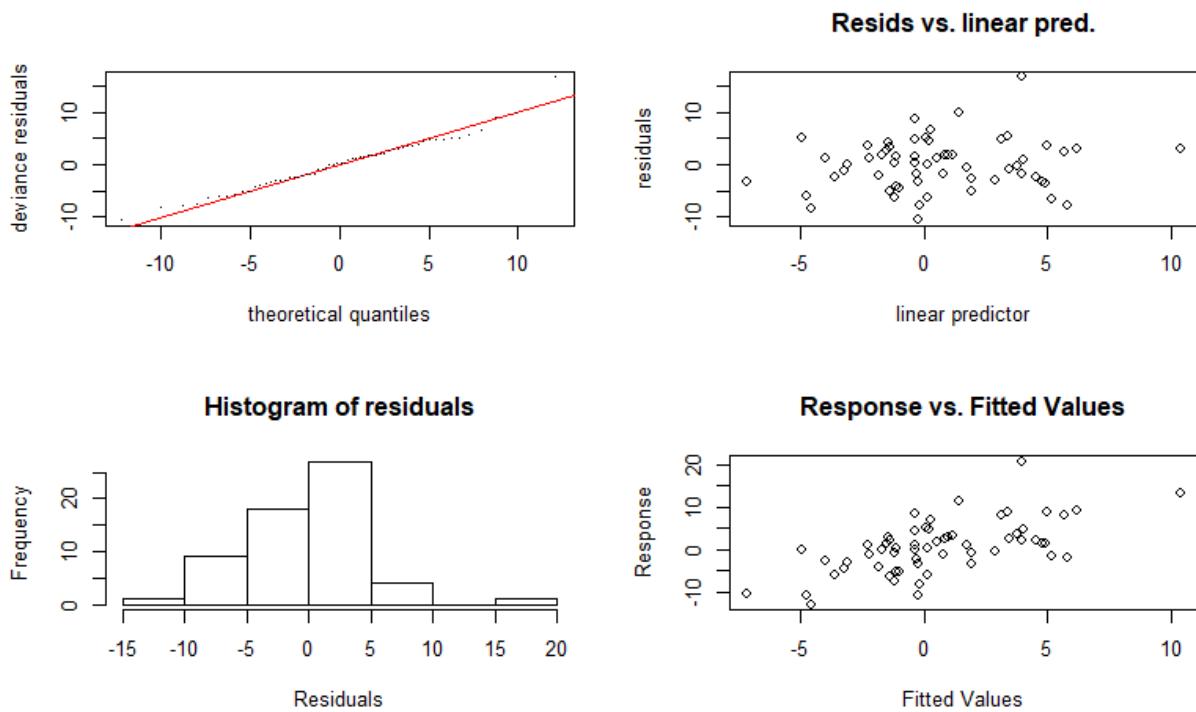


Figure A-52: Diagnostic Plots for TGT Fama-French 5-Factor GAM.

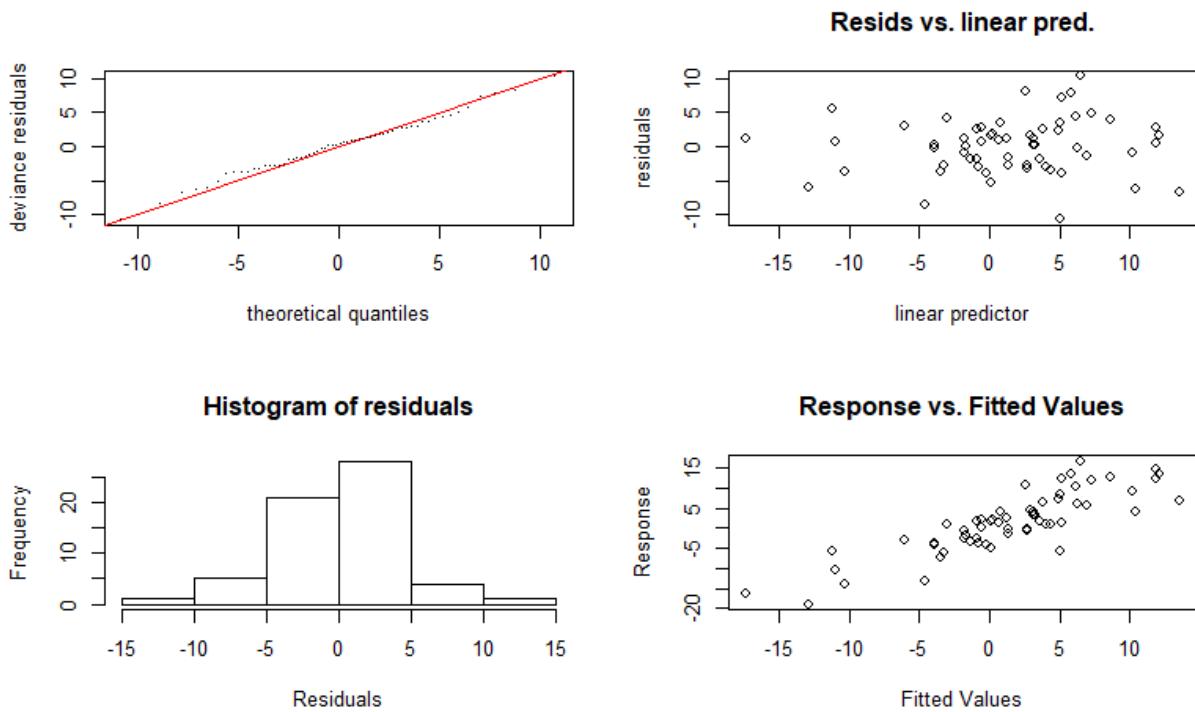


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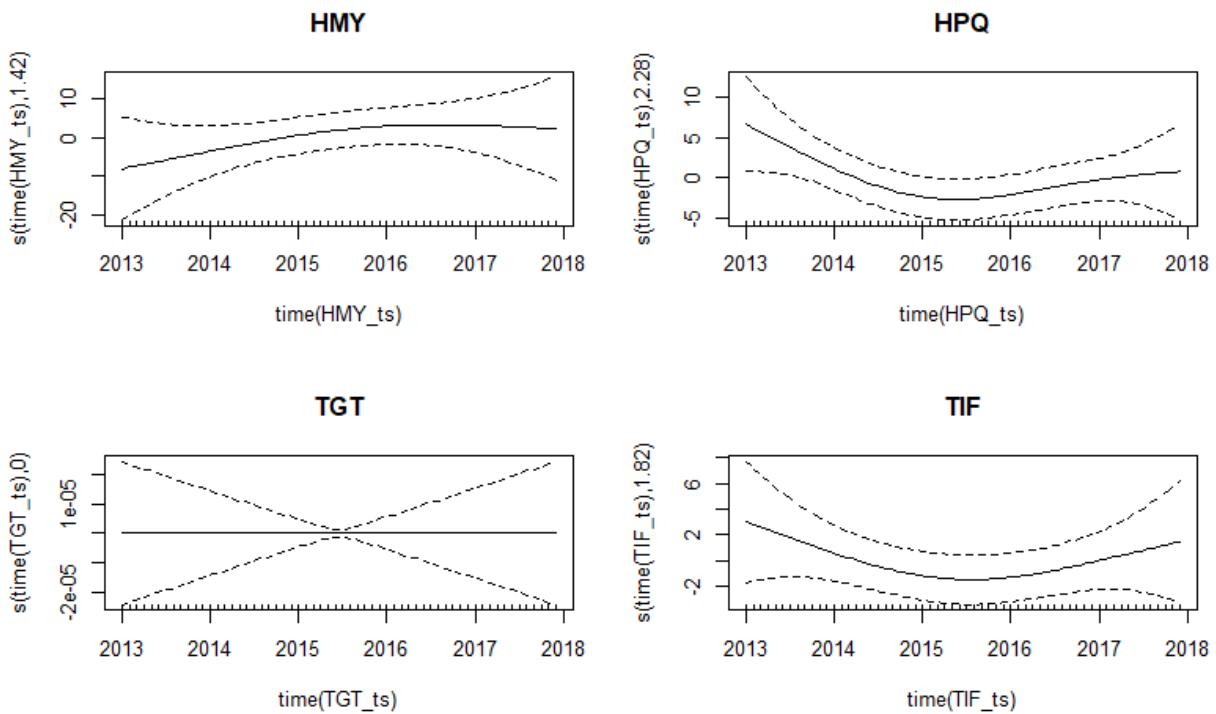


Figure A-54: Term Plots for Time-Only GAMs.

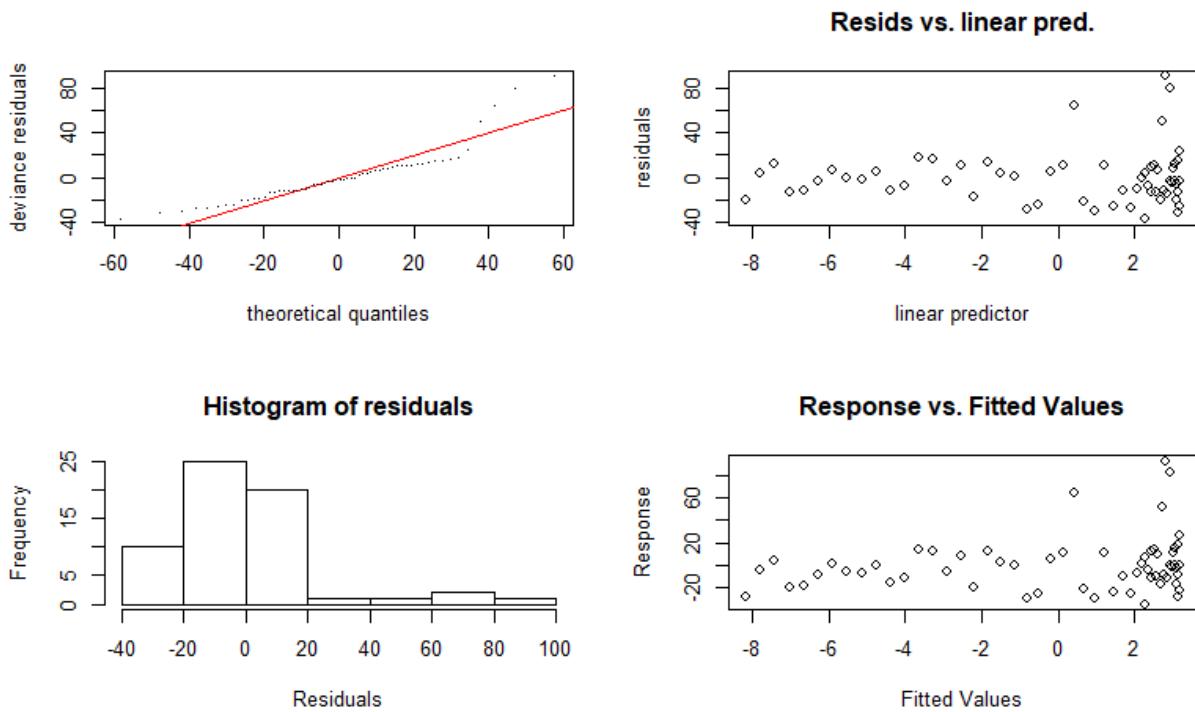


Figure A-55: Diagnostic Plots for HMY Time-Only GAM.

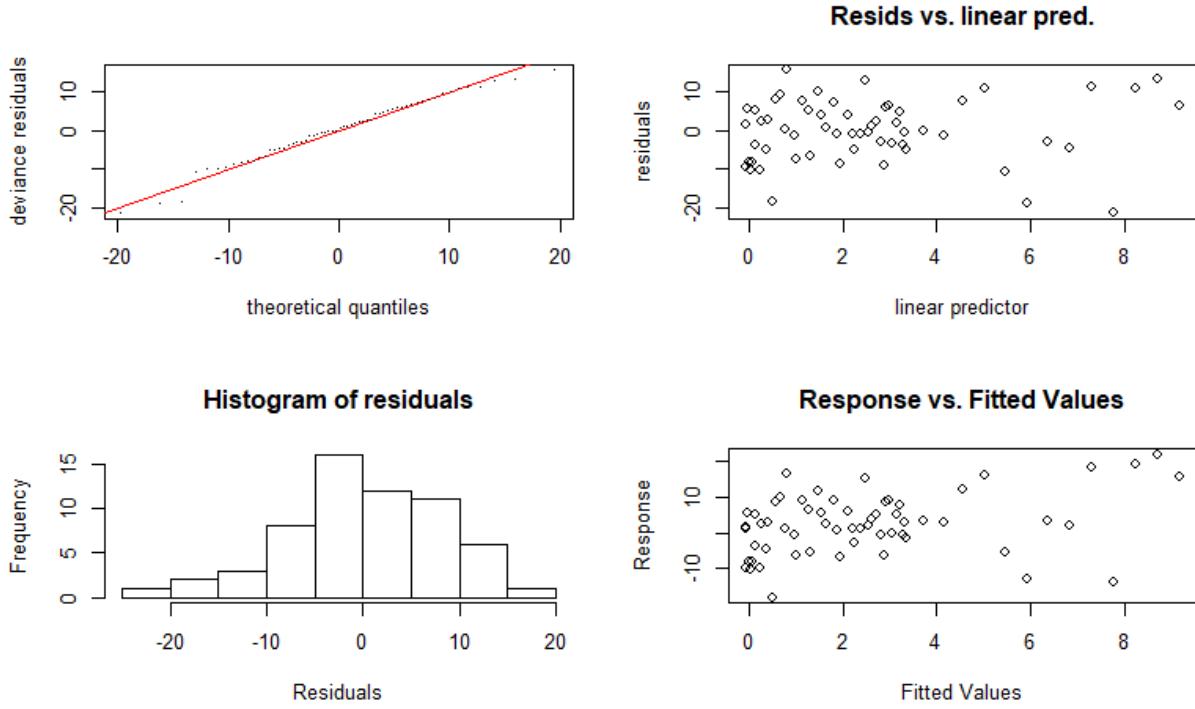


Figure A-56: Diagnostic Plots for HPQ Time-Only GAM.

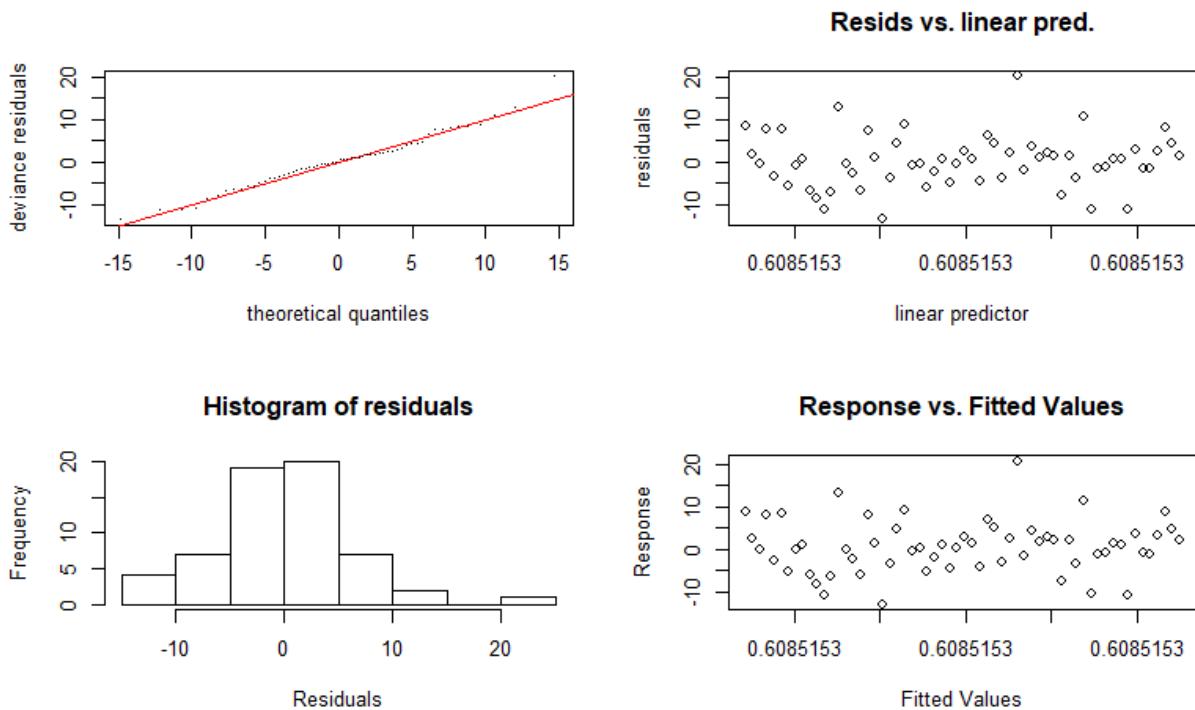


Figure A-57: Diagnostic Plots for TGT Time-Only GAM.

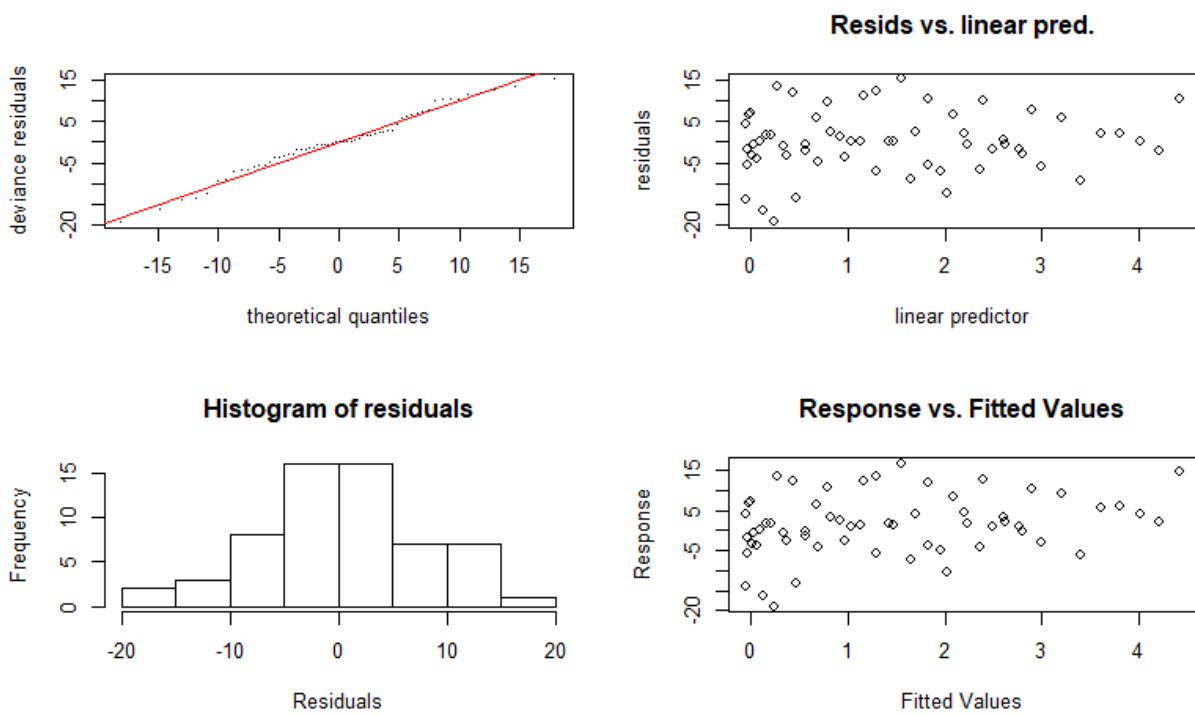


Figure A-58: Diagnostic Plots for TIF Time-Only GAM.

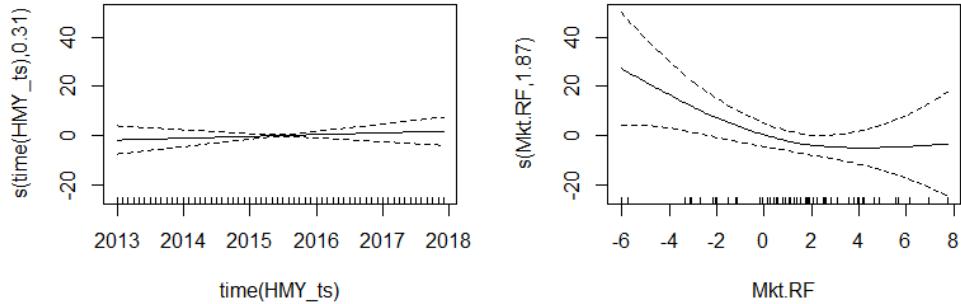


Figure A-59: Term Plots for HMY CAPM Time GAM.

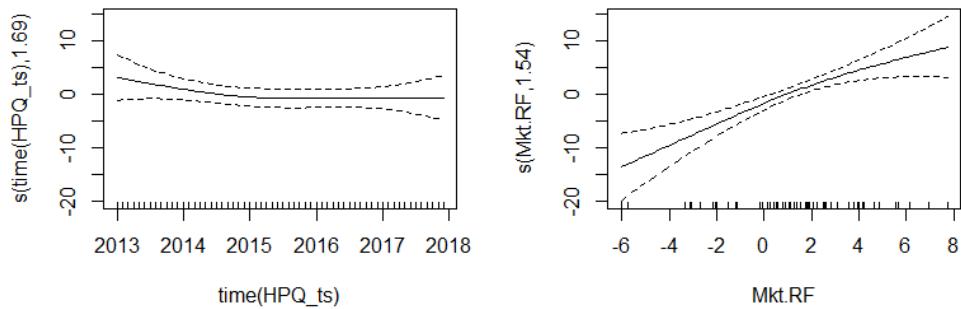


Figure A-60: Term Plots for HPQ CAPM Time GAM.

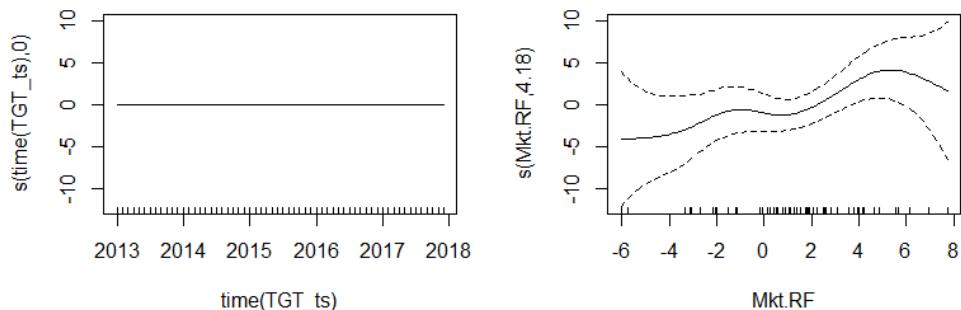


Figure A-61: Term Plots for TGT CAPM Time GAM.

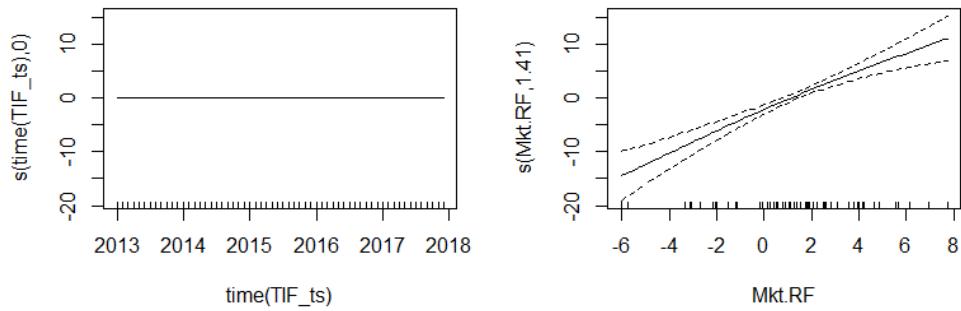


Figure A-62: Term Plots for TIF CAPM Time GAM.

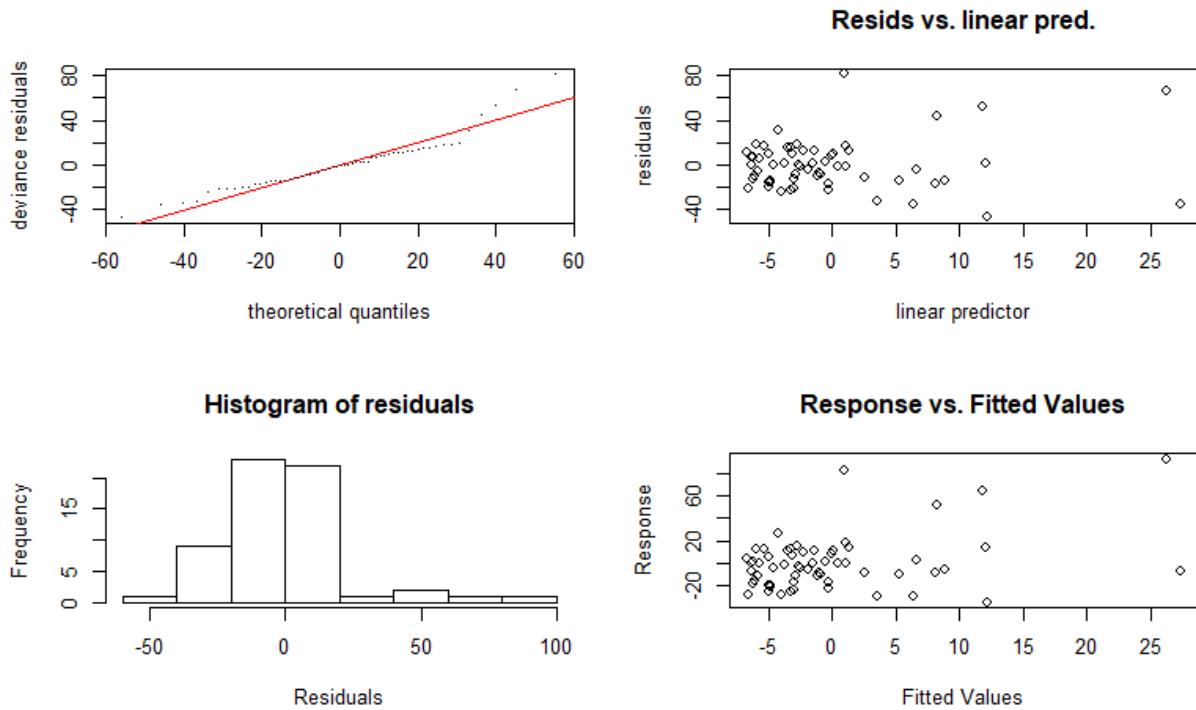


Figure A-63: Diagnostic Plots for HMY CAPM Time GAM.

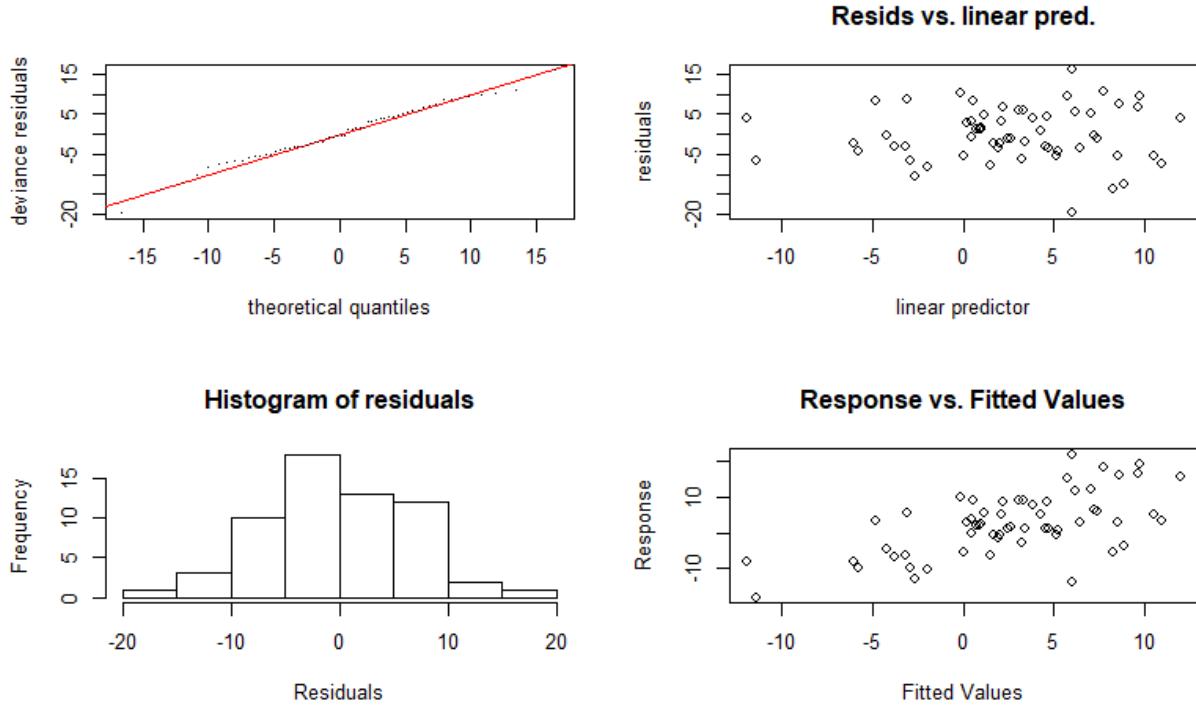


Figure A-64: Diagnostic Plots for HPQ CAPM Time GAM.

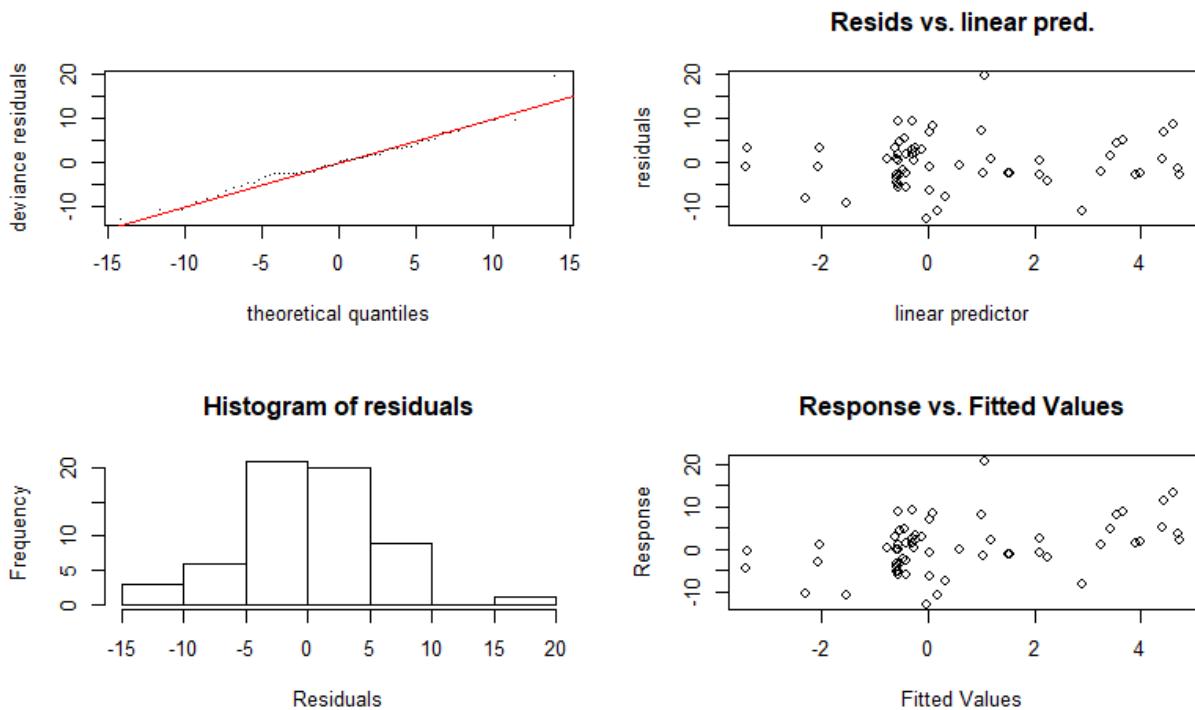


Figure A-65: Diagnostic Plots for TGT CAPM Time GAM.

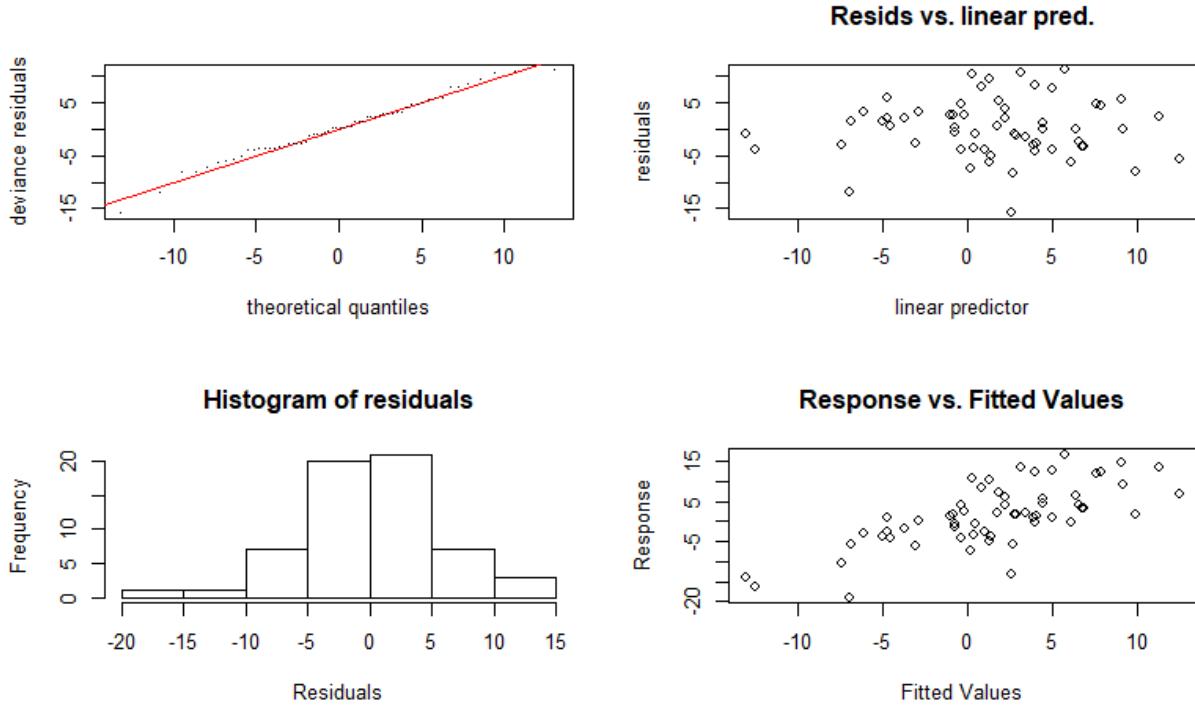


Figure A-66: Diagnostic Plots for TIF CAPM Time GAM.

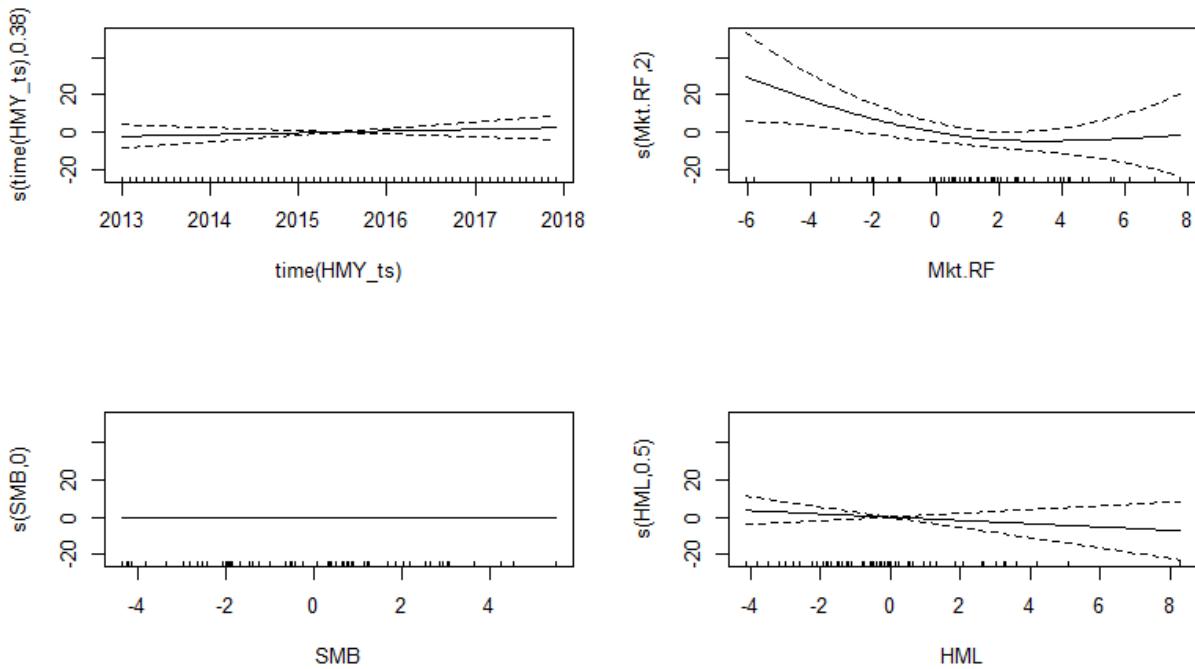


Figure A-67: Term Plots for HMY Fama-French 3-Factor Time GAM.

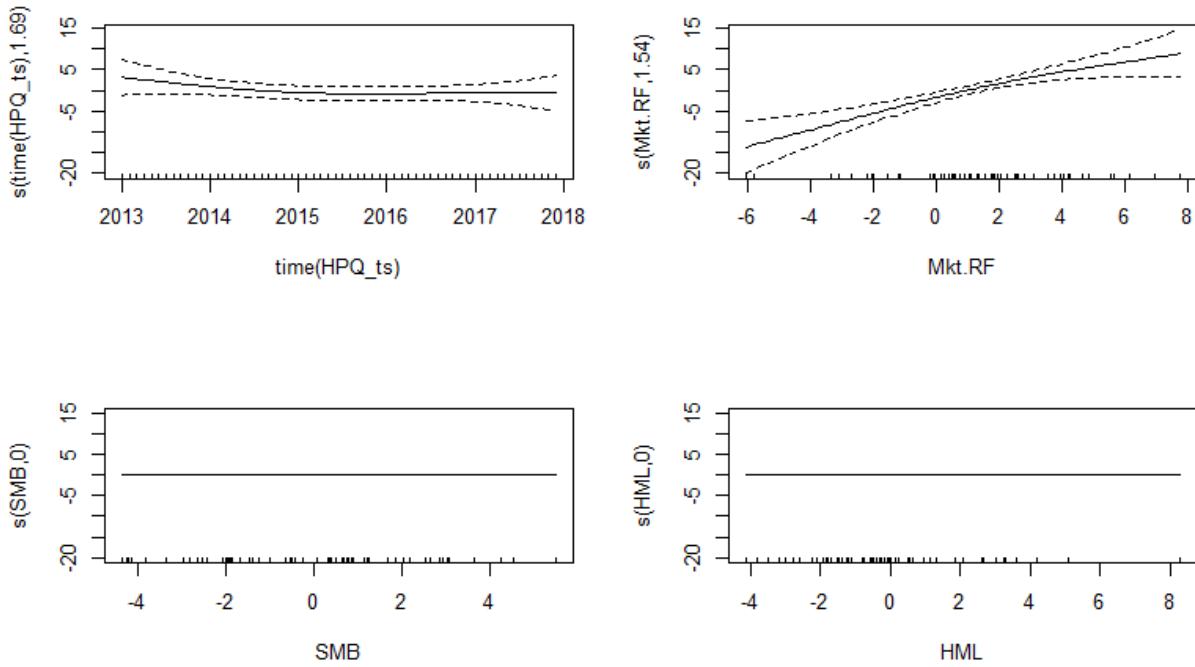


Figure A-68: Term Plots for HPQ Fama-French 3-Factor Time GAM.

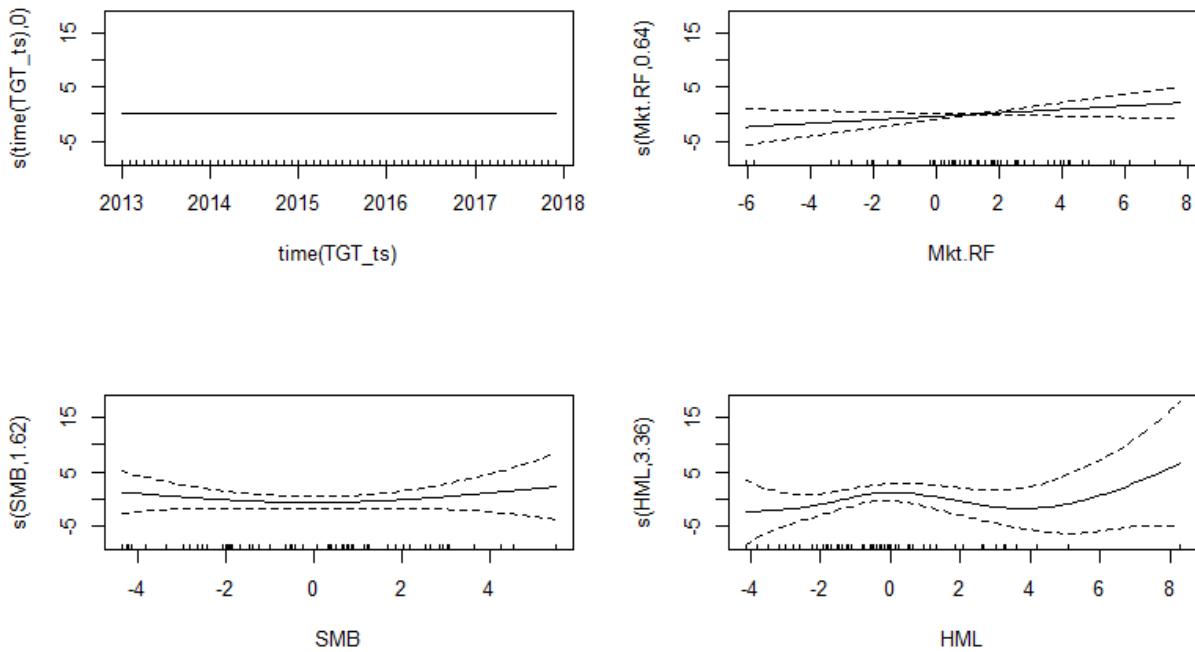


Figure A-69: Term Plots for TGT Fama-French 3-Factor Time GAM.

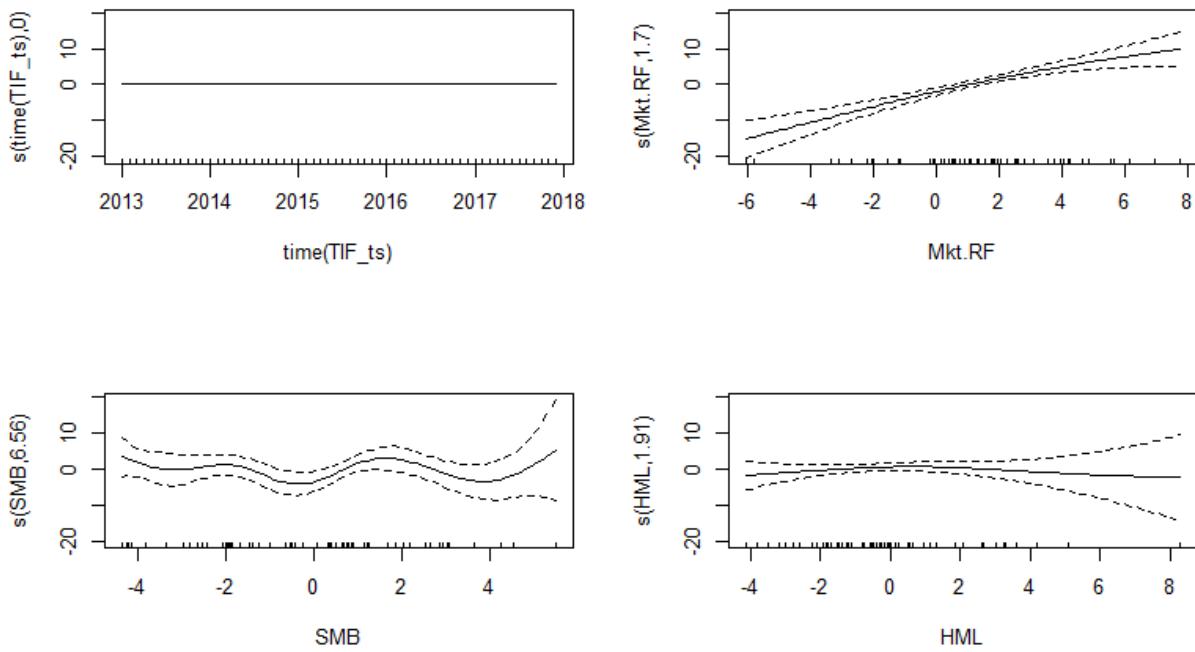


Figure A-70: Term Plots for TIF Fama-French 3-Factor Time GAM.

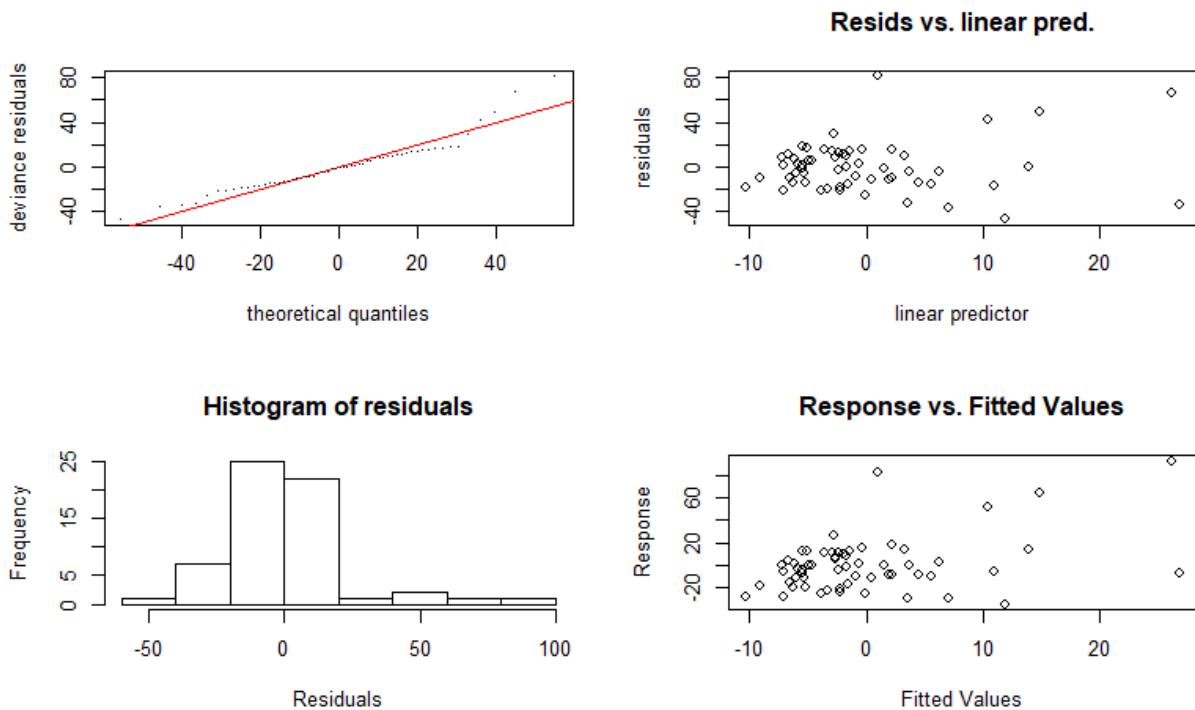


Figure A-71: Diagnostic Plots for HMY Fama-French 3-Factor Time GAM.

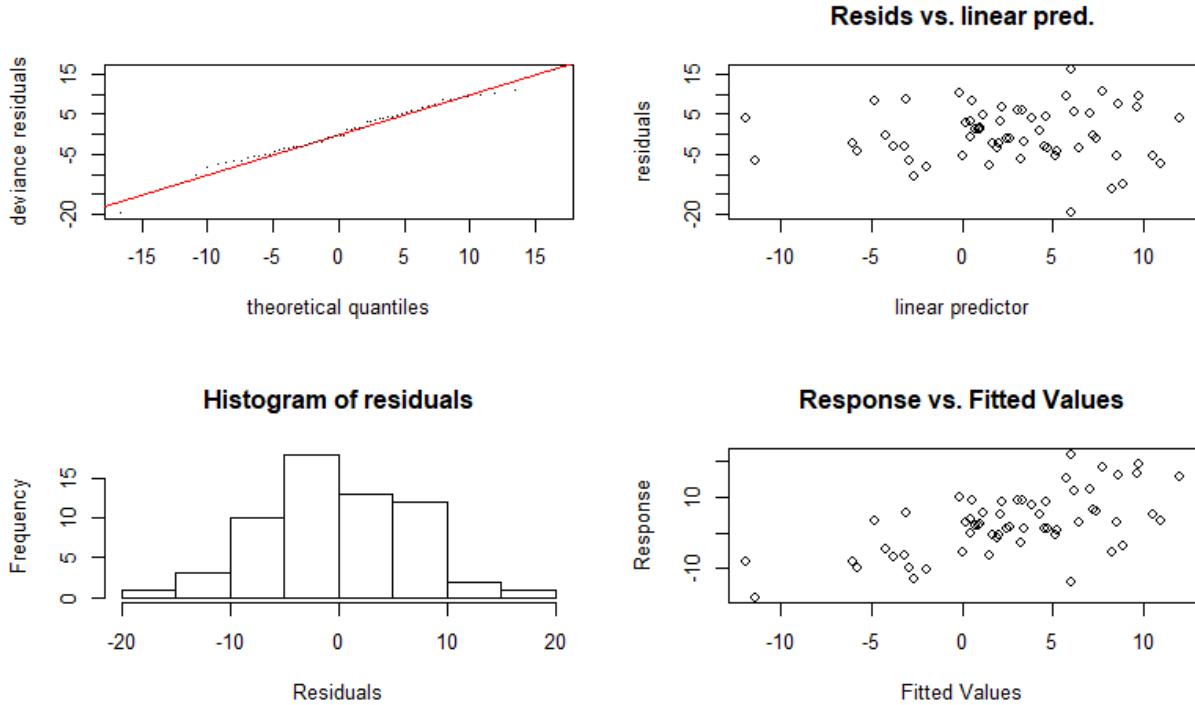


Figure A-72: Diagnostic Plots for HPQ Fama-French 3-Factor Time GAM.

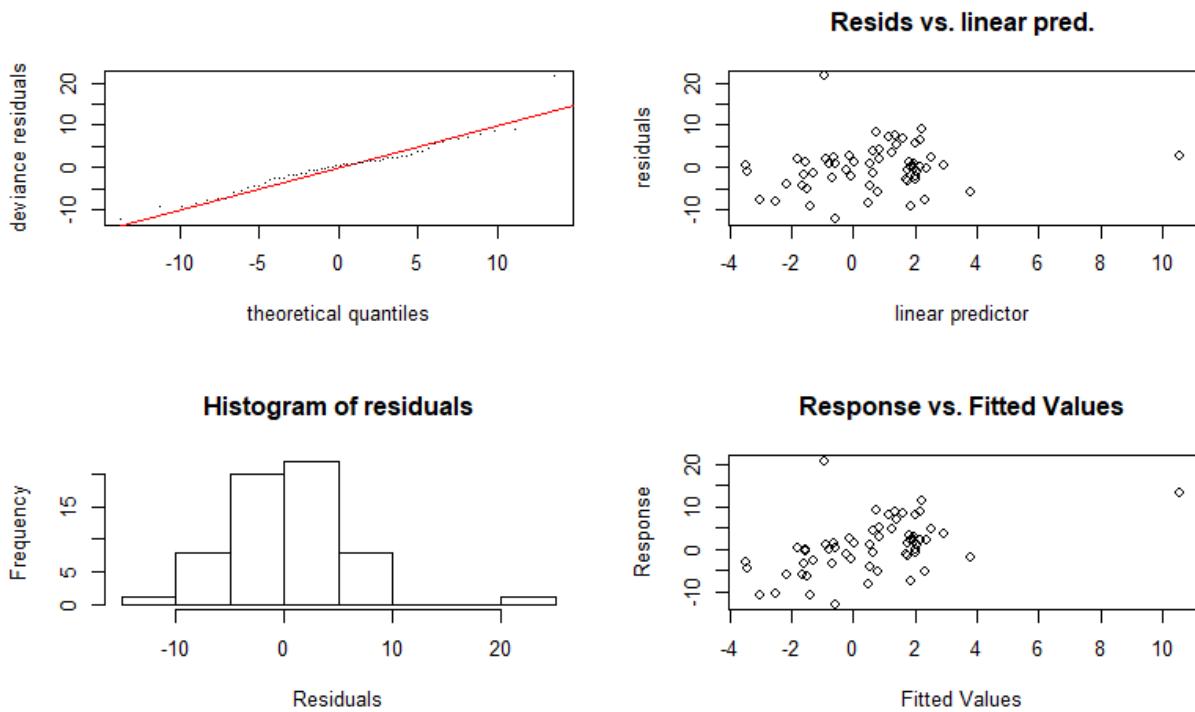


Figure A-73: Diagnostic Plots for TGT Fama-French 3-Factor Time GAM.

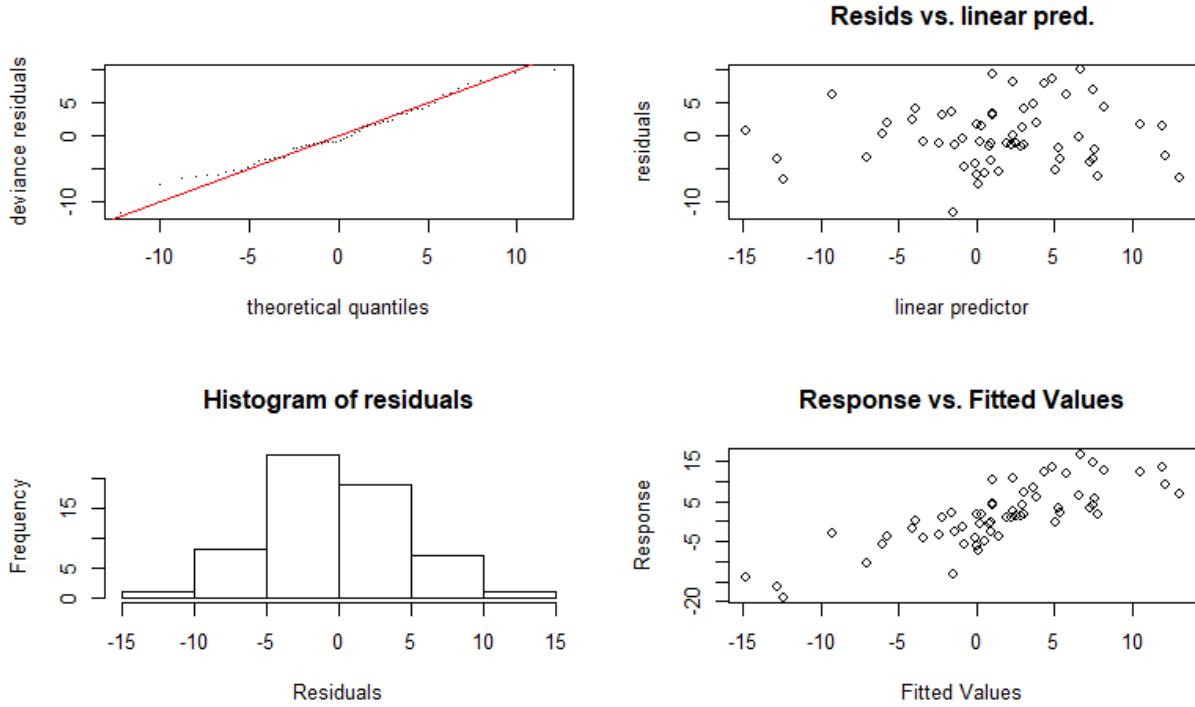


Figure A-74: Diagnostic Plots for TIF Fama-French 3-Factor Time GAM.

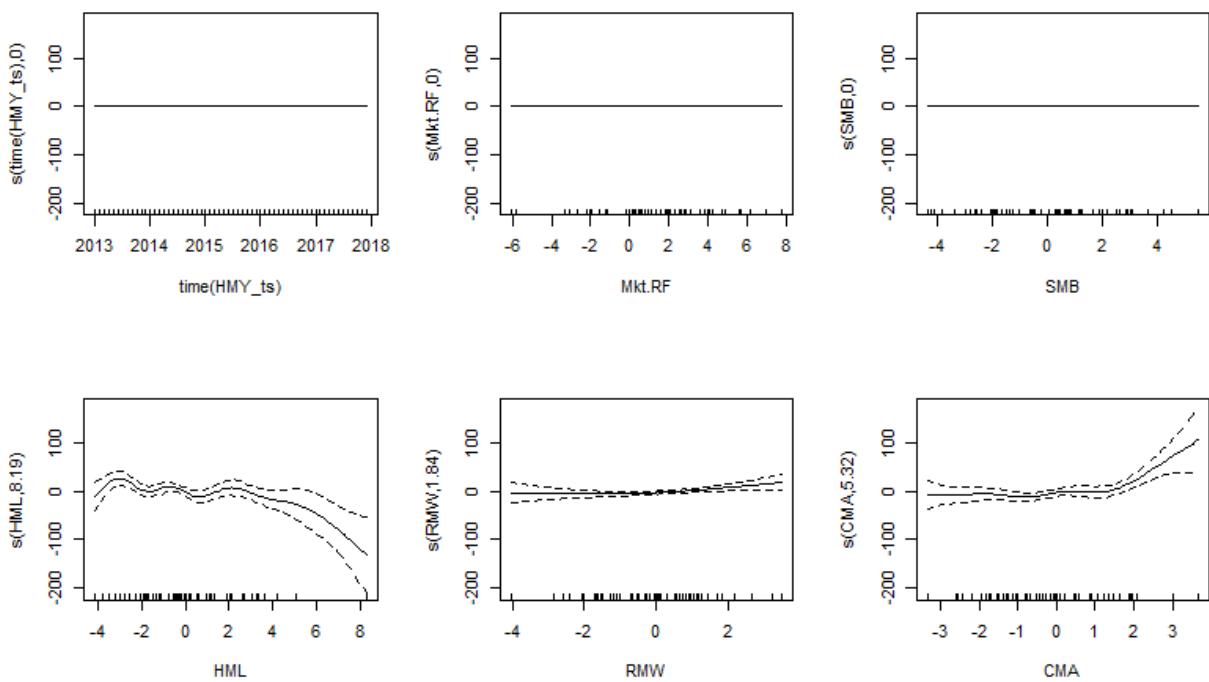


Figure A-75: Term Plots for HMY Fama-French 5-Factor Time GAM.

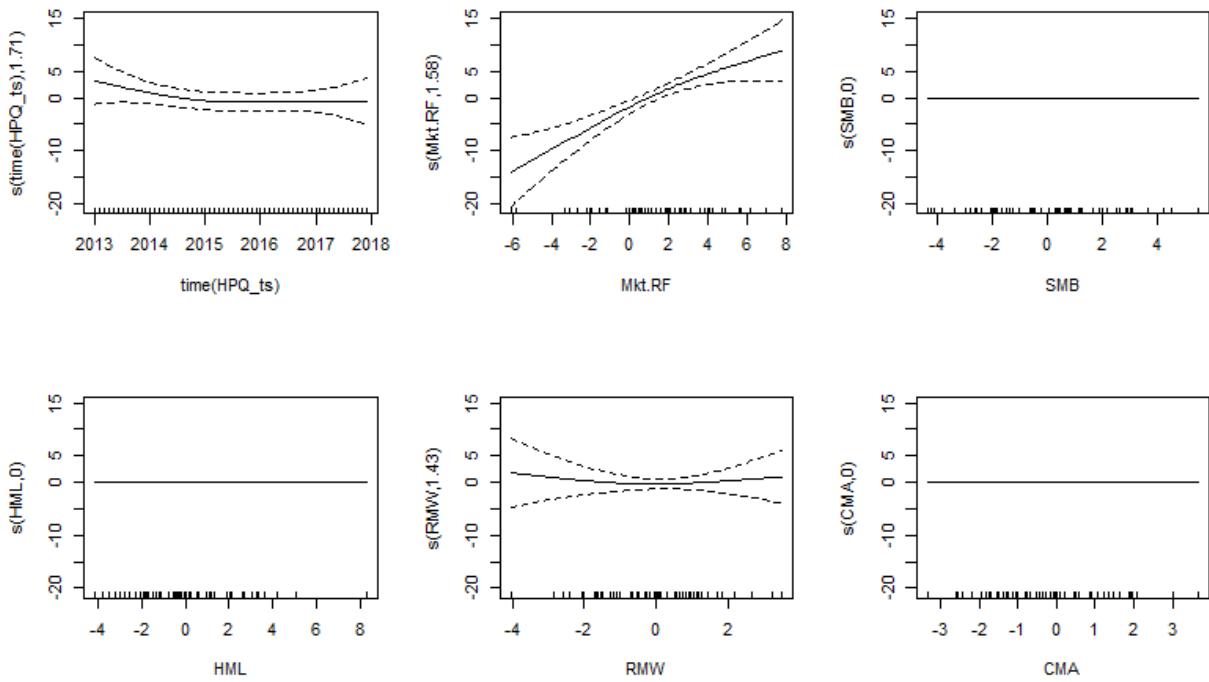


Figure A-76: Term Plots for HPQ Fama-French 5-Factor Time GAM.

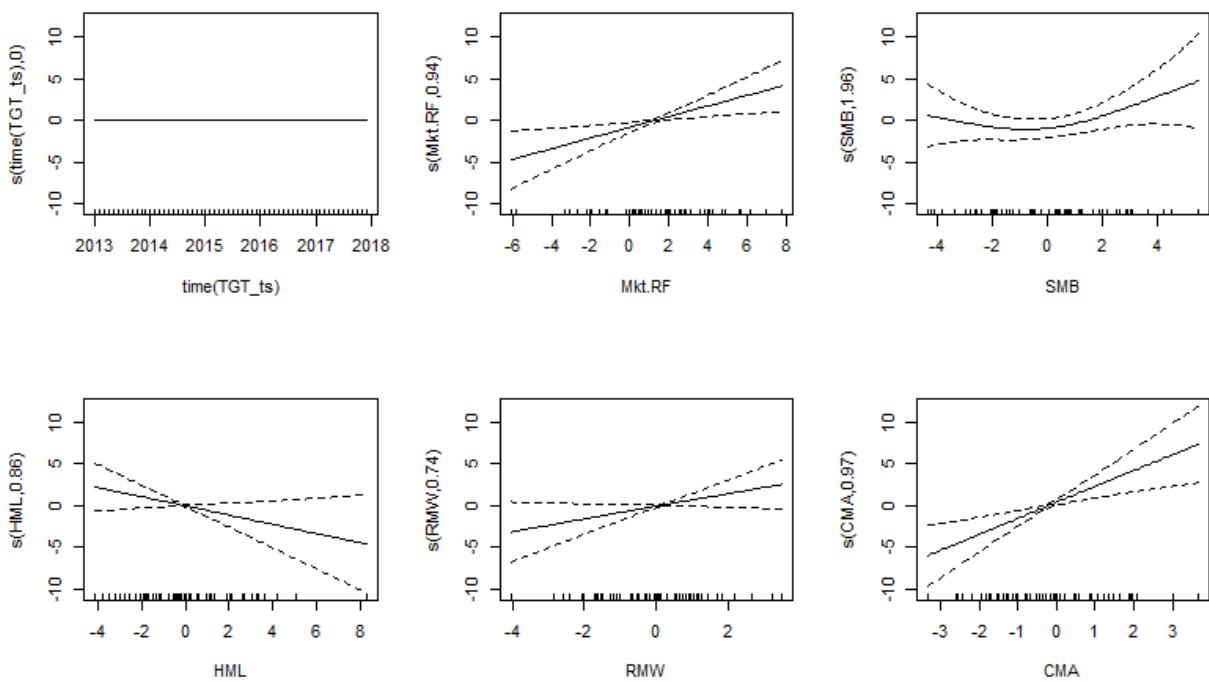


Figure A-77: Term Plots for TGT Fama-French 5-Factor Time GAM.

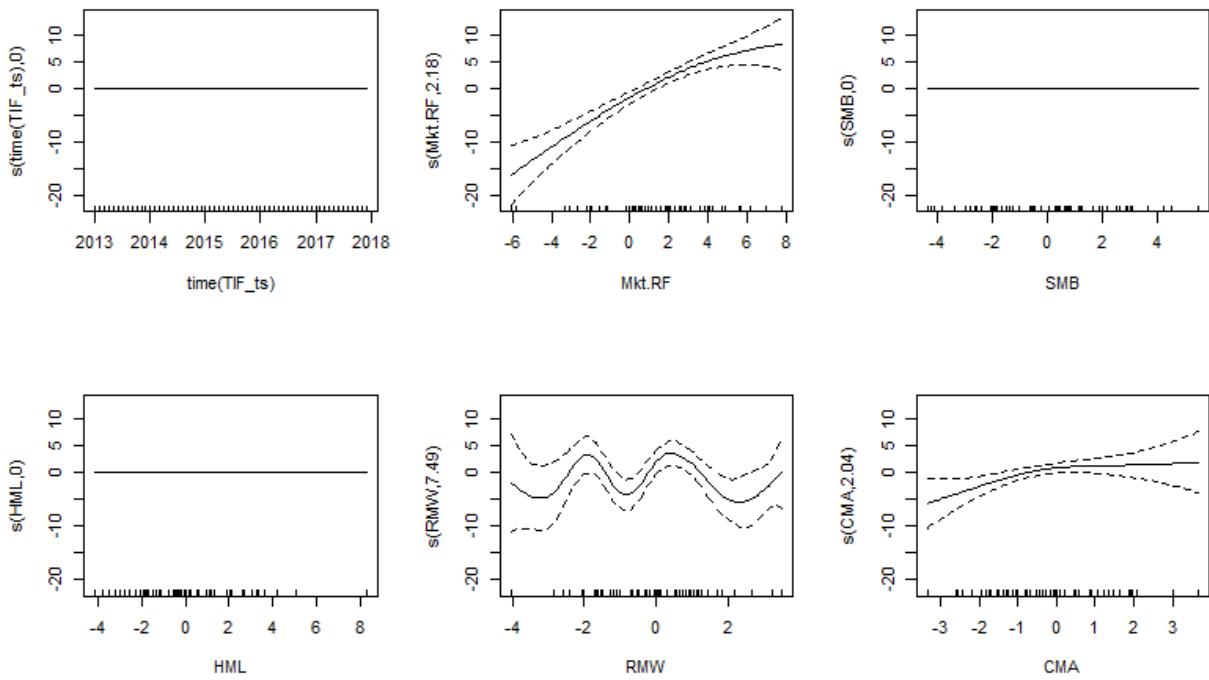


Figure A-78: Term Plots for TIF Fama-French 5-Factor Time GAM.

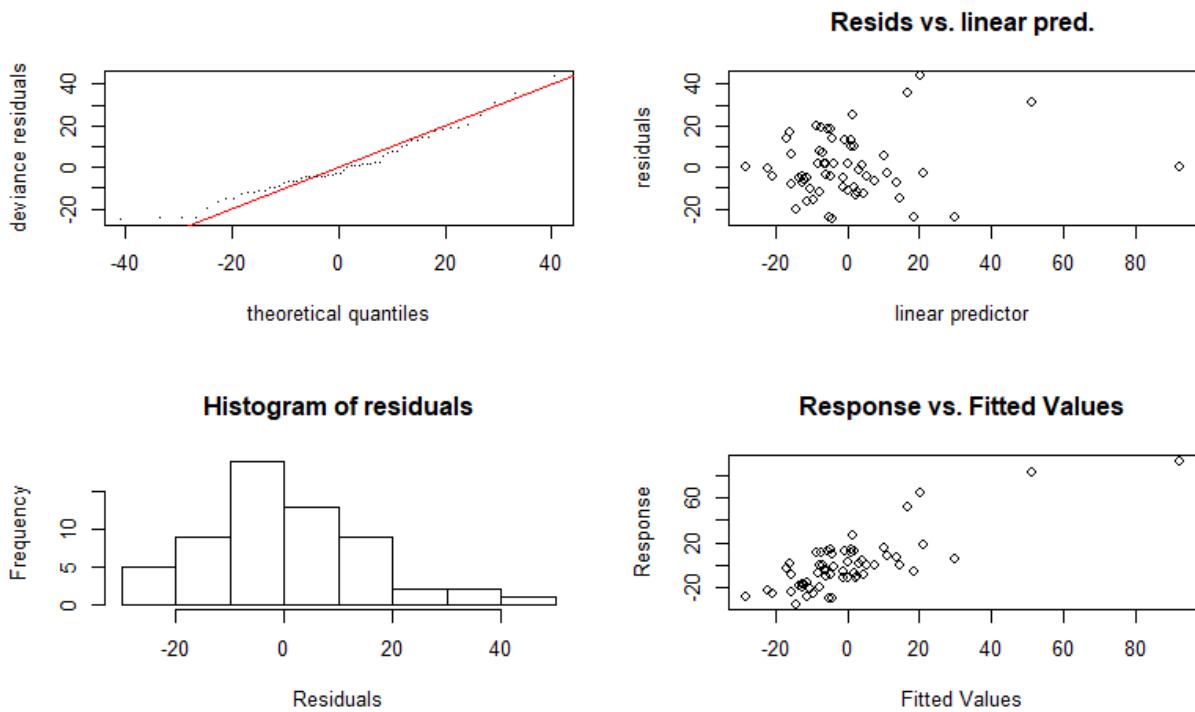


Figure A-79: Diagnostic Plots for HMY Fama-French 5-Factor Time GAM.

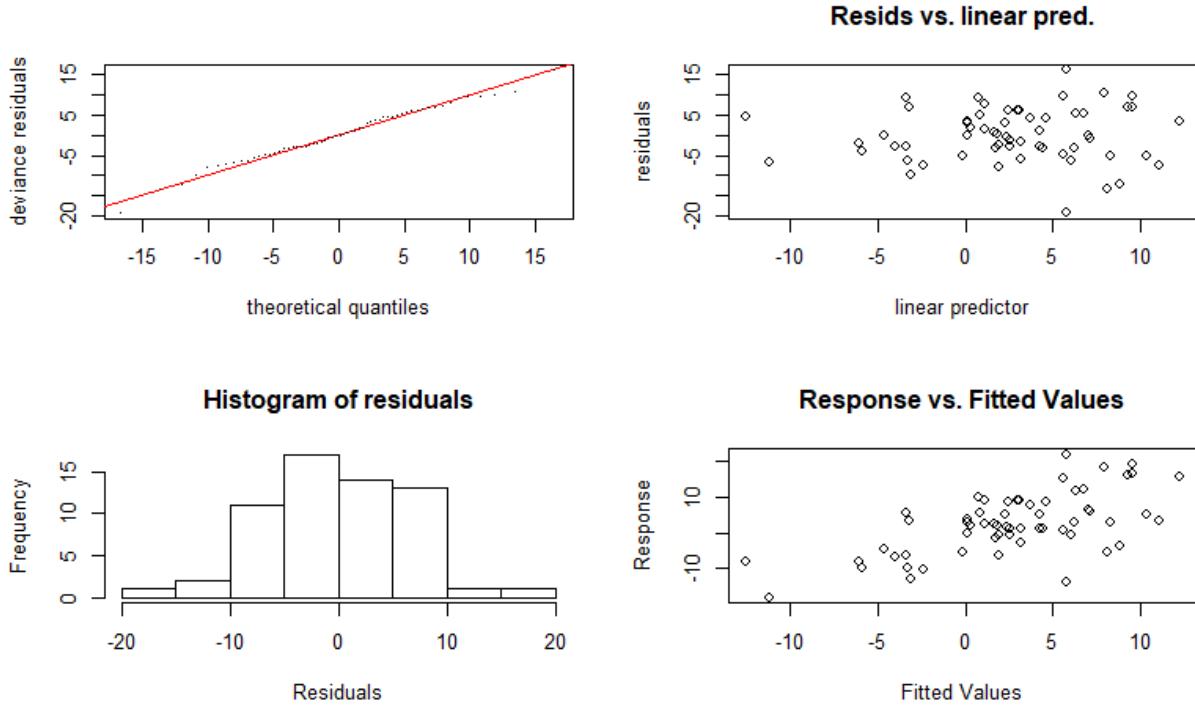


Figure A-80: Diagnostic Plots for HPQ Fama-French 5-Factor Time GAM.

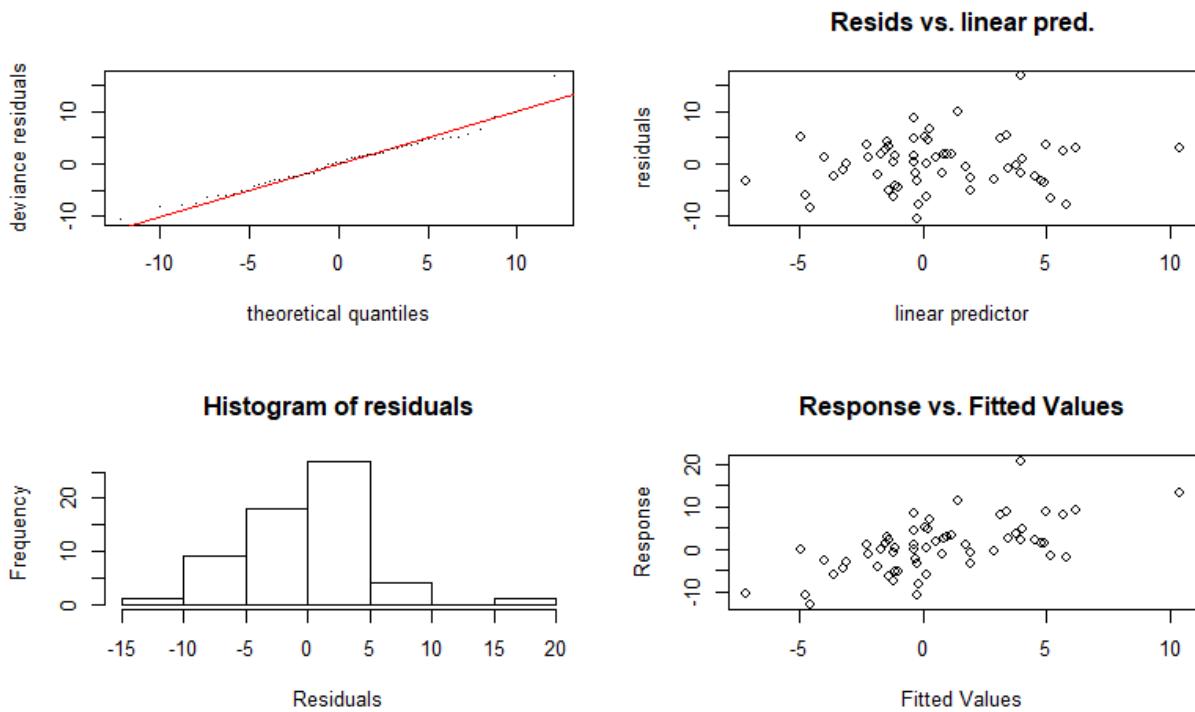


Figure A-81: Diagnostic Plots for TGT Fama-French 5-Factor Time GAM.

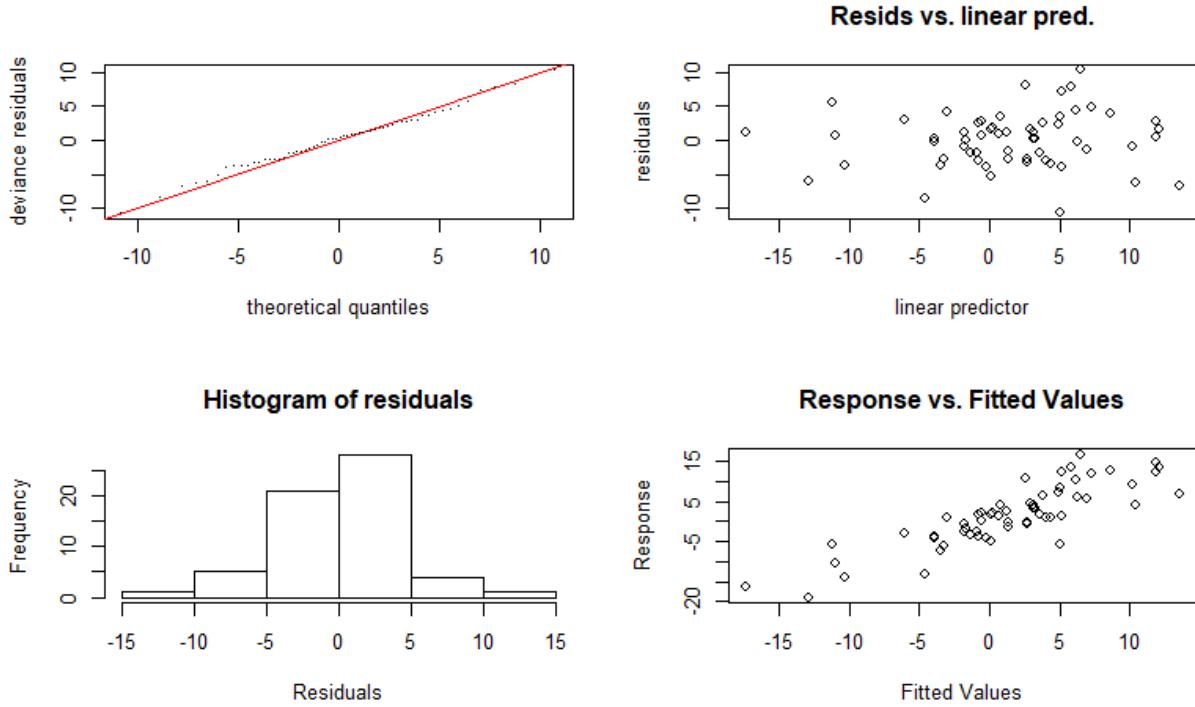


Figure A-82: Diagnostic Plots for TIF Fama-French 5-Factor Time GAM.

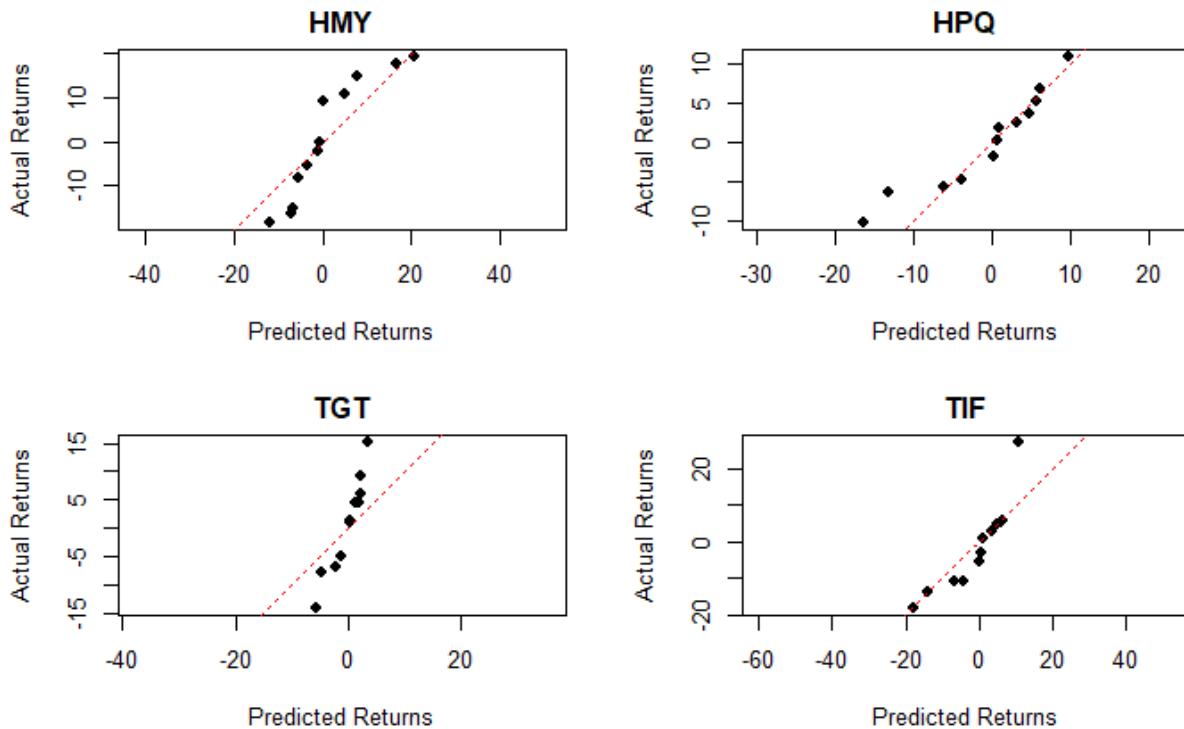


Figure A-83: Actual vs. Predicted Returns for Capital Asset Pricing Models.

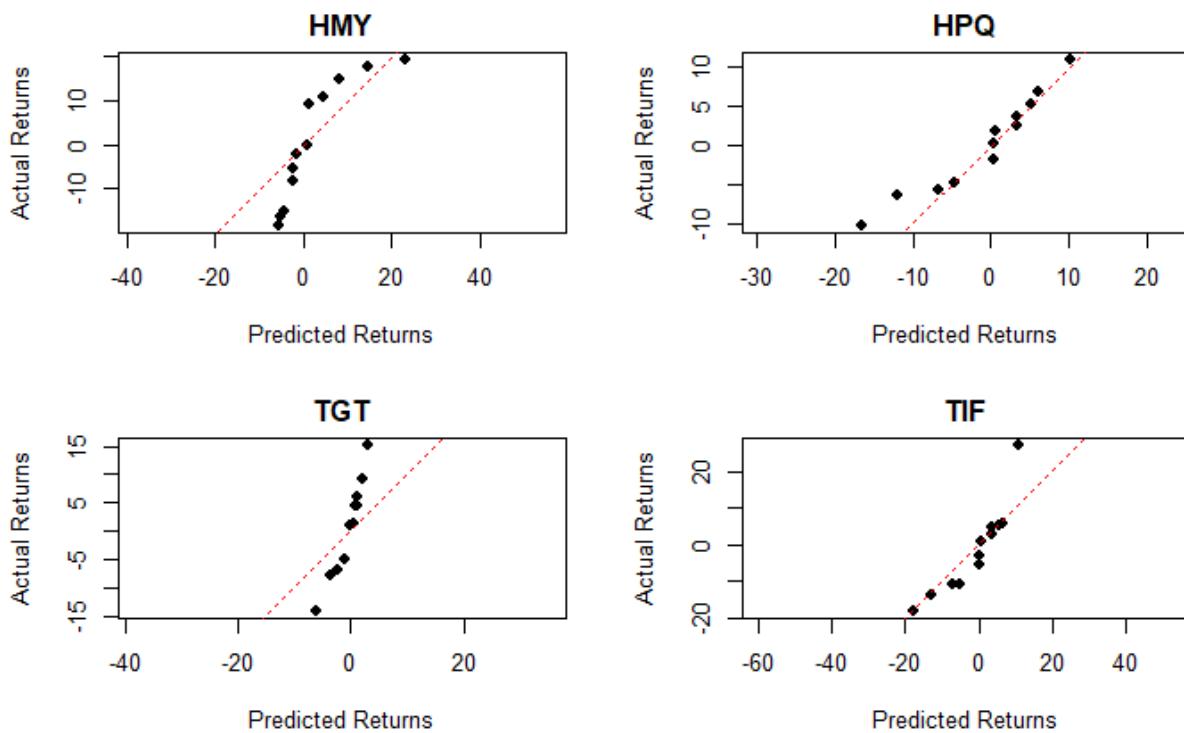


Figure A-84: Actual vs. Predicted Returns for Fama-French 3-Factor Models.

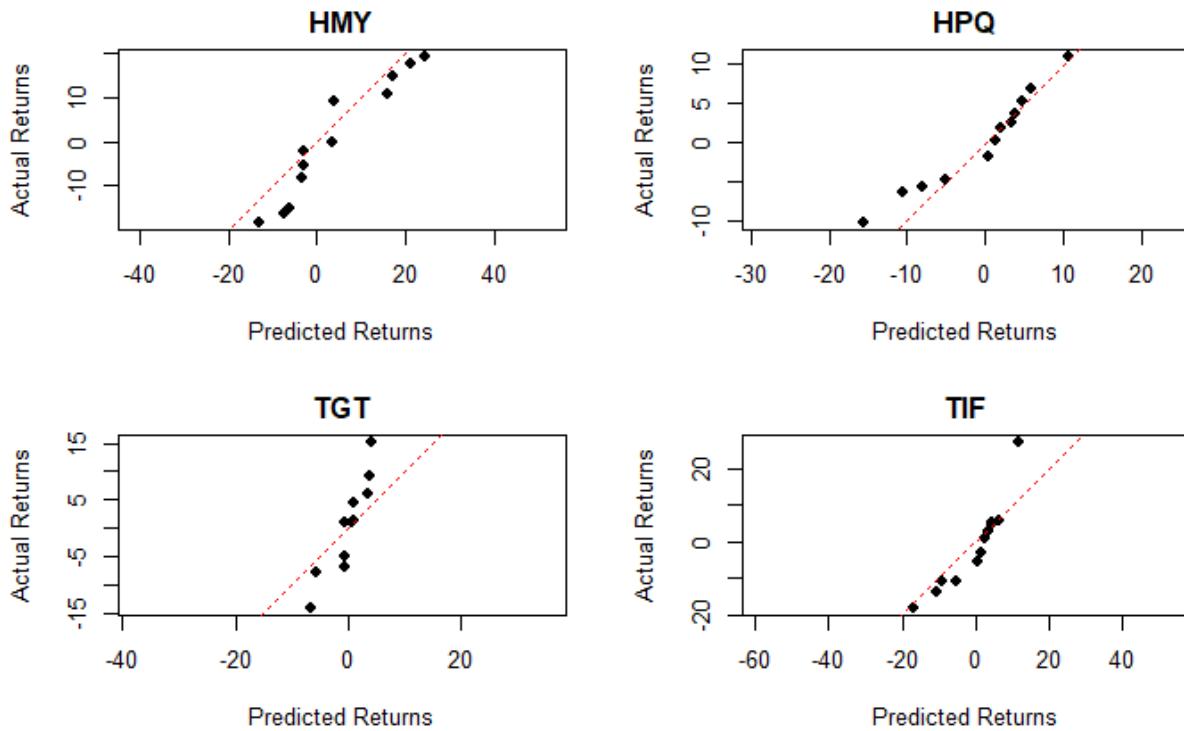


Figure A-85: Actual vs. Predicted Returns for Fama-French 5-Factor Models.

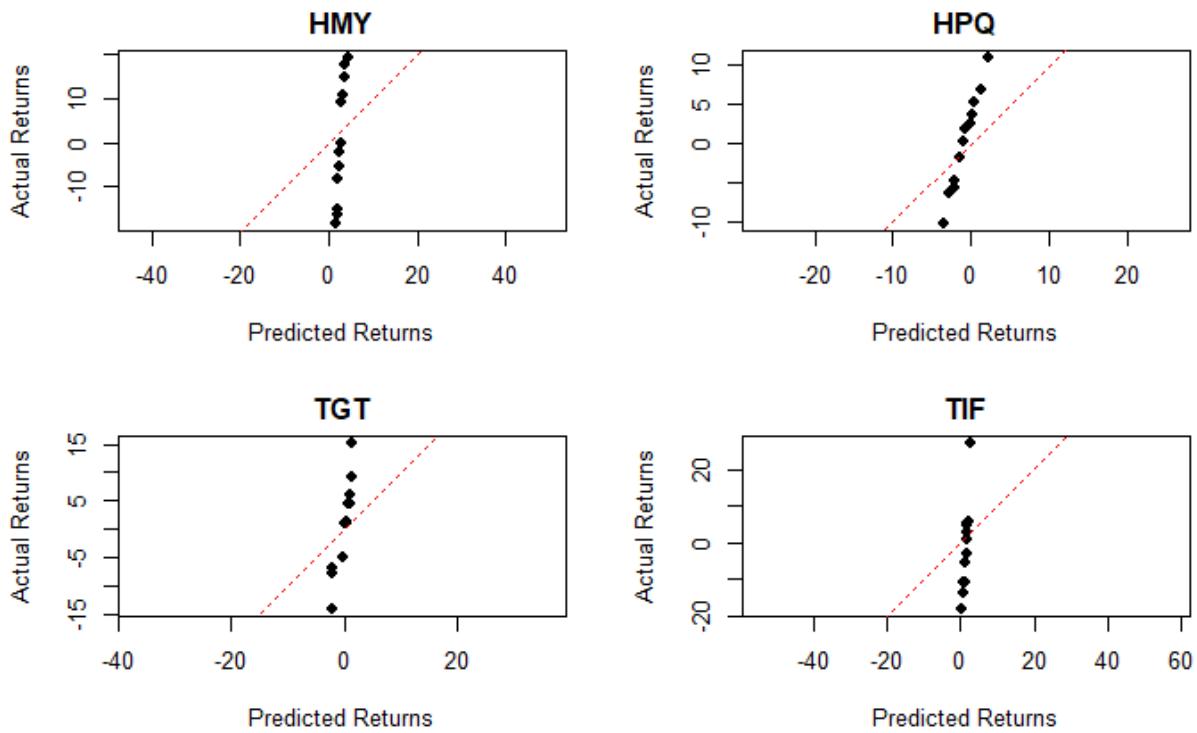


Figure A-86: Actual vs. Predicted Returns for ARMA Models.

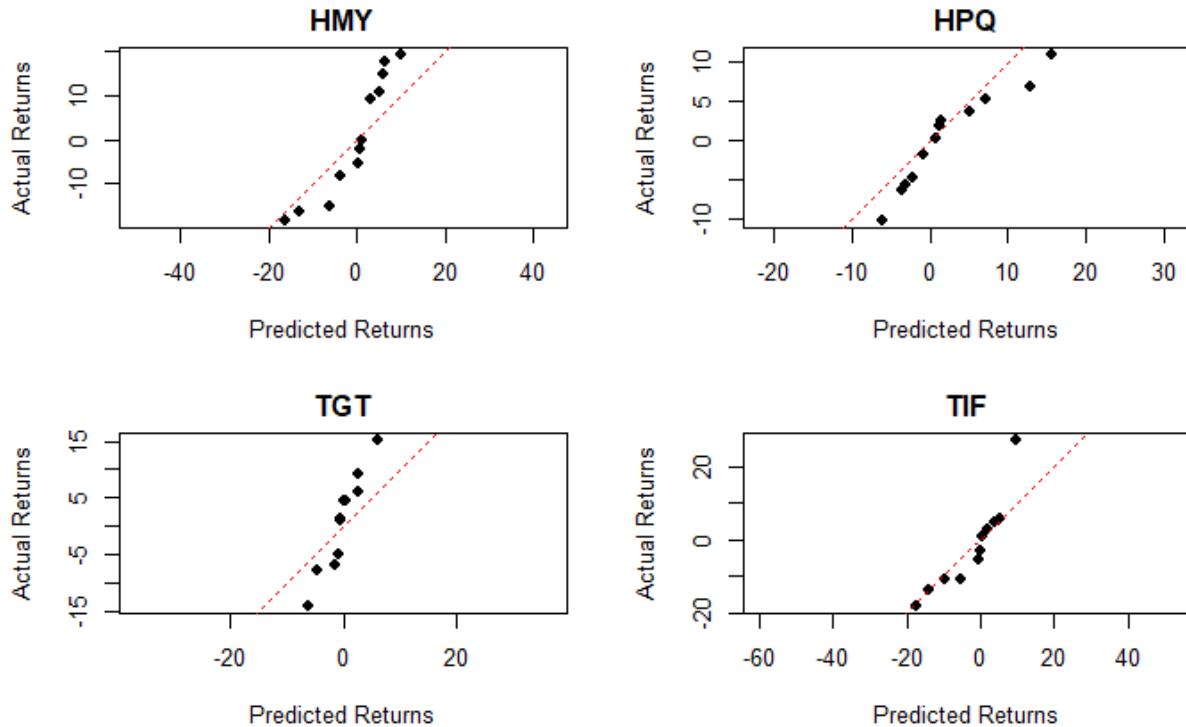


Figure A-87: Actual vs. Predicted Returns for ARMA Capital Asset Pricing Models.

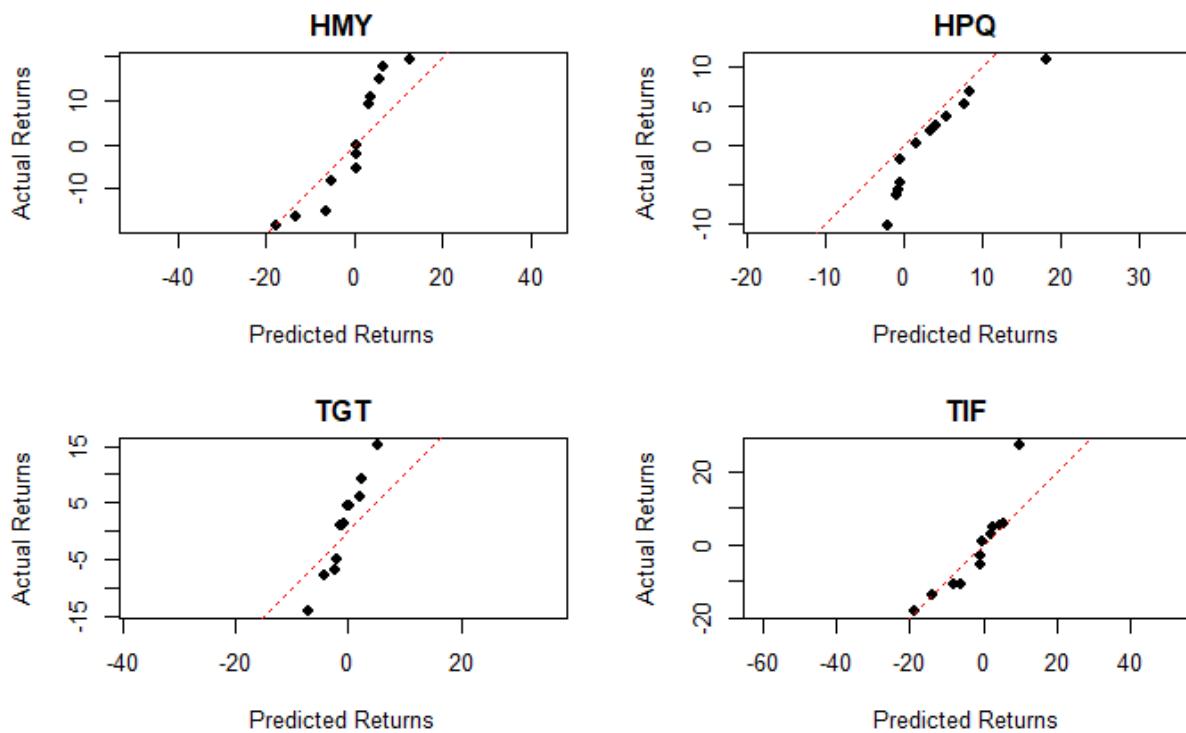


Figure A-88: Actual vs. Predicted Returns for ARMA Fama-French 3-Factor Models.

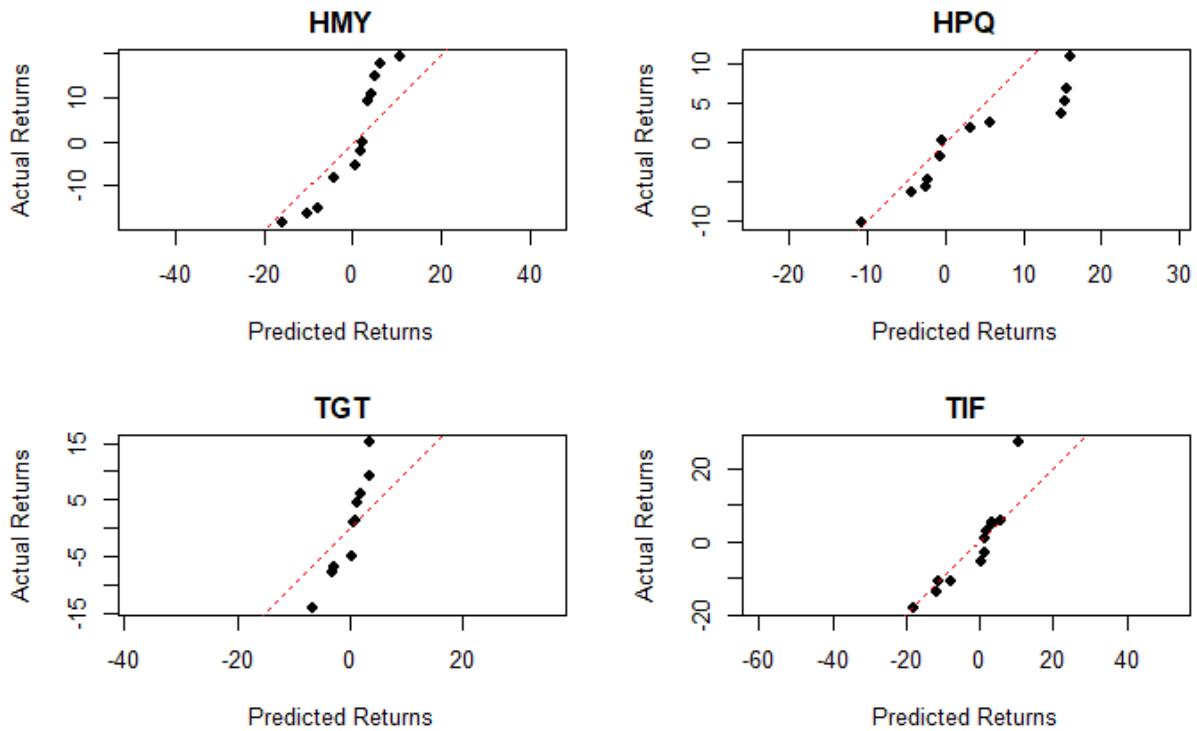


Figure A-89: Actual vs. Predicted Returns for ARMA Fama-French 5-Factor Models.

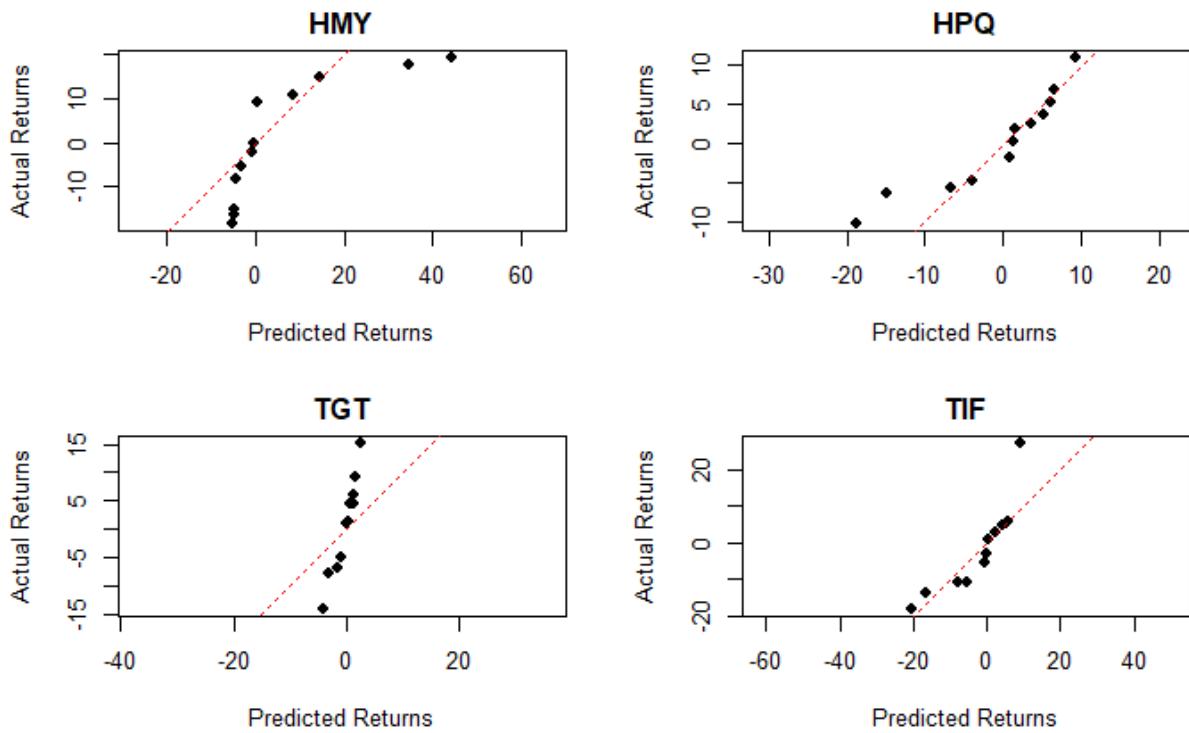


Figure A-90: Actual vs. Predicted Returns for CAPM GAMs.

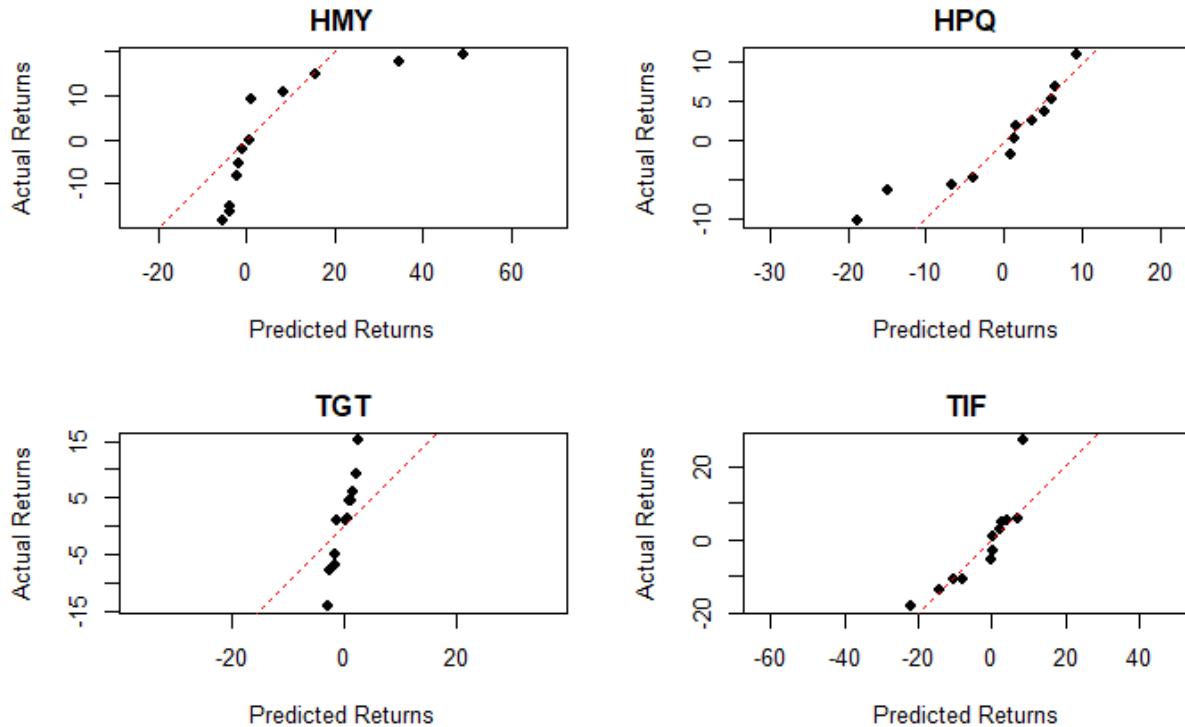


Figure A-91: Actual vs. Predicted Returns for Fama-French 3-Factor Models.

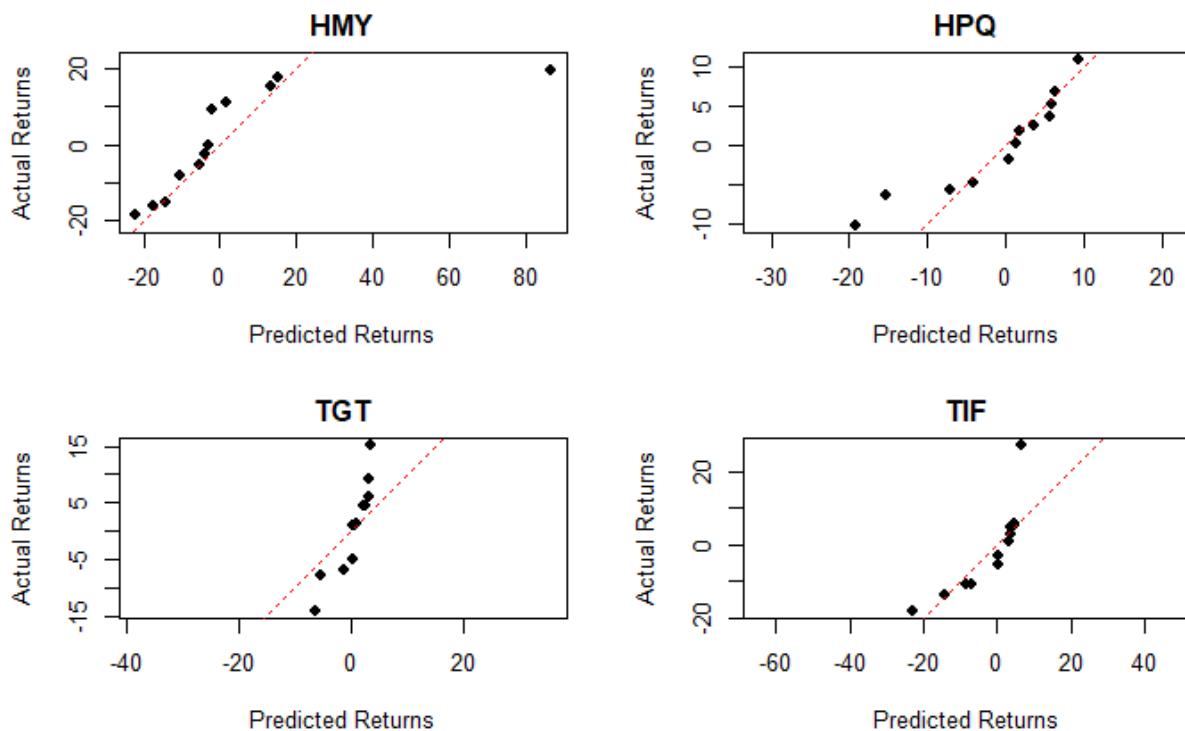


Figure A-92: Actual vs. Predicted Returns for Fama-French 5-Factor Models.

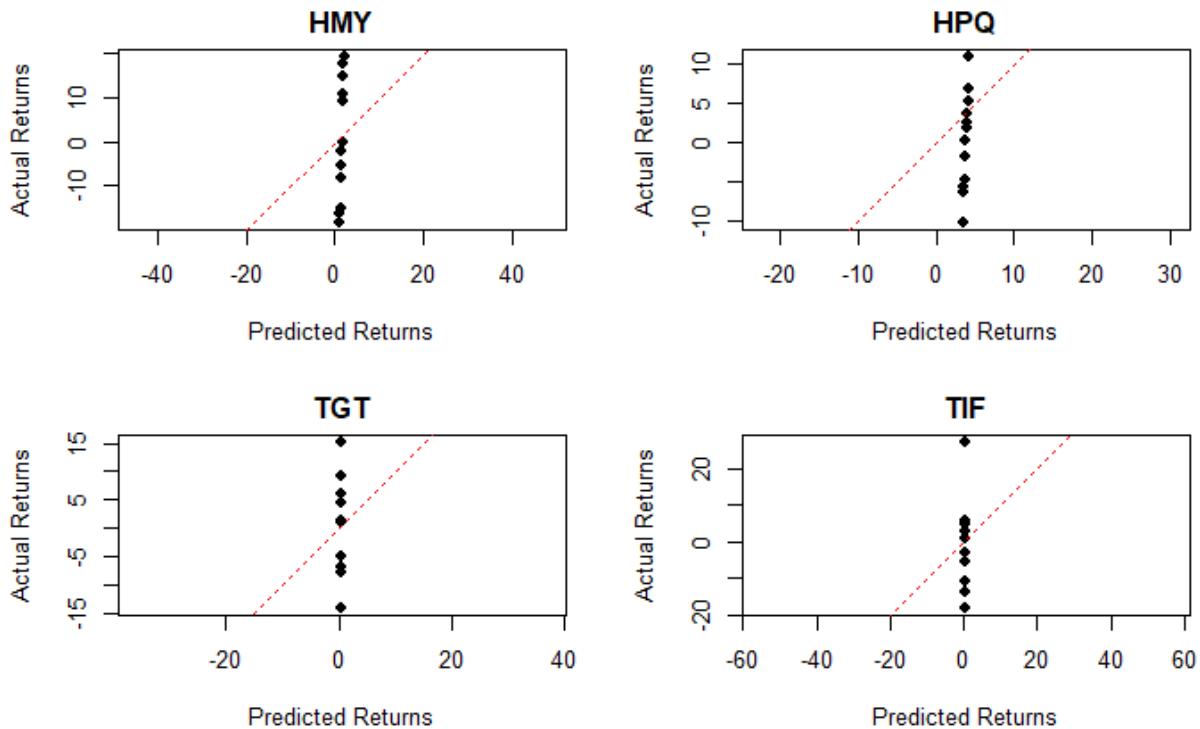


Figure A-93: Actual vs. Predicted Returns for Time-Only GAMs.

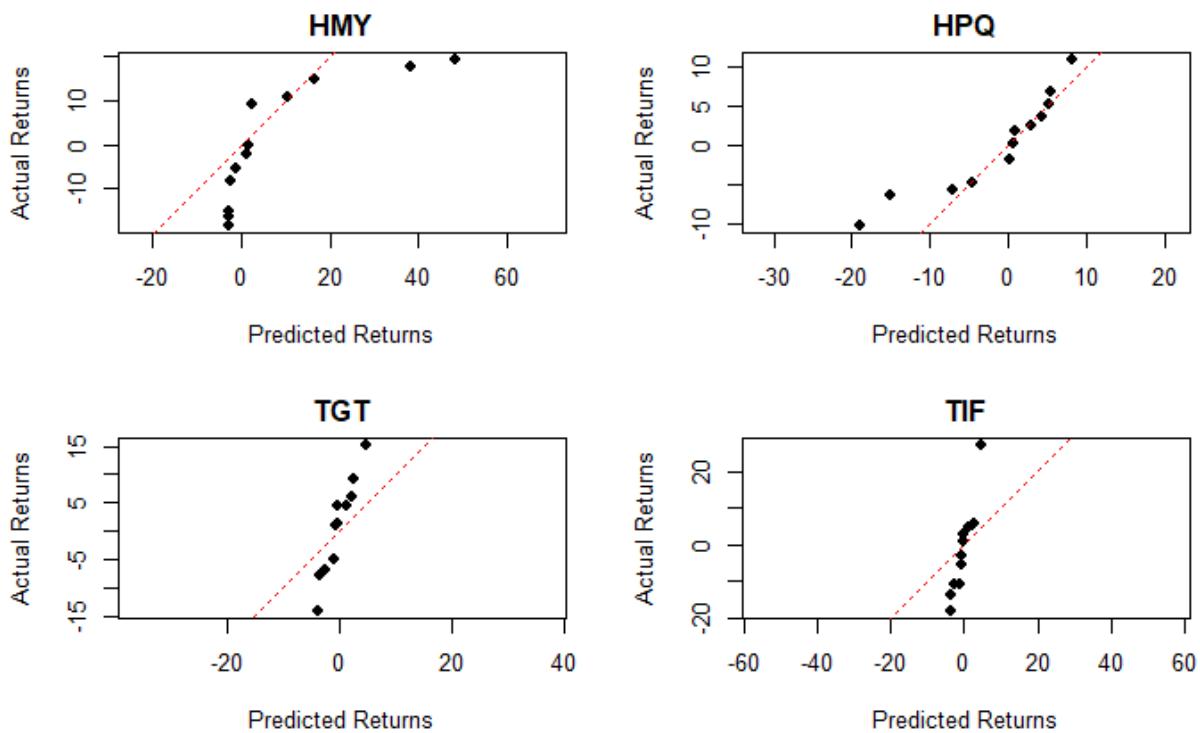


Figure A-94: Actual vs. Predicted Returns for CAPM Time GAMs.

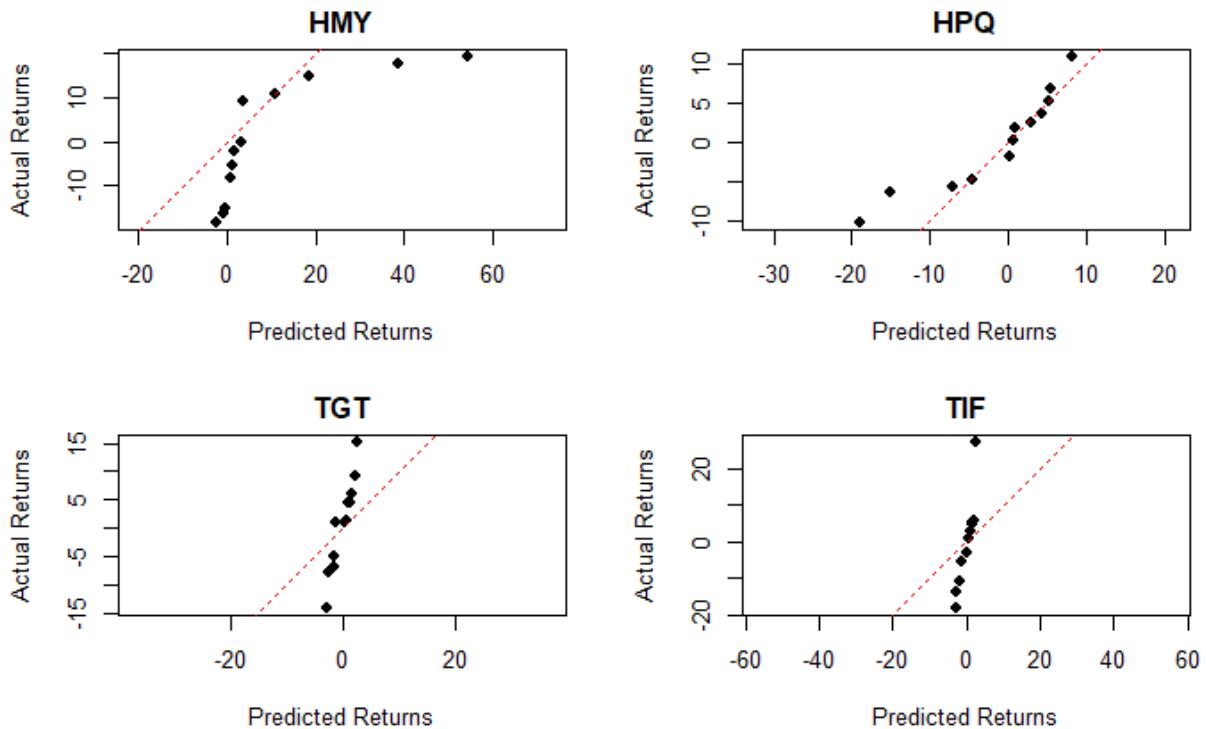


Figure A-95: Actual vs. Predicted Returns for Fama-French 3-Factor GAMs.

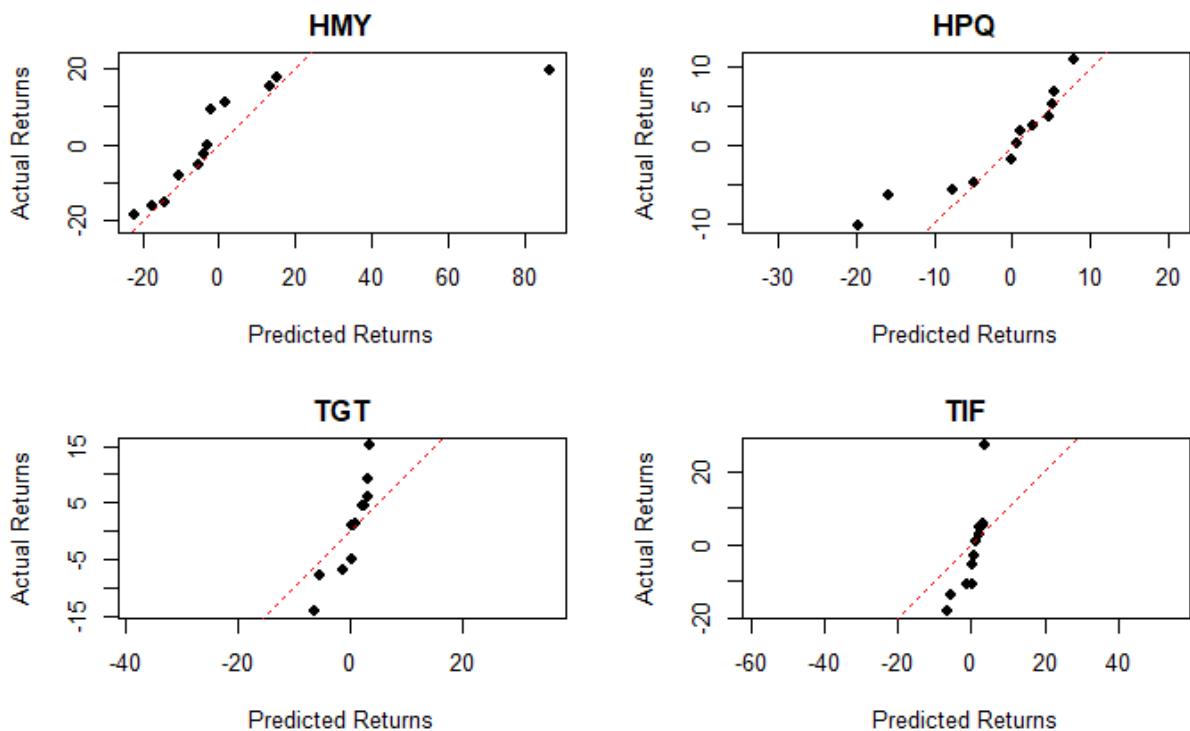


Figure A-96: Actual vs. Predicted Returns for Fama-French 5-Factor GAMs.

Code Appendix

```
library(tidyverse)
library(effects)
library(car)
library(TSA)
library(mgcv)

returns <- read.csv("Returns_2013-2017.csv", header=T)
returns$date <- as.character(returns$date)
returns <- transform(returns, Year=substr(date, 1, 4), Month=substr(date, 5, 6))
returns$Year <- as.numeric(as.character(returns$Year))
returns$Month <- as.numeric(as.character(returns$Month))
returns$new_date <- returns$Year + (returns$Month-1)/12
returns <- returns[order(returns$new_date),]
returns <- as.tibble(returns)
returns_long <- returns %>% gather(key=stock, value=return, HPQ:HMY)
cols <- c("TIF" = "aquamarine", "TGT" = "red", "HMY" = "black", "HPQ" =
"blue")
cols2 <- rbind(rep("aquamarine", 60), rep("red", 60), rep("blue", 60),
rep("black", 60))

ggplot(data=returns_long, aes(x=new_date, y=return)) +
  geom_line(aes(color=stock)) +
  scale_color_manual(values=cols) +
  xlab("Time") +
  ylab("Returns (%)")

pairs(returns_long[,c(2:4,7:8)], upper.panel=panel.cor)
plot(returns_long$return~returns_long$Mkt.RF,
      xlab="Expected Market Risk Premium", ylab="Returns (%)", col=cols,
      main="(a)", pch=16)
plot(returns_long$return~returns_long$SMB,
      xlab="Small Minus Big", ylab="Returns (%)", col=cols, main="(b)",
      pch=16)
plot(returns_long$return~returns_long$HML,
      xlab="High Minus Low", ylab="Returns (%)", col=cols, main="(c)",
      pch=16)
plot(returns_long$return~returns_long$RMW,
      xlab="Robust Minus Weak", ylab="Returns (%)", col=cols, main="(d)",
      pch=16)
plot(returns_long$return~returns_long$CMA,
      xlab="Conservative Minus Aggressive", ylab="Returns (%)", col=cols,
      main="(e)", pch=16)

HMY_MO <- lm(data=returns, HMY~1)
HPQ_MO <- lm(data=returns, HPQ~1)
```

```

TGT_MO <- lm(data=returns, TGT~1)
TIF_MO <- lm(data=returns, TIF~1)

CAPM_HMY <- lm(data=returns, HMY~Mkt.RF)
CAPM_HPQ <- lm(data=returns, HPQ~Mkt.RF)
CAPM_TGT <- lm(data=returns, TGT~Mkt.RF)
CAPM_TIF <- lm(data=returns, TIF~Mkt.RF)

CAPM_HMY_sum <- summary(CAPM_HMY)
CAPM_HPQ_sum <- summary(CAPM_HPQ)
CAPM_TGT_sum <- summary(CAPM_TGT)
CAPM_TIF_sum <- summary(CAPM_TIF)

plot(allEffects(CAPM_HMY, residuals=T))
plot(allEffects(CAPM_HPQ, residuals=T))
plot(allEffects(CAPM_TGT, residuals=T))
plot(allEffects(CAPM_TIF, residuals=T))

plot(CAPM_HMY)
plot(CAPM_HPQ)
plot(CAPM_TGT)
plot(CAPM_TIF)

FF3_HMY <- lm(data=returns, HMY~Mkt.RF + SMB + HML)
FF3_HPQ <- lm(data=returns, HPQ~Mkt.RF + SMB + HML)
FF3_TGT <- lm(data=returns, TGT~Mkt.RF + SMB + HML)
FF3_TIF <- lm(data=returns, TIF~Mkt.RF + SMB + HML)

plot(allEffects(FF3_HMY, residuals=T))
plot(allEffects(FF3_HPQ, residuals=T))
plot(allEffects(FF3_TGT, residuals=T))
plot(allEffects(FF3_TIF, residuals=T))

FF3_HMY_sum <- summary(FF3_HMY)
FF3_HPQ_sum <- summary(FF3_HPQ)
FF3_TGT_sum <- summary(FF3_TGT)
FF3_TIF_sum <- summary(FF3_TIF)

plot(FF3_HMY)
plot(FF3_HPQ)
plot(FF3_TGT)
plot(FF3_TIF)

vif(FF3_HMY)
vif(FF3_HPQ)
vif(FF3_TGT)

```

```

vif(FF3_TIF)

FF5_HMY <- lm(data=returns, HMY~Mkt.RF + SMB + HML + RMW + CMA)
FF5_HPQ <- lm(data=returns, HPQ~Mkt.RF + SMB + HML + RMW + CMA)
FF5_TGT <- lm(data=returns, TGT~Mkt.RF + SMB + HML + RMW + CMA)
FF5_TIF <- lm(data=returns, TIF~Mkt.RF + SMB + HML + RMW + CMA)

plot(allEffects(FF5_HMY, residuals=T))
plot(allEffects(FF5_HPQ, residuals=T))
plot(allEffects(FF5_TGT, residuals=T))
plot(allEffects(FF5_TIF, residuals=T))

FF5_HMY_sum <- summary(FF5_HMY)
FF5_HPQ_sum <- summary(FF5_HPQ)
FF5_TGT_sum <- summary(FF5_TGT)
FF5_TIF_sum <- summary(FF5_TIF)

plot(FF5_HMY)
plot(FF5_HPQ)
plot(FF5_TGT)
plot(FF5_TIF)

vif(FF5_HMY)
vif(FF5_HPQ)
vif(FF5_TGT)
vif(FF5_TIF)

HMY_ts <- ts(returns$HMY, start=c(2013,1), frequency=12)
HPQ_ts <- ts(returns$HPQ, start=c(2013,1), frequency=12)
TGT_ts <- ts(returns$TGT, start=c(2013,1), frequency=12)
TIF_ts <- ts(returns$TIF, start=c(2013,1), frequency=12)

acf(HMY_ts)
pacf(HMY_ts)

acf(HPQ_ts)
pacf(HPQ_ts)

acf(TGT_ts)
pacf(TGT_ts)

acf(TIF_ts)
pacf(TIF_ts)

plot(armasubsets(y=HMY_ts, nar=12, nma=12, y.name='test', ar.method='ols'))
plot(armasubsets(y=HPQ_ts, nar=12, nma=12, y.name='test', ar.method='ols'))

```

```

plot(armasubsets(y=TGT_ts, nar=12, nma=12, y.name='test', ar.method='ols'))
plot(armasubsets(y=TIF_ts, nar=12, nma=12, y.name='test', ar.method='ols'))

HPQ_ts_model <- arima(HMY_ts, order=c(0,0,12), fixed=c(rep(0,11),NA,NA))
HMY_ts_model <- arima(HPQ_ts, order=c(0,0,12), fixed=c(rep(0,11),NA,NA))
TGT_ts_model <- arima(TGT_ts, order=c(6,0,0), fixed=c(rep(0,5),NA,NA))
TIF_ts_model <- arima(TIF_ts, order=c(12,0,12),
                      fixed=c(0,NA,rep(0,9),NA,rep(0,9),NA,0,0,NA))

HMY_ts_CAPM <- ts(CAPM_HMY$residuals, start=c(2013,1), frequency=12)
HPQ_ts_CAPM <- ts(CAPM_HPQ$residuals, start=c(2013,1), frequency=12)
TGT_ts_CAPM <- ts(CAPM_TGT$residuals, start=c(2013,1), frequency=12)
TIF_ts_CAPM <- ts(CAPM_TIF$residuals, start=c(2013,1), frequency=12)

acf(HMY_ts_CAPM)
pacf(HMY_ts_CAPM)

acf(HPQ_ts_CAPM)
pacf(HPQ_ts_CAPM)

acf(TGT_ts_CAPM)
pacf(TGT_ts_CAPM)

acf(TIF_ts_CAPM)
pacf(TIF_ts_CAPM)

plot(armasubsets(y=HMY_ts_CAPM, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=HPQ_ts_CAPM, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=TGT_ts_CAPM, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=TIF_ts_CAPM, nar=12, nma=12, y.name='test',
                  ar.method='ols'))

HPQ_ts_model_CAPM <- arima(HMY_ts, order=c(1,0,0), xreg=returns$Mkt.RF)
HMY_ts_model_CAPM <- arima(HPQ_ts, order=c(0,0,9), xreg=returns$Mkt.RF,
                            fixed=c(rep(0,8), NA, NA, NA))
TGT_ts_model_CAPM <- arima(TGT_ts, order=c(0,0,11), xreg=returns$Mkt.RF,
                            fixed=c(rep(0,10), NA, NA, NA))
TIF_ts_model_CAPM <- arima(TIF_ts, order=c(2,0,0), xreg=returns$Mkt.RF,
                            fixed=c(0, NA, NA, NA))

HMY_ts_FF3 <- ts(FF3_HMY$residuals, start=c(2013,1), frequency=12)
HPQ_ts_FF3 <- ts(FF3_HPQ$residuals, start=c(2013,1), frequency=12)
TGT_ts_FF3 <- ts(FF3_TGT$residuals, start=c(2013,1), frequency=12)

```

```

TIF_ts_FF3 <- ts(FF3_TIF$residuals, start=c(2013,1), frequency=12)

acf(HMY_ts_FF3)
pacf(HMY_ts_FF3)

acf(HPQ_ts_FF3)
pacf(HPQ_ts_FF3)

acf(TGT_ts_FF3)
pacf(TGT_ts_FF3)

acf(TIF_ts_FF3)
pacf(TIF_ts_FF3)

plot(armasubsets(y=HMY_ts_FF3, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=HPQ_ts_FF3, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=TGT_ts_FF3, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=TIF_ts_FF3, nar=12, nma=12, y.name='test',
                  ar.method='ols'))

HPQ_ts_model_FF3 <- arima(HPQ_ts, order=c(1,0,0), xreg=cbind(returns$Mkt.RF,
                                                               returns$SMB, returns$HML))
HMY_ts_model_FF3 <- arima(HMY_ts, order=c(2,0,0), xreg=cbind(returns$Mkt.RF,
                                                               returns$SMB, returns$HML), fixed=c(0,NA,NA,NA,NA,NA))
TGT_ts_model_FF3 <- arima(TGT_ts, order=c(11,0,0), xreg=cbind(returns$Mkt.RF,
                                                               returns$SMB, returns$HML), fixed=c(rep(0,10),NA,NA,NA,NA,NA))
TIF_ts_model_FF3 <- arima(TIF_ts, order=c(0,0,12), xreg=cbind(returns$Mkt.RF,
                                                               returns$SMB, returns$HML), fixed=c(rep(0,11),NA,NA,NA,NA,NA))

HMY_ts_FF5 <- ts(FF5_HMY$residuals, start=c(2013,1), frequency=12)
HPQ_ts_FF5 <- ts(FF5_HPQ$residuals, start=c(2013,1), frequency=12)
TGT_ts_FF5 <- ts(FF5_TGT$residuals, start=c(2013,1), frequency=12)
TIF_ts_FF5 <- ts(FF5_TIF$residuals, start=c(2013,1), frequency=12)

acf(HMY_ts_FF5)
pacf(HMY_ts_FF5)

acf(HPQ_ts_FF5)
pacf(HPQ_ts_FF5)

acf(TGT_ts_FF5)
pacf(TGT_ts_FF5)

```

```

acf(TIF_ts_FF5)
pacf(TIF_ts_FF5)

plot(armasubsets(y=HMY_ts_FF5, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=HPQ_ts_FF5, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=TGT_ts_FF5, nar=12, nma=12, y.name='test',
                  ar.method='ols'))
plot(armasubsets(y=TIF_ts_FF5, nar=12, nma=12, y.name='test',
                  ar.method='ols'))

HPQ_ts_model_FF5 <- arima(HMY_ts, order=c(1,0,0), xreg=cbind(returns$Mkt.RF,
                     returns$SMB, returns$HML, returns$RMW, returns$CMA))
HMY_ts_model_FF5 <- arima(HPQ_ts, order=c(0,0,9), xreg=cbind(returns$Mkt.RF,
                     returns$SMB, returns$HML, returns$RMW, returns$CMA),
                     fixed=c(rep(0,8),NA,NA,NA,NA,NA,NA,NA))
TGT_ts_model_FF5 <- arima(TGT_ts, order=c(0,0,11), xreg=cbind(returns$Mkt.RF,
                     returns$SMB, returns$HML, returns$RMW, returns$CMA),
                     fixed=c(rep(0,10),NA,NA,NA,NA,NA,NA,NA,NA))
TIF_ts_model_FF5 <- arima(TIF_ts, order=c(6,0,0), xreg=cbind(returns$Mkt.RF,
                     returns$SMB, returns$HML, returns$RMW, returns$CMA),
                     fixed=c(rep(0,5),NA,NA,NA,NA,NA,NA,NA))
returns_long$Year <- as.factor(returns_long$Year)
ggplot(data=returns_long, aes(x=Month, y=return, colour=Year)) +
  geom_line() +
  facet_grid(.~stock)

HMY_gam_CAPM <- gam(HMY ~ s(Mkt.RF, bs='ts'), data=returns)
HPQ_gam_CAPM <- gam(HPQ ~ s(Mkt.RF, bs='ts'), data=returns)
TGT_gam_CAPM <- gam(TGT ~ s(Mkt.RF, bs='ts'), data=returns)
TIF_gam_CAPM <- gam(TIF ~ s(Mkt.RF, bs='ts'), data=returns)

plot(HMY_gam_CAPM, main="HMY")
plot(HPQ_gam_CAPM, main="HPQ")
plot(TGT_gam_CAPM, main="TGT")
plot(TIF_gam_CAPM, main="TIF")

gam.check(HMY_gam_CAPM)
gam.check(HPQ_gam_CAPM)
gam.check(TGT_gam_CAPM)
gam.check(TIF_gam_CAPM)

HMY_gam_FF3 <- gam(HMY ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
                     s(HML, bs='ts'), data=returns)
HPQ_gam_FF3 <- gam(HPQ ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +

```

```

s(HML, bs='ts'), data=returns)
TGT_gam_FF3 <- gam(TGT ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
s(HML, bs='ts'), data=returns)
TIF_gam_FF3 <- gam(TIF ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
s(HML, bs='ts'), data=returns)

plot(HMY_gam_FF3)
plot(HPQ_gam_FF3)
plot(TGT_gam_FF3)
plot(TIF_gam_FF3)

gam.check(HMY_gam_FF3)
gam.check(HPQ_gam_FF3)
gam.check(TGT_gam_FF3)
gam.check(TIF_gam_FF3)

HMY_gam_FF5 <- gam(HMY ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
s(HML, bs='ts') + s(RMW, bs='ts') +
s(CMA, bs='ts'), data=returns)
HPQ_gam_FF5 <- gam(HPQ ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
s(HML, bs='ts') + s(RMW, bs='ts') +
s(CMA, bs='ts'), data=returns)
TGT_gam_FF5 <- gam(TGT ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
s(HML, bs='ts') + s(RMW, bs='ts') +
s(CMA, bs='ts'), data=returns)
TIF_gam_FF5 <- gam(TIF ~ s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
s(HML, bs='ts') + s(RMW, bs='ts') +
s(CMA, bs='ts'), data=returns)

plot(HMY_gam_FF5)
plot(HPQ_gam_FF5)
plot(TGT_gam_FF5)
plot(TIF_gam_FF5)

gam.check(HMY_gam_FF5)
gam.check(HPQ_gam_FF5)
gam.check(TGT_gam_FF5)
gam.check(TIF_gam_FF5)

returns$YearFrac <- time(HMY_ts)
HMY_gam_ts <- gam(HMY ~ s(YearFrac, k=5, bs='ts'), data=returns)
HPQ_gam_ts <- gam(HPQ ~ s(YearFrac, k=5, bs='ts'), data=returns)
TGT_gam_ts <- gam(TGT ~ s(YearFrac, k=5, bs='ts'), data=returns)
TIF_gam_ts <- gam(TIF ~ s(YearFrac, k=5, bs='ts'), data=returns)

plot(HMY_gam_ts, main="HMY")

```

```

plot(HPQ_gam_ts, main="HPQ")
plot(TGT_gam_ts, main="TGT")
plot(TIF_gam_ts, main="TIF")

gam.check(HMY_gam_ts)
gam.check(HPQ_gam_ts)
gam.check(TGT_gam_ts)
gam.check(TIF_gam_ts)

HMY_gam_ts_CAPM <- gam(HMY_ts ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts'), data=returns)
HPQ_gam_ts_CAPM <- gam(HPQ ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts'), data=returns)
TGT_gam_ts_CAPM <- gam(TGT ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts'), data=returns)
TIF_gam_ts_CAPM <- gam(TIF ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts'), data=returns)

plot(HMY_gam_ts_CAPM)
plot(HPQ_gam_ts_CAPM)
plot(TGT_gam_ts_CAPM)
plot(TIF_gam_ts_CAPM)

gam.check(HMY_gam_ts_CAPM)
gam.check(HPQ_gam_ts_CAPM)
gam.check(TGT_gam_ts_CAPM)
gam.check(TIF_gam_ts_CAPM)

HMY_gam_ts_FF3 <- gam(HMY ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts'), data=returns)
HPQ_gam_ts_FF3 <- gam(HPQ ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts'), data=returns)
TGT_gam_ts_FF3 <- gam(TGT ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts'), data=returns)
TIF_gam_ts_FF3 <- gam(TIF ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts'), data=returns)

plot(HMY_gam_ts_FF3)
plot(HPQ_gam_ts_FF3)
plot(TGT_gam_ts_FF3)
plot(TIF_gam_ts_FF3)

```

```

gam.check(HMY_gam_ts_FF3)
gam.check(HPQ_gam_ts_FF3)
gam.check(TGT_gam_ts_FF3)
gam.check(TIF_gam_ts_FF3)

HMY_gam_ts_FF5 <- gam(HMY ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts') + s(RMW, bs='ts') +
  s(CMA, bs='ts'), data=returns)
HPQ_gam_ts_FF5 <- gam(HPQ ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts') + s(RMW, bs='ts') +
  s(CMA, bs='ts'), data=returns)
TGT_gam_ts_FF5 <- gam(TGT ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts') + s(RMW, bs='ts') +
  s(CMA, bs='ts'), data=returns)
TIF_gam_ts_FF5 <- gam(TIF ~ s(YearFrac, k=5, bs='ts') +
  s(Mkt.RF, bs='ts') + s(SMB, bs='ts') +
  s(HML, bs='ts') + s(RMW, bs='ts') +
  s(CMA, bs='ts'), data=returns)

plot(HMY_gam_ts_FF5)
plot(HPQ_gam_ts_FF5)
plot(TGT_gam_ts_FF5)
plot(TIF_gam_ts_FF5)

gam.check(HMY_gam_ts_FF5)
gam.check(HPQ_gam_ts_FF5)
gam.check(TGT_gam_ts_FF5)
gam.check(TIF_gam_ts_FF5)

HMY_MO_pred <- rep(summary(HMY_MO)$coefficients[1], 12)
HPQ_MO_pred <- rep(summary(HPQ_MO)$coefficients[1], 12)
TGT_MO_pred <- rep(summary(TGT_MO)$coefficients[1], 12)
TIF_MO_pred <- rep(summary(TIF_MO)$coefficients[1], 12)

returns_2018 <- read.csv("Returns_2018.csv", header=TRUE)

CAPM_HMY_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*CAPM_HMY_sum$coefficients[2]
CAPM_HPQ_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*CAPM_HPQ_sum$coefficients[2]
CAPM_TGT_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*CAPM_TGT_sum$coefficients[2]
CAPM_TIF_pred <- returns_2018$RF +

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```

returns_2018$Mkt.RF*CAPM_TIF_sum$coefficients[2]

qqplot(CAPM_HMY_pred, returns_2018$HMY, main="HMY", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(CAPM_HPQ_pred, returns_2018$HPQ, main="HPQ", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(CAPM_TGT_pred, returns_2018$TGT, main="TGT", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(CAPM_TIF_pred, returns_2018$TIF, main="TIF", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

FF3_HMY_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*FF3_HMY_sum$coefficients[2] +
  returns_2018$SMB*FF3_HMY_sum$coefficients[3] +
  returns_2018$HML*FF3_HMY_sum$coefficients[4]
FF3_HPQ_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*FF3_HPQ_sum$coefficients[2] +
  returns_2018$SMB*FF3_HPQ_sum$coefficients[3] +
  returns_2018$HML*FF3_HPQ_sum$coefficients[4]
FF3_TGT_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*FF3_TGT_sum$coefficients[2] +
  returns_2018$SMB*FF3_TGT_sum$coefficients[3] +
  returns_2018$HML*FF3_TGT_sum$coefficients[4]
FF3_TIF_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*FF3_TIF_sum$coefficients[2] +
  returns_2018$SMB*FF3_TIF_sum$coefficients[3] +
  returns_2018$HML*FF3_TIF_sum$coefficients[4]

qqplot(FF3_HMY_pred, returns_2018$HMY, main="HMY", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(FF3_HPQ_pred, returns_2018$HPQ, main="HPQ", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(FF3_TGT_pred, returns_2018$TGT, main="TGT", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(FF3_TIF_pred, returns_2018$TIF, main="TIF", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

FF5_HMY_pred <- returns_2018$RF +

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returns_2018$Mkt.RF*FF5_HMY_sum$coefficients[2] +
returns_2018$SMB*FF5_HMY_sum$coefficients[3] +
returns_2018$HML*FF5_HMY_sum$coefficients[4] +
returns_2018$RMW*FF5_HMY_sum$coefficients[5] +
returns_2018$CMA*FF5_HMY_sum$coefficients[6]

FF5_HPQ_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*FF5_HPQ_sum$coefficients[2] +
  returns_2018$SMB*FF5_HPQ_sum$coefficients[3] +
  returns_2018$HML*FF5_HPQ_sum$coefficients[4] +
  returns_2018$RMW*FF5_HPQ_sum$coefficients[5] +
  returns_2018$CMA*FF5_HPQ_sum$coefficients[6]

FF5_TGT_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*FF5_TGT_sum$coefficients[2] +
  returns_2018$SMB*FF5_TGT_sum$coefficients[3] +
  returns_2018$HML*FF5_TGT_sum$coefficients[4] +
  returns_2018$RMW*FF5_TGT_sum$coefficients[5] +
  returns_2018$CMA*FF5_TGT_sum$coefficients[6]

FF5_TIF_pred <- returns_2018$RF +
  returns_2018$Mkt.RF*FF5_TIF_sum$coefficients[2] +
  returns_2018$SMB*FF5_TIF_sum$coefficients[3] +
  returns_2018$HML*FF5_TIF_sum$coefficients[4] +
  returns_2018$RMW*FF5_TIF_sum$coefficients[5] +
  returns_2018$CMA*FF5_TIF_sum$coefficients[6]

qqplot(FF5_HMY_pred, returns_2018$HMY, main="HMY", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(FF5_HPQ_pred, returns_2018$HPQ, main="HPQ", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(FF5_TGT_pred, returns_2018$TGT, main="TGT", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(FF5_TIF_pred, returns_2018$TIF, main="TIF", xlab="Predicted Returns",
       ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_ts_pred <- predict(HMY_ts_model, n.ahead=12)
HPQ_ts_pred <- predict(HPQ_ts_model, n.ahead=12)
TGT_ts_pred <- predict(TGT_ts_model, n.ahead=12)
TIF_ts_pred <- predict(TIF_ts_model, n.ahead=12)

qqplot(unlist(HMY_ts_pred$pred), returns_2018$HMY, main="HMY",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(HPQ_ts_pred$pred), returns_2018$HPQ, main="HPQ",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)

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abline(0,1, col="red", lty=3)
qqplot(unlist(TGT_ts_pred$pred), returns_2018$TGT, main="TGT",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(TIF_ts_pred$pred), returns_2018$TIF, main="TIF",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_ts_pred_CAPM <- predict(HMY_ts_model_CAPM, n.ahead=12,
                             newxreg=returns_2018$Mkt.RF)
HPQ_ts_pred_CAPM <- predict(HPQ_ts_model_CAPM, n.ahead=12,
                             newxreg=returns_2018$Mkt.RF)
TGT_ts_pred_CAPM <- predict(TGT_ts_model_CAPM, n.ahead=12,
                             newxreg=returns_2018$Mkt.RF)
TIF_ts_pred_CAPM <- predict(TIF_ts_model_CAPM, n.ahead=12,
                             newxreg=returns_2018$Mkt.RF)

qqplot(unlist(HMY_ts_pred_CAPM$pred), returns_2018$HMY, main="HMY",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(HPQ_ts_pred_CAPM$pred), returns_2018$HPQ, main="HPQ",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(TGT_ts_pred_CAPM$pred), returns_2018$TGT, main="TGT",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(TIF_ts_pred_CAPM$pred), returns_2018$TIF, main="TIF",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_ts_pred_FF3 <- predict(HMY_ts_model_FF3, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML))
HPQ_ts_pred_FF3 <- predict(HPQ_ts_model_FF3, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML))
TGT_ts_pred_FF3 <- predict(TGT_ts_model_FF3, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML))
TIF_ts_pred_FF3 <- predict(TIF_ts_model_FF3, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML))

qqplot(unlist(HMY_ts_pred_FF3$pred), returns_2018$HMY, main="HMY",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(HPQ_ts_pred_FF3$pred), returns_2018$HPQ, main="HPQ",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

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qqplot(unlist(TGT_ts_pred_FF3$pred), returns_2018$TGT, main="TGT",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(TIF_ts_pred_FF3$pred), returns_2018$TIF, main="TIF",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_ts_pred_FF5 <- predict(HMY_ts_model_FF5, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML,
                            returns_2018$RMW, returns_2018$CMA))
HPQ_ts_pred_FF5 <- predict(HPQ_ts_model_FF5, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML,
                            returns_2018$RMW, returns_2018$CMA))
TGT_ts_pred_FF5 <- predict(TGT_ts_model_FF5, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML,
                            returns_2018$RMW, returns_2018$CMA))
TIF_ts_pred_FF5 <- predict(TIF_ts_model_FF5, n.ahead=12,
                            newxreg=cbind(returns_2018$Mkt.RF, returns_2018$SMB, returns_2018$HML,
                            returns_2018$RMW, returns_2018$CMA))

qqplot(unlist(HMY_ts_pred_FF5$pred), returns_2018$HMY, main="HMY",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(HPQ_ts_pred_FF5$pred), returns_2018$HPQ, main="HPQ",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(TGT_ts_pred_FF5$pred), returns_2018$TGT, main="TGT",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(unlist(TIF_ts_pred_FF5$pred), returns_2018$TIF, main="TIF",
       xlab="Predicted Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_CAPM_pred_GAM <- predict(HMY_gam_CAPM, newdata=returns_2018)
HPQ_CAPM_pred_GAM <- predict(HPQ_gam_CAPM, newdata=returns_2018)
TGT_CAPM_pred_GAM <- predict(TGT_gam_CAPM, newdata=returns_2018)
TIF_CAPM_pred_GAM <- predict(TIF_gam_CAPM, newdata=returns_2018)

qqplot(HMY_CAPM_pred_GAM, returns_2018$HMY, main="HMY", xlab="Predicted
    Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(HPQ_CAPM_pred_GAM, returns_2018$HPQ, main="HPQ", xlab="Predicted
    Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TGT_CAPM_pred_GAM, returns_2018$TGT, main="TGT", xlab="Predicted
    Returns", ylab="Actual Returns", asp=1, pch=16)

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abline(0,1, col="red", lty=3)
qqplot(TIF_CAPM_pred_GAM, returns_2018$TIF, main="TIF", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_FF3_pred_GAM <- predict(HMY_gam_FF3, newdata=returns_2018)
HPQ_FF3_pred_GAM <- predict(HPQ_gam_FF3, newdata=returns_2018)
TGT_FF3_pred_GAM <- predict(TGT_gam_FF3, newdata=returns_2018)
TIF_FF3_pred_GAM <- predict(TIF_gam_FF3, newdata=returns_2018)

qqplot(HMY_FF3_pred_GAM, returns_2018$HMY, main="HMY", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(HPQ_FF3_pred_GAM, returns_2018$HPQ, main="HPQ", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TGT_FF3_pred_GAM, returns_2018$TGT, main="TGT", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TIF_FF3_pred_GAM, returns_2018$TIF, main="TIF", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_FF5_pred_GAM <- predict(HMY_gam_FF5, newdata=returns_2018)
HPQ_FF5_pred_GAM <- predict(HPQ_gam_FF5, newdata=returns_2018)
TGT_FF5_pred_GAM <- predict(TGT_gam_FF5, newdata=returns_2018)
TIF_FF5_pred_GAM <- predict(TIF_gam_FF5, newdata=returns_2018)

qqplot(HMY_FF5_pred_GAM, returns_2018$HMY, main="HMY", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(HPQ_FF5_pred_GAM, returns_2018$HPQ, main="HPQ", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TGT_FF5_pred_GAM, returns_2018$TGT, main="TGT", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TIF_FF5_pred_GAM, returns_2018$TIF, main="TIF", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

returns_2018$YearFrac <- time(ts(returns_2018$RF, start=c(2018,1),
frequency=12))

HMY_t_pred_GAM <- predict(HMY_gam_ts, newdata=returns_2018)
HPQ_t_pred_GAM <- predict(HPQ_gam_ts, newdata=returns_2018)

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TGT_t_pred_GAM <- predict(TGT_gam_ts, newdata=returns_2018)
TIF_t_pred_GAM <- predict(TGT_gam_ts, newdata=returns_2018)

qqplot(HMY_t_pred_GAM, returns_2018$HMY, main="HMY", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(HPQ_t_pred_GAM, returns_2018$HPQ, main="HPQ", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TGT_t_pred_GAM, returns_2018$TGT, main="TGT", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TIF_t_pred_GAM, returns_2018$TIF, main="TIF", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_CAPMt_pred_GAM <- predict(HMY_gam_ts_CAPM, newdata=returns_2018)
HPQ_CAPMt_pred_GAM <- predict(HPQ_gam_ts_CAPM, newdata=returns_2018)
TGT_CAPMt_pred_GAM <- predict(TGT_gam_ts_CAPM, newdata=returns_2018)
TIF_CAPMt_pred_GAM <- predict(TGT_gam_ts_CAPM, newdata=returns_2018)

qqplot(HMY_CAPMt_pred_GAM, returns_2018$HMY, main="HMY", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(HPQ_CAPMt_pred_GAM, returns_2018$HPQ, main="HPQ", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TGT_CAPMt_pred_GAM, returns_2018$TGT, main="TGT", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TIF_CAPMt_pred_GAM, returns_2018$TIF, main="TIF", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_FF3t_pred_GAM <- predict(HMY_gam_ts_FF3, newdata=returns_2018)
HPQ_FF3t_pred_GAM <- predict(HPQ_gam_ts_FF3, newdata=returns_2018)
TGT_FF3t_pred_GAM <- predict(TGT_gam_ts_FF3, newdata=returns_2018)
TIF_FF3t_pred_GAM <- predict(TGT_gam_ts_FF3, newdata=returns_2018)

qqplot(HMY_FF3t_pred_GAM, returns_2018$HMY, main="HMY", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(HPQ_FF3t_pred_GAM, returns_2018$HPQ, main="HPQ", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

```

```

qqplot(TGT_FF3t_pred_GAM, returns_2018$TGT, main="TGT", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TIF_FF3t_pred_GAM, returns_2018$TIF, main="TIF", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)

HMY_FF5t_pred_GAM <- predict(HMY_gam_ts_FF5, newdata=returns_2018)
HPQ_FF5t_pred_GAM <- predict(HPQ_gam_ts_FF5, newdata=returns_2018)
TGT_FF5t_pred_GAM <- predict(TGT_gam_ts_FF5, newdata=returns_2018)
TIF_FF5t_pred_GAM <- predict(TGT_gam_ts_FF5, newdata=returns_2018)

qqplot(HMY_FF5t_pred_GAM, returns_2018$HMY, main="HMY", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(HPQ_FF5t_pred_GAM, returns_2018$HPQ, main="HPQ", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TGT_FF5t_pred_GAM, returns_2018$TGT, main="TGT", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
qqplot(TIF_FF5t_pred_GAM, returns_2018$TIF, main="TIF", xlab="Predicted
      Returns", ylab="Actual Returns", asp=1, pch=16)
abline(0,1, col="red", lty=3)
plot(returns_2018$HMY~returns_2018>Date, main="HMY", xlab="Time",
      ylab="Returns", type="l", lty=1, ylim=c(-30,30), col="red", lwd=2,
      xaxt='n')
axis(1, at=c(201801, 201802, 201803, 201804, 201805, 201806, 201807, 201808,
            201809, 201810, 201811, 201812), labels=c("Jan", "Feb", "Mar", "Apr",
            "May", "Jun", "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"))
lines(HMY_MO_pred~returns_2018>Date, col="grey")
lines(CAPM_HMY_pred~returns_2018>Date, col="grey")
lines(FF3_HMY_pred~returns_2018>Date, col="purple", lwd=2)
lines(FF5_HMY_pred~returns_2018>Date, col="grey")
lines(unlist(HMY_ts_pred$pred)~returns_2018>Date, col="grey")
lines(unlist(HMY_ts_pred_CAPM$pred)~returns_2018>Date, col="grey")
lines(unlist(HMY_ts_pred_FF3$pred)~returns_2018>Date, col="grey")
lines(unlist(HMY_ts_pred_FF5$pred)~returns_2018>Date, col="grey")
lines(HMY_CAPM_pred_GAM~returns_2018>Date, col="grey")
lines(HMY_FF3_pred_GAM~returns_2018>Date, col="grey")
lines(HMY_FF5_pred_GAM~returns_2018>Date, col="grey")
lines(HMY_t_pred_GAM~returns_2018>Date, col="grey")
lines(HMY_CAPMt_pred_GAM~returns_2018>Date, col="grey")
lines(HMY_FF3t_pred_GAM~returns_2018>Date, col="grey")
lines(HMY_FF5t_pred_GAM~returns_2018>Date, col="grey")

```

```

plot(returns_2018$HPQ~returns_2018$date, main="HPQ", xlab="Time",
      ylab="Returns", type="l", lty=1, ylim=c(-20,20), col="red", lwd=2,
      xaxt='n')
axis(1, at=c(201801, 201802, 201803, 201804, 201805, 201806, 201807, 201808,
           201809, 201810, 201811, 201812), labels=c("Jan", "Feb", "Mar", "Apr",
           "May", "Jun", "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"))
lines(HPQ_MO_pred~returns_2018$date, col="grey")
lines(CAPM_HPQ_pred~returns_2018$date, col="grey")
lines(FF3_HPQ_pred~returns_2018$date, col="purple", lwd=2)
lines(FF5_HPQ_pred~returns_2018$date, col="grey")
lines(unlist(HPQ_ts_pred$pred)~returns_2018$date, col="grey")
lines(unlist(HPQ_ts_pred_CAPM$pred)~returns_2018$date, col="grey")
lines(unlist(HPQ_ts_pred_FF3$pred)~returns_2018$date, col="grey")
lines(unlist(HPQ_ts_pred_FF5$pred)~returns_2018$date, col="grey")
lines(HPQ_CAPM_pred_GAM~returns_2018$date, col="grey")
lines(HPQ_FF3_pred_GAM~returns_2018$date, col="grey")
lines(HPQ_FF5_pred_GAM~returns_2018$date, col="grey")
lines(HPQ_t_pred_GAM~returns_2018$date, col="grey")
lines(HPQ_CAPMt_pred_GAM~returns_2018$date, col="grey")
lines(HPQ_FF3t_pred_GAM~returns_2018$date, col="grey")
lines(HPQ_FF5t_pred_GAM~returns_2018$date, col="grey")

plot(returns_2018$TGT~returns_2018$date, main="TGT", xlab="Time",
      ylab="Returns", type="l", lty=1, ylim=c(-20,20), col="red", lwd=2,
      xaxt='n')
axis(1, at=c(201801, 201802, 201803, 201804, 201805, 201806, 201807, 201808,
           201809, 201810, 201811, 201812), labels=c("Jan", "Feb", "Mar", "Apr",
           "May", "Jun", "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"))
lines(TGT_MO_pred~returns_2018$date, col="grey")
lines(CAPM_TGT_pred~returns_2018$date, col="grey")
lines(FF3_TGT_pred~returns_2018$date, col="grey")
lines(FF5_TGT_pred~returns_2018$date, col="grey")
lines(unlist(TGT_ts_pred$pred)~returns_2018$date, col="grey")
lines(unlist(TGT_ts_pred_CAPM$pred)~returns_2018$date, col="purple", lwd=2)
lines(unlist(TGT_ts_pred_FF3$pred)~returns_2018$date, col="grey")
lines(unlist(TGT_ts_pred_FF5$pred)~returns_2018$date, col="grey")
lines(TGT_CAPM_pred_GAM~returns_2018$date, col="grey")
lines(TGT_FF3_pred_GAM~returns_2018$date, col="grey")
lines(TGT_FF5_pred_GAM~returns_2018$date, col="grey")
lines(TGT_t_pred_GAM~returns_2018$date, col="grey")
lines(TGT_CAPMt_pred_GAM~returns_2018$date, col="grey")
lines(TGT_FF3t_pred_GAM~returns_2018$date, col="grey")
lines(TGT_FF5t_pred_GAM~returns_2018$date, col="grey")

```

```

plot(returns_2018$TIF~returns_2018>Date, main="TIF", xlab="Time",
      ylab="Returns", type="l", lty=1, ylim=c(-30,30), col="red", lwd=2,
      xaxt='n')
axis(1, at=c(201801, 201802, 201803, 201804, 201805, 201806, 201807, 201808,
           201809, 201810, 201811, 201812), labels=c("Jan", "Feb", "Mar", "Apr",
           "May", "Jun", "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"))
lines(TIF_MO_pred~returns_2018>Date, col="grey")
lines(CAPM_TIF_pred~returns_2018>Date, col="grey")
lines(FF3_TIF_pred~returns_2018>Date, col="grey")
lines(FF5_TIF_pred~returns_2018>Date, col="grey")
lines(unlist(TIF_ts_pred$pred)~returns_2018>Date, col="grey")
lines(unlist(TIF_ts_pred_CAPM$pred)~returns_2018>Date, col="grey")
lines(unlist(TIF_ts_pred_FF3$pred)~returns_2018>Date, col="grey")
lines(unlist(TIF_ts_pred_FF5$pred)~returns_2018>Date, col="grey")
lines(TIF_CAPM_pred_GAM~returns_2018>Date, col="grey")
lines(TIF_FF3_pred_GAM~returns_2018>Date, col="grey")
lines(TIF_FF5_pred_GAM~returns_2018>Date, col="purple", lwd=2)
lines(TIF_t_pred_GAM~returns_2018>Date, col="grey")
lines(TIF_CAPMt_pred_GAM~returns_2018>Date, col="grey")
lines(TIF_FF3t_pred_GAM~returns_2018>Date, col="grey")
lines(TIF_FF5t_pred_GAM~returns_2018>Date, col="grey")

MAD_calc <- function(data1, data2){
  absdiff <- abs(data1-data2)
  mean(absdiff)
}

HMY_MAD <- cbind(MAD_calc(returns_2018$HMY, HMY_MO_pred),
                  MAD_calc(returns_2018$HMY, CAPM_HMY_pred),
                  MAD_calc(returns_2018$HMY, FF3_HMY_pred),
                  MAD_calc(returns_2018$HMY, FF5_HMY_pred),
                  MAD_calc(returns_2018$HMY, unlist(HMY_ts_pred$pred)),
                  MAD_calc(returns_2018$HMY, unlist(HMY_ts_pred_CAPM$pred)),
                  MAD_calc(returns_2018$HMY, unlist(HMY_ts_pred_FF3$pred)),
                  MAD_calc(returns_2018$HMY, unlist(HMY_ts_pred_FF5$pred)),
                  MAD_calc(returns_2018$HMY, HMY_CAPM_pred_GAM),
                  MAD_calc(returns_2018$HMY, HMY_FF3_pred_GAM),
                  MAD_calc(returns_2018$HMY, HMY_FF5_pred_GAM),
                  MAD_calc(returns_2018$HMY, HMY_t_pred_GAM),
                  MAD_calc(returns_2018$HMY, HMY_CAPMt_pred_GAM),
                  MAD_calc(returns_2018$HMY, HMY_FF3t_pred_GAM),
                  MAD_calc(returns_2018$HMY, HMY_FF5t_pred_GAM))

HPQ_MAD <- cbind(MAD_calc(returns_2018$HPQ, HPQ_MO_pred),
                  MAD_calc(returns_2018$HPQ, CAPM_HPQ_pred),
                  MAD_calc(returns_2018$HPQ, FF3_HPQ_pred),

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MAD_calc(returns_2018$HPQ, FF5_HPO_pred),
MAD_calc(returns_2018$HPQ, unlist(HPO_ts_pred$pred)),
MAD_calc(returns_2018$HPQ, unlist(HPO_ts_pred_CAPM$pred)),
MAD_calc(returns_2018$HPQ, unlist(HPO_ts_pred_FF3$pred)),
MAD_calc(returns_2018$HPQ, unlist(HPO_ts_pred_FF5$pred)),
MAD_calc(returns_2018$HPQ, HPQ_CAPM_pred_GAM),
MAD_calc(returns_2018$HPQ, HPQ_FF3_pred_GAM),
MAD_calc(returns_2018$HPQ, HPQ_FF5_pred_GAM),
MAD_calc(returns_2018$HPQ, HPQ_t_pred_GAM),
MAD_calc(returns_2018$HPQ, HPQ_CAPMt_pred_GAM),
MAD_calc(returns_2018$HPQ, HPQ_FF3t_pred_GAM),
MAD_calc(returns_2018$HPQ, HPQ_FF5t_pred_GAM)

TGT_MAD <- cbind(MAD_calc(returns_2018$TGT, TGT_MO_pred),
MAD_calc(returns_2018$TGT, CAPM_TGT_pred),
MAD_calc(returns_2018$TGT, FF3_TGT_pred),
MAD_calc(returns_2018$TGT, FF5_TGT_pred),
MAD_calc(returns_2018$TGT, unlist(TGT_ts_pred$pred)),
MAD_calc(returns_2018$TGT, unlist(TGT_ts_pred_CAPM$pred)),
MAD_calc(returns_2018$TGT, unlist(TGT_ts_pred_FF3$pred)),
MAD_calc(returns_2018$TGT, unlist(TGT_ts_pred_FF5$pred)),
MAD_calc(returns_2018$TGT, TGT_CAPM_pred_GAM),
MAD_calc(returns_2018$TGT, TGT_FF3_pred_GAM),
MAD_calc(returns_2018$TGT, TGT_FF5_pred_GAM),
MAD_calc(returns_2018$TGT, TGT_t_pred_GAM),
MAD_calc(returns_2018$TGT, TGT_CAPMt_pred_GAM),
MAD_calc(returns_2018$TGT, TGT_FF3t_pred_GAM),
MAD_calc(returns_2018$TGT, TGT_FF5t_pred_GAM))

TIF_MAD <- cbind(MAD_calc(returns_2018$TIF, TIF_MO_pred),
MAD_calc(returns_2018$TIF, CAPM_TIF_pred),
MAD_calc(returns_2018$TIF, FF3_TIF_pred),
MAD_calc(returns_2018$TIF, FF5_TIF_pred),
MAD_calc(returns_2018$TIF, unlist(TIF_ts_pred$pred)),
MAD_calc(returns_2018$TIF, unlist(TIF_ts_pred_CAPM$pred)),
MAD_calc(returns_2018$TIF, unlist(TIF_ts_pred_FF3$pred)),
MAD_calc(returns_2018$TIF, unlist(TIF_ts_pred_FF5$pred)),
MAD_calc(returns_2018$TIF, TIF_CAPM_pred_GAM),
MAD_calc(returns_2018$TIF, TIF_FF3_pred_GAM),
MAD_calc(returns_2018$TIF, TIF_FF5_pred_GAM),
MAD_calc(returns_2018$TIF, TIF_t_pred_GAM),
MAD_calc(returns_2018$TIF, TIF_CAPMt_pred_GAM),
MAD_calc(returns_2018$TIF, TIF_FF3t_pred_GAM),
MAD_calc(returns_2018$TIF, TIF_FF5t_pred_GAM))

MSE_calc <- function(data1, data2){

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```

sqrdiff <- (data1-data2)^2
mean(sqrdiff)
}

HMY_MSE <- cbind(MSE_calc(returns_2018$HMY, HMY_MO_pred),
  MSE_calc(returns_2018$HMY, CAPM_HMY_pred),
  MSE_calc(returns_2018$HMY, FF3_HMY_pred),
  MSE_calc(returns_2018$HMY, FF5_HMY_pred),
  MSE_calc(returns_2018$HMY, unlist(HMY_ts_pred$pred)),
  MSE_calc(returns_2018$HMY, unlist(HMY_ts_pred_CAPM$pred)),
  MSE_calc(returns_2018$HMY, unlist(HMY_ts_pred_FF3$pred)),
  MSE_calc(returns_2018$HMY, unlist(HMY_ts_pred_FF5$pred)),
  MSE_calc(returns_2018$HMY, HMY_CAPM_pred_GAM),
  MSE_calc(returns_2018$HMY, HMY_FF3_pred_GAM),
  MSE_calc(returns_2018$HMY, HMY_FF5_pred_GAM),
  MSE_calc(returns_2018$HMY, HMY_t_pred_GAM),
  MSE_calc(returns_2018$HMY, HMY_CAPMt_pred_GAM),
  MSE_calc(returns_2018$HMY, HMY_FF3t_pred_GAM),
  MSE_calc(returns_2018$HMY, HMY_FF5t_pred_GAM))

HPQ_MSE <- cbind(MSE_calc(returns_2018$HPQ, HPQ_MO_pred),
  MSE_calc(returns_2018$HPQ, CAPM_HPQ_pred),
  MSE_calc(returns_2018$HPQ, FF3_HPQ_pred),
  MSE_calc(returns_2018$HPQ, FF5_HPQ_pred),
  MSE_calc(returns_2018$HPQ, unlist(HPQ_ts_pred$pred)),
  MSE_calc(returns_2018$HPQ, unlist(HPQ_ts_pred_CAPM$pred)),
  MSE_calc(returns_2018$HPQ, unlist(HPQ_ts_pred_FF3$pred)),
  MSE_calc(returns_2018$HPQ, unlist(HPQ_ts_pred_FF5$pred)),
  MSE_calc(returns_2018$HPQ, HPQ_CAPM_pred_GAM),
  MSE_calc(returns_2018$HPQ, HPQ_FF3_pred_GAM),
  MSE_calc(returns_2018$HPQ, HPQ_FF5_pred_GAM),
  MSE_calc(returns_2018$HPQ, HPQ_t_pred_GAM),
  MSE_calc(returns_2018$HPQ, HPQ_CAPMt_pred_GAM),
  MSE_calc(returns_2018$HPQ, HPQ_FF3t_pred_GAM),
  MSE_calc(returns_2018$HPQ, HPQ_FF5t_pred_GAM))

TGT_MSE <- cbind(MSE_calc(returns_2018$TGT, TGT_MO_pred),
  MSE_calc(returns_2018$TGT, CAPM_TGT_pred),
  MSE_calc(returns_2018$TGT, FF3_TGT_pred),
  MSE_calc(returns_2018$TGT, FF5_TGT_pred),
  MSE_calc(returns_2018$TGT, unlist(TGT_ts_pred$pred)),
  MSE_calc(returns_2018$TGT, unlist(TGT_ts_pred_CAPM$pred)),
  MSE_calc(returns_2018$TGT, unlist(TGT_ts_pred_FF3$pred)),
  MSE_calc(returns_2018$TGT, unlist(TGT_ts_pred_FF5$pred)),
  MSE_calc(returns_2018$TGT, TGT_CAPM_pred_GAM),
  MSE_calc(returns_2018$TGT, TGT_FF3_pred_GAM),

```

```

MSE_calc(returns_2018$TGT, TGT_FF5_pred_GAM),
MSE_calc(returns_2018$TGT, TGT_t_pred_GAM),
MSE_calc(returns_2018$TGT, TGT_CAPMt_pred_GAM),
MSE_calc(returns_2018$TGT, TGT_FF3t_pred_GAM),
MSE_calc(returns_2018$TGT, TGT_FF5t_pred_GAM))

TIF_MSE <- cbind(MSE_calc(returns_2018$TIF, TIF_MO_pred),
MSE_calc(returns_2018$TIF, CAPM_TIF_pred),
MSE_calc(returns_2018$TIF, FF3_TIF_pred),
MSE_calc(returns_2018$TIF, FF5_TIF_pred),
MSE_calc(returns_2018$TIF, unlist(TIF_ts_pred$pred)),
MSE_calc(returns_2018$TIF, unlist(TIF_ts_pred_CAPM$pred)),
MSE_calc(returns_2018$TIF, unlist(TIF_ts_pred_FF3$pred)),
MSE_calc(returns_2018$TIF, unlist(TIF_ts_pred_FF5$pred)),
MSE_calc(returns_2018$TIF, TIF_CAPM_pred_GAM),
MSE_calc(returns_2018$TIF, TIF_FF3_pred_GAM),
MSE_calc(returns_2018$TIF, TIF_FF5_pred_GAM),
MSE_calc(returns_2018$TIF, TIF_t_pred_GAM),
MSE_calc(returns_2018$TIF, TIF_CAPMt_pred_GAM),
MSE_calc(returns_2018$TIF, TIF_FF3t_pred_GAM),
MSE_calc(returns_2018$TIF, TIF_FF5t_pred_GAM))

```